

# Model Solutions

Please write clearly in	block capitals.		
Centre number		Candidate number	
Surname			
Forename(s)			
Candidate signature			

# GCSE MATHEMATICS

Н

Higher Tier

Paper 2 Calculator

Thursday 8 November 2018 Morning Time allowed: 1 hour 30 minutes

### **Materials**

For this paper you must have:

- a calculator
- · mathematical instruments.



#### Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer all questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work you do not want to be marked.

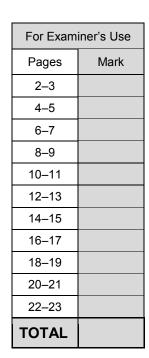
# Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper.
   These must be tagged securely to this answer book.

#### **Advice**

In all calculations, show clearly how you work out your answer.





# Answer all questions in the spaces provided

1 What does  $(A \cap B)$  represent in  $P(A \cap B)$ ? N = intersect = AND Circle your answer.

[1 mark]

A or B or both

A but not B

not A and not B



2 P is (4, 9) and Q is (-2, 1) Circle the midpoint of PQ.

 $midpoint = \left(\frac{4-2}{2}, \frac{9+1}{2}\right)$ 

(3, 4)

(3, 5)

(6, 8)

Which of these is a geometric progression? 3

Circle your answer.

to calculate next term

[1 mark]

1 3 5 7 9

1 3 6 10 15

1 4 9 16 25

x3 x3 x3....

4 The bearing of A from B is 310°

Circle the bearing of B from A.

[1 mark]

050°

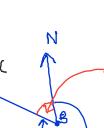
110°



220°

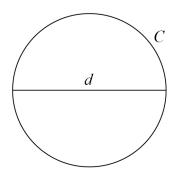
cointerior:

180-50 =



\_ 360 - 310 = 50

5 A circle has circumference C and diameter d.



C = kd

What  ${\bf value}$  does the constant k represent?



[1 mark]

Answer

**6** Here is some information about 20 trains leaving a station.

Number of minutes late, <i>t</i>	Number of trains	Midpoint ≃ ∞	fx	
0 ≤ <i>t</i> < 5	12	2.5	30	25×12
5 ≤ <i>t</i> < 10	7	7.5	52.5	7·5×7
10 ≤ <i>t</i> < 15	1	12.5	12.5	12.5×1
<i>t</i> ≥ 15	0 —		,	
Total	20		95	-

**6** (a) Work out an estimate of the mean number of minutes late.

[3 marks]

Answer 4.75 minutes

**6 (b)** The station manager looks at the information in more detail.

Number of minutes late, <i>t</i>	Number of trains	
0 ≤ <i>t</i> < 2	12	midpoint=1, before 2.5
2 ≤ <i>t</i> < 4	0	•
4 ≤ <i>t</i> < 6	7	— Midpoint=5, before 7.5
6 ≤ <i>t</i> < 8	0	9
8 ≤ <i>t</i> < 10	0	
10 ≤ <i>t</i> < 12	1 -	_ Our midpoint was 12.5, here it is

He works out an estimate of the mean using this information.

How does his estimate compare with the answer to part (a)? Tick **one** box.

[1 mark]

11

Higher than part (a)

Turn over for the next question

\_\_\_\_\_

7 Work out the values of a and b in the identity

$$5(7x + 8) + 3(2x + b) = ax + 13$$

[4 marks]

$$35x + 40 + 6x + 3b = ax + 13$$

$$41x + (40+3b) = ax + 13$$

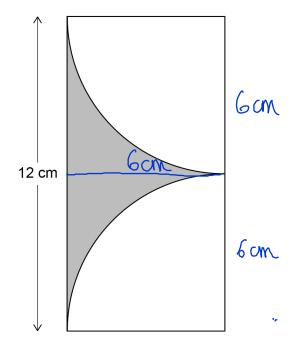
$$ax = 41x$$
  $40 + 3b = 13$ 

$$a = 41$$
  $3b = -27$ 

$$a =$$
  $b =$   $Q$ 



8 Two identical quarter circles are cut from a rectangle as shown.



Not drawn accurately

Work out the shaded area.

[4 marks]

Area of rectangle =  $12 \times 6 = 72 \text{ cm}^2$ Area of Quarter Circle:  $\frac{1}{4} \times 7 \times 6^2 = 97$ of 2: 187

Shaded = 72-18T = 15.45...

Answer

**9** The diagrams show the position of a tap when off and fully on.

The tap is fully on when the angle of turn is 180°



When fully on, water flows out of the tap at 14 litres per minute.

The rate at which water flows out is in direct proportion to the angle of turn.

The tap is turned 135°



The water flows into a tank with a capacity of 79.8 litres.

Will it take **less than**  $7\frac{1}{2}$  minutes to fill the tank?

You **must** show your working.

$$\frac{135}{180} \text{ switched on}$$

$$\frac{135}{180} \times 14 \times 7.5 = 78.75 \text{ full}$$

$$\frac{135}{180} \times 14 \text{ per min}$$

$$\frac{135}{180} \times 14 \times 7.5 = 78.75 \text{ full}$$

$$\frac{135}{180} \times 14 \times 7.5 = 78.75 \text{ full}$$

$$\frac{135}{180} \times 14 \times 7.5 = 78.75 \text{ full}$$

$$\frac{135}{180} \times 14 \times 7.5 = 78.75 \text{ full}$$

$$\frac{135}{180} \times 14 \times 7.5 = 78.75 \text{ full}$$

$$\frac{135}{180} \times 14 \times 7.5 = 78.75 \text{ full}$$

$$\frac{135}{180} \times 14 \times 7.5 = 78.75 \text{ full}$$

$$\frac{135}{180} \times 14 \times 7.5 = 78.75 \text{ full}$$

$$\frac{135}{180} \times 14 \times 7.5 = 78.75 \text{ full}$$

$$\frac{135}{180} \times 14 \times 7.5 = 78.75 \text{ full}$$

$$\frac{135}{180} \times 14 \times 7.5 = 78.75 \text{ full}$$

$$\frac{135}{180} \times 14 \times 7.5 = 78.75 \text{ full}$$

$$\frac{135}{180} \times 14 \times 7.5 = 78.75 \text{ full}$$

$$\frac{135}{180} \times 14 \times 7.5 = 78.75 \text{ full}$$

$$\frac{135}{180} \times 14 \times 7.5 = 78.75 \text{ full}$$

$$\frac{135}{180} \times 14 \times 7.5 = 78.75 \text{ full}$$

$$\frac{135}{180} \times 14 \times 7.5 = 78.75 \text{ full}$$

$$\frac{135}{180} \times 14 \times 7.5 = 78.75 \text{ full}$$

$$\frac{135}{180} \times 14 \times 7.5 = 78.75 \text{ full}$$

$$\frac{135}{180} \times 14 \times 7.5 = 78.75 \text{ full}$$

$$\frac{135}{180} \times 14 \times 7.5 = 78.75 \text{ full}$$

$$\frac{135}{180} \times 14 \times 7.5 = 78.75 \text{ full}$$

$$\frac{135}{180} \times 14 \times 7.5 = 78.75 \text{ full}$$

$$\frac{135}{180} \times 14 \times 7.5 = 78.75 \text{ full}$$

$$\frac{135}{180} \times 14 \times 7.5 = 78.75 \text{ full}$$

$$\frac{135}{180} \times 14 \times 7.5 = 78.75 \text{ full}$$

$$\frac{135}{180} \times 14 \times 7.5 = 78.75 \text{ full}$$

$$\frac{135}{180} \times 14 \times 7.5 = 78.75 \text{ full}$$

$$\frac{135}{180} \times 14 \times 7.5 = 78.75 \text{ full}$$

$$\frac{135}{180} \times 14 \times 7.5 = 78.75 \text{ full}$$

$$\frac{135}{180} \times 14 \times 7.5 = 78.75 \text{ full}$$

$$\frac{135}{180} \times 14 \times 7.5 = 78.75 \text{ full}$$

$$\frac{135}{180} \times 14 \times 7.5 = 78.75 \text{ full}$$

$$\frac{135}{180} \times 14 \times 7.5 = 78.75 \text{ full}$$

$$\frac{135}{180} \times 14 \times 7.5 = 78.75 \text{ full}$$

$$\frac{135}{180} \times 14 \times 7.5 = 78.75 \text{ full}$$

$$\frac{135}{180} \times 14 \times 7.5 = 78.75 \text{ full}$$

$$\frac{135}{180} \times 14 \times 7.5 = 78.75 \text{ full}$$

$$\frac{135}{180} \times 14 \times 7.5 = 78.75 \text{ full}$$

$$\frac{135}{180} \times 14 \times 7.5 = 78.75 \text{ full}$$

$$\frac{135}{180} \times 14 \times 7.5 = 78.75 \text{ full}$$

$$\frac{135}{180} \times 14 \times 7.5 = 78.75 \text{ full}$$

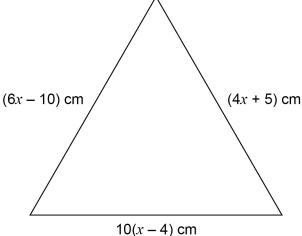
$$\frac{135}{180} \times 14 \times 7.5 = 78.75 \text{ full}$$

$$\frac{135}{180} \times 14 \times 7.5 = 78.75 \text{ full}$$

$$\frac{135}{180} \times 14 \times 7.5 = 78.75 \times 7.5$$

10

This triangle is equilateral. \ All side lengths are equal



Not drawn accurately

Is the perimeter of the triangle greater than one metre?

You must show your working.

[5 marks]

Find x: 
$$43c + 5 = 6x - 10$$

-4x

 $5 = 2x - 10$ 

+10

 $15 = 2x$ 
 $15 = 2x$ 
 $15 = 2x$ 

All sides

35 an

 $) + 5 = 35 \, \text{cm}$ One side length:  $\frac{105 \text{ cm} = 1.05 \text{m}}{100}$ Three

The perimeter is greater than Im

11 An approximation for the value of  $\pi$  is given by

$$4\bigg(1-\frac{22}{57}+\frac{22}{85}-\frac{22}{105}+\frac{22}{117}-\frac{22}{242}\bigg)$$

Use your calculator to show that this approximation is within 0.1 of 3.14

[2 marks]

12 Work out 
$$\frac{9.12 \times 10^{10}}{3.2 \times 10^4}$$

Give your answer in standard form.

[2 marks]

$$\frac{9.12 \times 10^{16}}{3.2}$$

Answer 
$$2.85 \times 10^6$$

13 Ashraf is going to put boxes into a crate.

> The crate is a cuboid measuring 2.5 m by 2 m by 1.2 m Each box is a cube of length 50 cm

He does these calculations.

volume of crate = 
$$2.5 \times 2 \times 1.2^{\circ}$$

$$= 6 m^3 5$$

volume of one box = 
$$0.5 \times 0.5 \times 0.5$$

$$= 0.125 \,\mathrm{m}^3$$

number of boxes = 
$$6 \div 0.125$$

He claims,

"I can put 48 boxes in the crate."

Evaluate Ashraf's method and claim.

[2 marks]

His method doesn't work as 1.2m

does not divide exactly into 50cm.

This means his claim is wrong

14 The cross section of a prism has n sides.

Circle the expression for the number of edges of the prism.

[1 mark]

$$n + 2$$

$$2n + 3$$

15 The volume of a medal is 45 cm<sup>3</sup>

The medal is made from copper and tin.

volume of copper: volume of tin = 22:3

The density of copper is 8.96 g/cm<sup>3</sup>

The density of tin is 7.31 g/cm<sup>3</sup>

Density = Mass volume

Work out the mass of the medal.

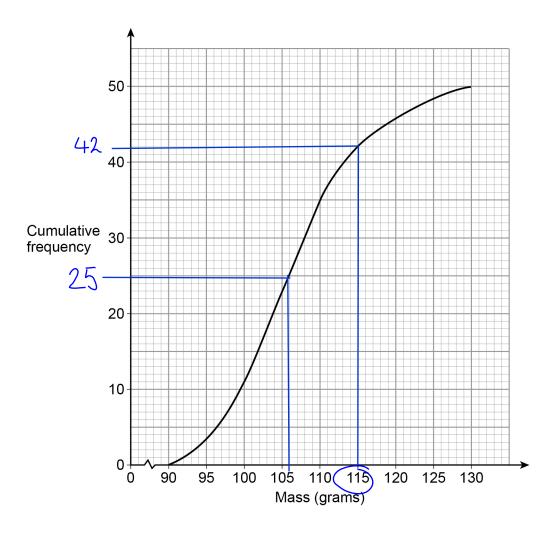
[4 marks]

Mass of (opper: 
$$8.96 \times 39.6 = 354.8169$$
of Tin:  $7.31 \times 5.4 = 39.4749$ 
Total =  $394.29$ 

Answer 394.29 grams

Do not write outside the box

The cumulative frequency graph shows information about the masses of 50 apples.



**16 (a)** Use the graph to estimate the median mass of the apples.

[1 mark]

$$\frac{50}{2} = 25$$

Answer \_\_\_\_\_ grams

**16 (b)** Estimate the proportion of the apples that have a mass greater than <u>115 grams.</u>

[2 marks]

50 - 42 = 8

Answer  $\frac{8}{50}$   $\left(\frac{4}{25}\right)$ 

.

Turn over ▶



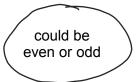
17 a is a prime number.

b is an even number.

$$N = a^2 + ab$$

Circle the correct statement about N.

[1 mark]



always even

always prime

always odd

- 18 A bag contains 20 discs. 10 are red, 7 are blue and 3 are green.
- Marnie takes a disc at random before putting it back in the bag. 18 (a) Nick then takes a disc at random before putting it back in the bag. Olly then takes a disc at random.

Work out the probability that they all take a red disc.

[2 marks]

Red:  $\frac{10}{20} = \frac{1}{2}$ 3 people so  $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$ Answer  $\frac{1}{8}$ 



18 (b) All 20 discs are in the bag.

Reggie takes three discs at random, one after the other.

After he takes a disc he does **not** put it back in the bag.

Reggie's first disc is blue.

Work out the probability that all three discs are different colours.

$$\frac{3}{10^{-1}} = \frac{3}{19}$$

$$\frac{10 \times 3}{342} = \frac{30}{342}$$

$$\frac{3 \times 10}{10 \times 18} = \frac{30}{347}$$

$$\frac{30}{342} + \frac{30}{342} = \frac{60}{342} = \frac{10}{57}$$

Do not write outside the box

19

#### Lunch

Choose one starter and one main course

There are four starters and ten main courses to choose from.

Two of the starters and three of the main courses are suitable for vegans.

What percentage of the possible lunches have both courses suitable for vegans?

[3 marks]

Total Number of: 4 × 10 = 40

combinations

Answer

20 n is a positive integer.

Prove algebraically that 
$$2n^2\left(\frac{3}{n}+n\right)+6n(n^2-1)$$
 is a cube number.

[3 marks]

 $+2n^3+6n^3-6n$ 

$$= 8n^3$$

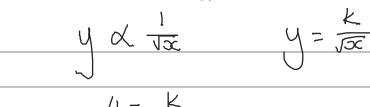
$$= (2n)^3 = \text{cube number}$$



[3 marks]

$$y = 4$$
 when  $x = 9$ 

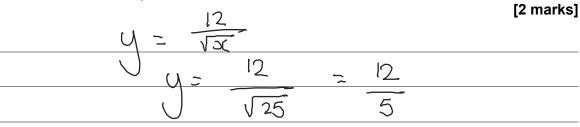
**21 (a)** Work out an equation connecting y and x.



$$\frac{\sqrt{9}}{4 = \frac{K}{3} (\times 3) K = 12}$$

Answer  $\frac{12}{\sqrt{x}}$ 

**21 (b)** Work out the value of y when x = 25



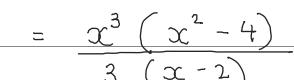
Turn over for the next question

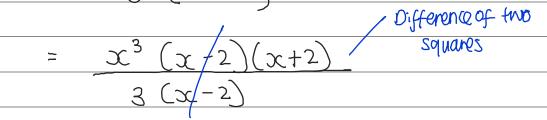
Turn over ▶

Do not write outside the box

[3 marks]

Simplify fully 
$$\frac{x^5 - 4x^3}{3x - 6}$$





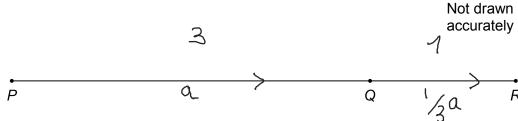
$$\frac{x^3(x+2)}{3}$$
Answer

# 23

PQR is a straight line.

$$PQ:QR = 3:1$$

$$\overrightarrow{PQ} = \mathbf{a}$$



Circle the vector  $\overrightarrow{RQ} \sim 0$  pposite direction

[1 mark]

$$\frac{1}{3}a$$

$$\frac{1}{4}a$$

$$\left(-\frac{1}{3}\mathbf{a}\right)$$

$$-\frac{1}{4}a$$

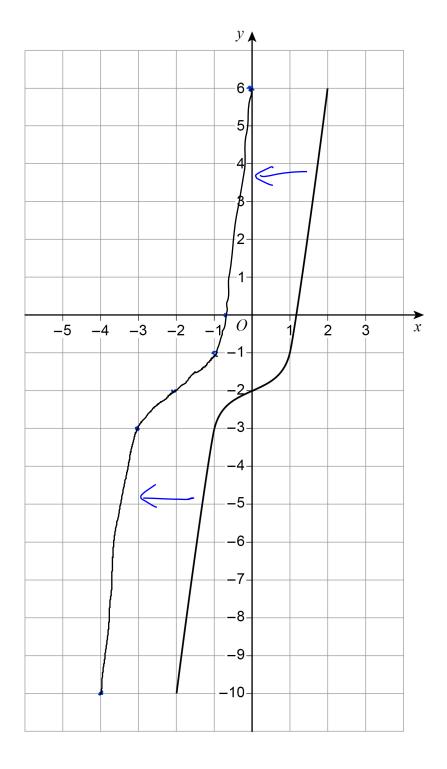


Do not write outside the box

24 Here is a sketch of y = f(x)

The curve passes through the points

$$(-2, -10)$$
  $(-1, -3)$   $(0, -2)$   $(1, -1)$   $(2, 6)$ 



On the grid, sketch the curve y = f(x + 2) Move to the left 2

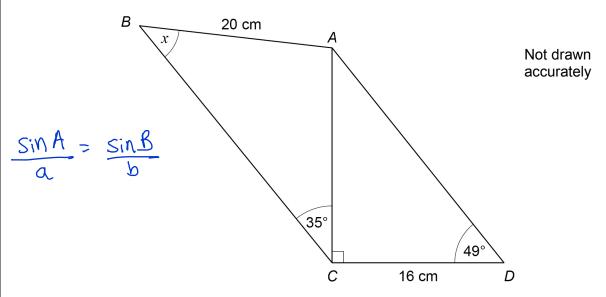
[2 marks]

Turn over ▶





25 ABC and ACD are triangles.



Work out the size of angle x.

tand= opp adj

[5 marks]

Find AC: 
$$6an 49 = \frac{AC}{16}$$

$$AC = 16 + an 49$$

Sine Rule to 
$$\sin x = \sin 35$$
  
find x 16 tan 49 20

$$Sin \propto = \frac{16 \tan 49 \times \sin 35}{20}$$
  
 $\propto = Sin^{-1} \left( 0.527859... \right)$   
 $3C = 31.86...$ 

Answer 31.9 degrees (3sf)

26

$$f(x) = \frac{x}{x+2}$$
  $g(x) = x^2 - 2$ 

$$g(x) = x^2 - 2$$

Work out fg(x)

Give your answer in the form  $a + bx^n$  where a, b and n are integers.

[3 marks]

$$fg(x) = \frac{x^2 - 2}{x^2 - 2 + 2}$$

$$= \frac{x^2 - 2}{x^2 - 2 + 2} = \frac{3c^2 - 2}{3c^2}$$

$$= \frac{x^2 - 2}{x^2}$$

$$= \frac{1 - 2x}{x^2}$$

Answer 
$$1 - 2 \times 2$$

The point  $\left(3, \frac{1}{64}\right)$  lies on the curve  $y = k^x$  where k is a constant. 27

Show that the point  $\left(\frac{1}{2}, \frac{1}{2}\right)$  lies on the curve.

[3 marks]

$$\frac{1}{64} = \frac{1}{64} = \frac{3}{64} = \frac{1}{4}$$

When 
$$3c = \frac{1}{2}$$
 $y = \frac{1}{4} = \frac{1}{2}$ 

Therefore (1/2,1/2) his on the curre



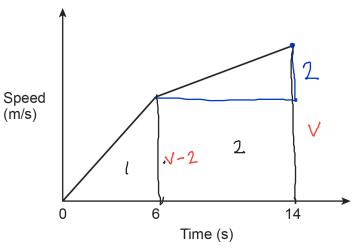
28 Izzy runs an 80-metre race in 14 seconds.

During the first 6 seconds her speed increases at a constant rate.

During the last 8 seconds her speed increases at a different constant rate.

Her speed at 14 seconds is 2 m/s more than her speed at 6 seconds.

Here is a sketch of her speed-time graph.



Not drawn accurately

gradient

**28 (a)** Work out her acceleration during the last 8 seconds.

State the units of your answer.

$$\mathcal{M} = \frac{y_2 - y_1}{x_2 - x_1}$$

 $=\frac{2}{8}=\frac{1}{4}$ 

[2 marks]

Answer

28 (b) When Izzy finishes the 80-metre race, her speed is v m/s

Work out the value of v.

[4 marks]

(1) 
$$\frac{1}{2} \times (v-2) \times 6 = 3(v-2) = 3v-6$$

(1) 
$$\frac{1}{2} \times (v-2) \times 6 = 3(v-2) = 3v-6$$
  
(2)  $\frac{1}{2} \times (v-2+v) \times 8 = 4(2v-2) = 8v-8$ 

$$3v - 6 + 8v - 8 = 80$$

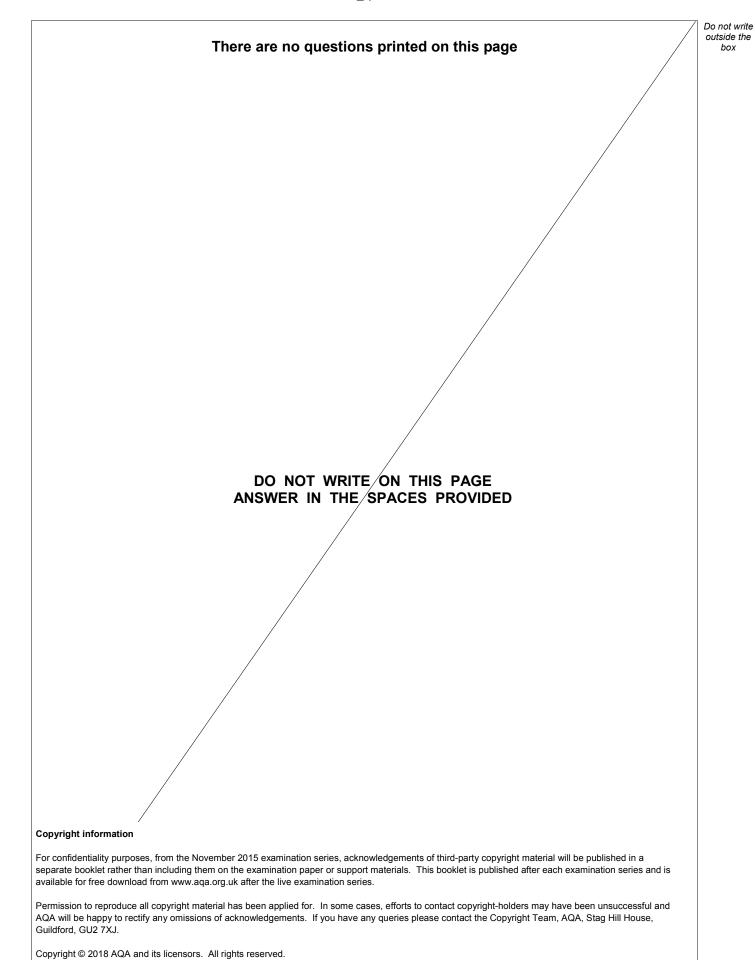
$$11v - 14 = 80$$

$$11V = 94$$
 $V = 8.54$ 

Answer 
$$= 8.55$$

## **END OF QUESTIONS**





2 4