MODEL ANSWERS

Write your name here

Surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Further Mathematics

Advanced Subsidiary
Further Mathematics options
26: Further Mechanics 2
(Part of option J only)

Thursday 17 May 2018 – Afternoon

Paper Reference **8FM0-26**

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 4 questions.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

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Pearson

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8\,\mathrm{m\,s^{-2}}$ and give your answer to either 2 significant figures or 3 significant figures.

Answer ALL questions. Write your answers in the spaces provided.

1.

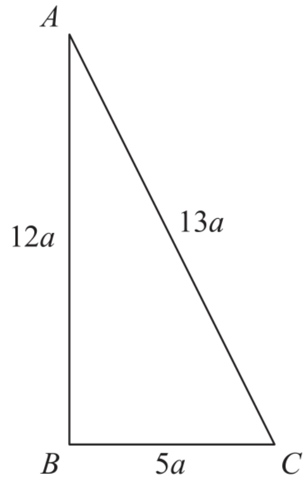


Figure 1

A thin uniform rod, of total length $30a$ and mass M, is bent to form a frame. The frame is in the shape of a triangle ABC, where $AB = 12a$, $BC = 5a$ and $CA = 13a$, as shown in Figure 1.

(a) Show that the centre of mass of the frame is $\frac{3}{2}a$ from AB.

(4)

The frame is freely suspended from A. A horizontal force of magnitude kMg, where k is a constant, is applied to the frame at B. The line of action of the force lies in the vertical plane containing the frame. The frame hangs in equilibrium with AB vertical.

(b) Find the value of k.

(3)

a) Take B as origin :

Side	Length	Coordinates of c.o.m
A B	12a	$\begin{pmatrix} 0 \\ 6a \end{pmatrix}$
B C	5a	$\begin{pmatrix} 5a/2 \\ 0 \end{pmatrix}$
A C	13a	$\begin{pmatrix} 5a/2 \\ 6a \end{pmatrix}$

Question 1 continued

$$\underline{\sum m_i x_i = \bar{x} \sum m_i}$$ cancel out M from LHS & RHS then cancel out a

$$12\cancel{a}\begin{pmatrix} 0 \\ 6a \end{pmatrix} + 5\cancel{a}\begin{pmatrix} 5a/2 \\ 0 \end{pmatrix} + 13\cancel{a}\begin{pmatrix} 5a/2 \\ 6a \end{pmatrix} = 30\cancel{a}\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}$$

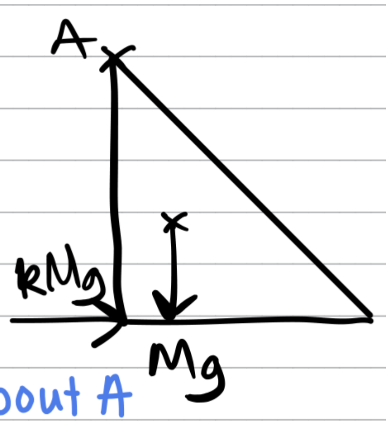
$$\begin{pmatrix} 25a/2 + 65a/2 \\ 72a + 78a \end{pmatrix} = \begin{pmatrix} 30\bar{x} \\ 30\bar{y} \end{pmatrix}$$

$$\begin{pmatrix} 45a \\ 150a \end{pmatrix} = \begin{pmatrix} 30\bar{x} \\ 30\bar{y} \end{pmatrix}$$

$$\therefore \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} \frac{45a}{30} \\ \frac{150a}{30} \end{pmatrix} = \begin{pmatrix} 3a/2 \\ 5a \end{pmatrix}$$

$\therefore$ $\bar{x}$ = Distance of C.O.M from AB = $\frac{3a}{2}$ //

b)



take moments about A

$\underline{M(A)}$: $\cancel{Mg}\left(\frac{3\cancel{a}}{2}\right) = k\cancel{Mg}(12\cancel{a})$

$\frac{3}{2} = 12k \quad \therefore \boxed{k = \frac{1}{8}}$

(Total for Question 1 is 7 marks)

Turn over

P 6 0 2 0 7 A 0 3 1 6

2. A car moves round a bend which is banked at a constant angle of $\theta°$ to the horizontal.

When the car is travelling at a constant speed of $80\,\text{km}\,\text{h}^{-1}$ there is no sideways frictional force on the car. The car is modelled as a particle moving in a horizontal circle of radius 500 m.

(a) Find the value of θ.

(7)

(b) Identify one limitation of this model.

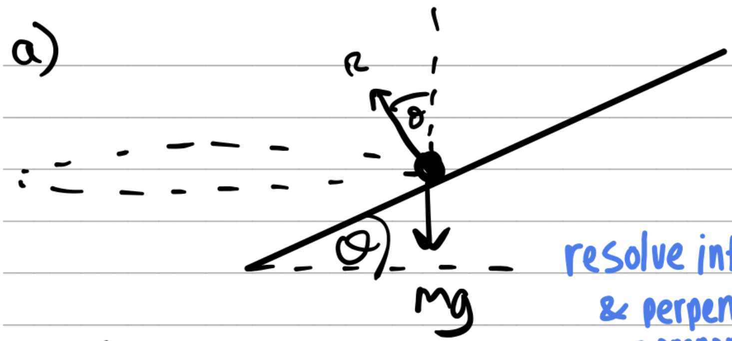
(1)

The speed of the car is increased so that it is now travelling at a constant speed of $90\,\text{km}\,\text{h}^{-1}$
The car is still modelled as a particle moving in a horizontal circle of radius 500 m.

(c) Describe the extra force that will now be acting on the car, stating the direction of this force.

(1)

a)



convert from kmh^{-1} to ms^{-1}

$80\text{kmh}^{-1} = \frac{80000}{3600}\text{ m/s}$

$= \frac{200}{9}\text{ m/s}$

resolve into parallel & perpendicular components

N2L ($\leftarrow +$): $R\sin\theta = \frac{mv^2}{r} = \frac{m\left(\frac{200}{9}\right)^2}{500} = R\sin\theta$

vertical normal force = centripetal force

$\therefore R\sin\theta = \frac{80}{81}m$ — 1

$R(\updownarrow)$: $R\cos\theta = mg$ — 2

$1^2 + 2^2$: $R^2\sin^2\theta + R^2\cos^2\theta = m^2g^2 + \frac{6400m^2}{6561}$

$R^2(\cancel{\sin^2\theta + \cos^2\theta}) = m^2(97.015\ldots)$ (1)

$\therefore R = m\sqrt{97.015\ldots} = 9.85m$

$\cos\theta = \frac{mg}{R} = \frac{\cancel{m}g}{9.85\cancel{m}} = 0.995\ldots$

$\theta = \cos^{-1}(0.995)$

$= \boxed{5.8°}$

Question 2 continued

b) We've assumed there's only one point of contact with the road, through which the weight acts, which isn't accurate

c) Friction between the tyres and the road, acting down the slope of the road.

(friction opposes motion)

3.

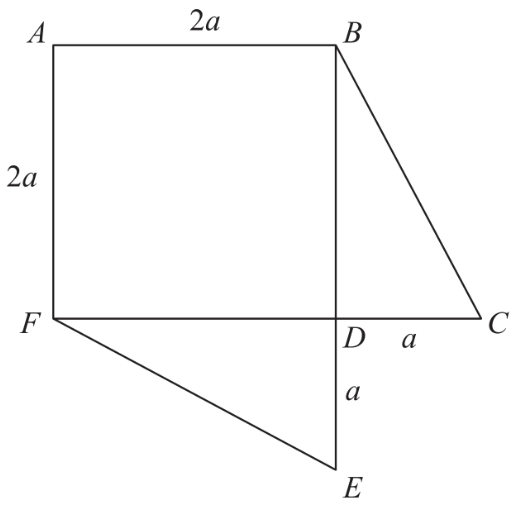


Figure 2

The lamina L, shown in Figure 2, consists of a uniform square lamina $ABDF$ and two uniform triangular laminas BDC and FDE. The square has sides of length $2a$. The two triangles are identical.

The straight lines BDE and FDC are perpendicular with $BD = DF = 2a$ and $DC = DE = a$.
The mass per unit of area of the square is M.
The mass per unit area of each triangle is $3M$.
The centre of mass of L is at the point G.

(a) Without doing any calculations, explain why G lies on AD.

(1)

(b) Show that the distance of G from D is $\dfrac{\sqrt{2}}{2}a$

(7)

The lamina L is freely suspended from B and hangs in equilibrium.

(c) Find the size of the angle between BE and the downward vertical.

(3)

a) L is symmetrical about AD, and ΔDFE and ΔBCD have equal mass per unit area.

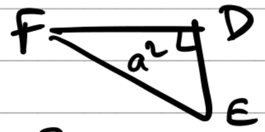
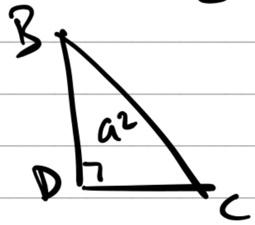
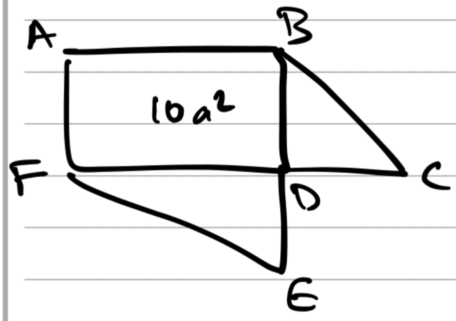
Question 3 continued

$\nearrow$ (= mass per unit area × area)

$\longrightarrow +$ define axis

$\downarrow +$

b)

Shape	Mass	Distance of c.o.m from AF	Distance of c.o.m from AB
	$M(4a^2)$	a	a
	$3M(a^2)$	$\frac{2}{3}(2a) = \frac{4a}{3}$	$2a + \frac{a}{3} = \frac{7a}{3}$
	$3M(a^2)$	$2a + \frac{a}{3} = \frac{7a}{3}$	$\frac{2}{3}(2a) = \frac{4a}{3}$
	$M(10a^2)$	$\bar{X}$	$\bar{Y}$

$$\underline{\sum m_i x_i = \bar{x} \sum m_i}$$

$$4\cancel{M}\cancel{a^2}\begin{pmatrix} a \\ a \end{pmatrix} + 3\cancel{M}\cancel{a^2}\begin{pmatrix} 4a/3 \\ 7a/3 \end{pmatrix} + 3\cancel{M}\cancel{a^2}\begin{pmatrix} 7a/3 \\ 4a/3 \end{pmatrix} = \cancel{M}\begin{pmatrix} \bar{X} \\ \bar{Y} \end{pmatrix}(10\cancel{a^2})$$

$$\begin{pmatrix} 4a + 4a + 7a \\ 4a + 4a + 7a \end{pmatrix} = 10\begin{pmatrix} \bar{X} \\ \bar{Y} \end{pmatrix} = \begin{pmatrix} 15a \\ 15a \end{pmatrix}$$

$$\therefore \begin{pmatrix} \bar{X} \\ \bar{Y} \end{pmatrix} = \begin{pmatrix} 3a/2 \\ 3a/2 \end{pmatrix}$$

$\therefore$ Distance of c.o.m from A $= \sqrt{\left(\frac{3a}{2}\right)^2 + \left(\frac{3a}{2}\right)^2} = \frac{3a\sqrt{2}}{2}$ //

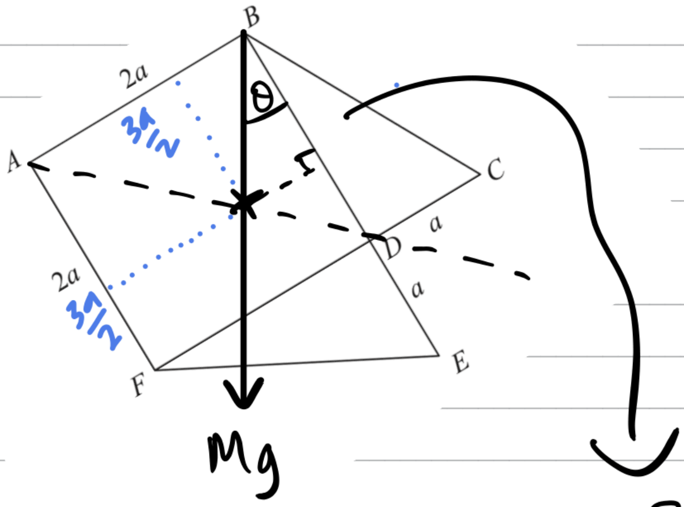
Question 3 continued

Distance AD $= \sqrt{(2a)^2+(2a)^2} = 2a\sqrt{2}$

so Distance of G from D $= 2a\sqrt{2} - \frac{3a\sqrt{2}}{2}$

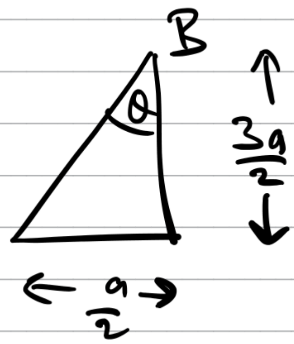
$= \boxed{\frac{\sqrt{2}}{2}a}$

c)



weight acts through C.O.M.

Using position of C.O.M. from A:
$\left(\frac{3a}{2}, \frac{3a}{2}\right)$



$\tan\theta = \frac{\frac{a}{2}}{\frac{3a}{2}} = \frac{1}{3}$

$\therefore \theta = \tan^{-1}\left(\frac{1}{3}\right) = \boxed{18.4^\circ}$

P 6 0 2 0 7 A 0 1 0 1 6

4. A particle, P, moves on the x-axis. At time t seconds, $t \geqslant 0$, the velocity of P is $v\,\text{m s}^{-1}$ in the direction of x increasing and the acceleration of P is $a\,\text{m s}^{-2}$ in the direction of x increasing.

When $t = 0$ the particle is at rest at the origin O.

Given that $a = \frac{5}{2}(5 - v)$

(a) show that $v = 5(1 - e^{-2.5t})$

(5)

(b) state the limiting value of v as t increases.

(1)

At the instant when $v = 2.5$, the particle is d metres from O.

(c) Show that $d = 2\ln 2 - 1$

(7)

a) $a = \frac{dv}{dt} = \frac{5}{2}(5-v)$

$\frac{2}{5}\frac{dv}{dt} = 5 - v$

$\left(\frac{\frac{2}{5}}{5-v}\right)\frac{dv}{dt} = 1$

$\therefore \frac{2}{5}\int \frac{1}{5-v}\,dv = \int (1)\,dt$

$-\frac{2}{5}\ln|5-v| = t + c$

$t=0, v=0:\quad -\frac{2}{5}\ln 5 = c$ //

$\therefore -\frac{2}{5}\ln|5-v| = t - \frac{2}{5}\ln 5$

$\frac{2}{5}\ln 5 - \frac{2}{5}\ln|5-v| = t$

$\frac{2}{5}\ln\left|\frac{5}{5-v}\right| = t \qquad \therefore \ln\left|\frac{5}{5-v}\right| = \frac{5t}{2}$ //

P 6 0 2 0 7 A 0 1 2 1 6

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Question 4 continued

so $e^{\frac{5t}{2}} = \frac{5}{5-v}$ $\therefore \frac{5-v}{5} = e^{-\frac{5t}{2}}$

$\Rightarrow 5 - v = 5e^{-\frac{5t}{2}}$

$\Rightarrow v = 5 - 5e^{-\frac{5t}{2}} = 5\left(1 - e^{-\frac{5t}{2}}\right)$ //

b) $\lim_{t\to\infty}\left[5\left(1-e^{-\frac{5t}{2}}\right)\right] = 5(1-0) = \boxed{5}$ m/s

c) $v = 2.5$, $x = d$.

$2.5 = 5\left(1 - e^{-\frac{5t}{2}}\right)$

$\frac{1}{2} = 1 - e^{-\frac{5t}{2}}$

$\frac{1}{2} = e^{-\frac{5t}{2}}$

$\therefore \ln\frac{1}{2} = -\frac{5t}{2}$

this is when $x = d$

$\therefore -\frac{2}{5}\ln\frac{1}{2} = t = \frac{2}{5}\ln 2$ //

$x = \int v\,dt = 5\int 1 - e^{-\frac{5t}{2}}\,dt$

$x = 5\left[t + \frac{2}{5}e^{-\frac{5t}{2}}\right] + c$

$\underline{t=0,\ x=0}$: $0 = 5\left[\frac{2}{5}(1)\right] + c$

$\therefore c = -2$ //

so $x = 5t + 2e^{-\frac{5t}{2}} - 2$ //

P 6 0 2 0 7 A 0 1 3 1 6

Turn over

Question 4 continued

at $t = \frac{2}{5}\ln 2$, $x = d$.

$\Rightarrow d = 5\left(\frac{2}{5}\ln 2\right) + 2e^{-\frac{5}{2}\left(\frac{2}{5}\ln 2\right)} - 2$

$\Rightarrow d = 2\ln 2 + 2e^{-\ln 2} - 2$

$\Rightarrow d = 2\ln 2 + 2e^{\ln\frac{1}{2}} - 2$

$\Rightarrow d = 2\ln 2 + 2\left(\frac{1}{2}\right) - 2$

$\Rightarrow \boxed{d = 2\ln 2 - 1}$