MODEL ANSWERS

AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname ______________________

Forename(s) ______________________

Candidate signature ______________________

AS
MATHEMATICS
Paper 1

Wednesday 16 May 2018   Morning   Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
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Section A

Answer **all** questions in the spaces provided.

1 Three of the following points lie on the same straight line.

Which point does **not** lie on this line?

Tick **one** box.

[1 mark]

(−2, 14)	☐
(−1, 8)	☑
(1, −1)	☐
(2, −6)	☐

2 A circle has equation $(x-2)^2+(y+3)^2=13$

Find the gradient of the tangent to this circle at the origin.

Circle your answer.

[1 mark]

$-\frac{3}{2}$ $\quad$ $-\frac{2}{3}$ $\quad$ $\frac{2}{3}$ $\quad$ $\frac{3}{2}$

Origin $(2,-3)$

$2x + 2y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{x}{y}$

At $(2,-3)$, $\frac{dy}{dx} = -\frac{(2)}{-3} = \frac{2}{3}$.

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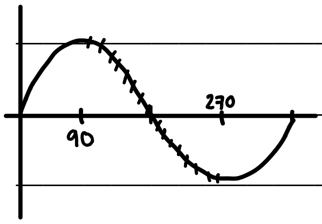
3 State the interval for which $\sin x$ is a decreasing function for $0° \leq x \leq 360°$

[2 marks]



Decreasing from $90° < x < 270°$.

Turn over for the next question

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4 (a) Find the first three terms in the expansion of $(1-3x)^4$ in ascending powers of x.

[3 marks]

$(1-3x)^4 = \binom{4}{0}(-3x)^0 + \binom{4}{1}(-3x)^1 + \binom{4}{2}(-3x)^2 + \cdots$

$= 1 + 4x(-3x) + 6(9x^2) + \cdots$

$= 1 - 12x + 54x^2 + \cdots$

4 (b) Using your expansion, approximate $(0.994)^4$ to six decimal places.

[2 marks]

$(1-3x)^4 = 0.994^4$

$1-3x = 0.994$

$3x = 0.006$

$x = 0.002$

Substitute $x=0.002$ into our answer from part (a):

$= 1 - 12(0.002) + 54(0.002)^2 = 0.976216$

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5 Point C has coordinates $(c, 2)$ and point D has coordinates $(6, d)$.

The line $y + 4x = 11$ is the perpendicular bisector of CD.

Find c and d.

[5 marks]

The gradient of CD is the negative reciprocal of the gradient of its bisector. So the gradient of CD is $1/4$.

Gradient $= \frac{\text{change in } y}{\text{change in } x} = \frac{d-2}{6-c}$.

So $\frac{d-2}{6-c} = \frac{1}{4} \Rightarrow 4d - 8 = 6 - c \Rightarrow 4d + c = 14$ ①

The midpoint of CD is $\left(\frac{6+c}{2}, \frac{d+2}{2}\right)$.

These coordinates will satisfy the equation for the bisector, so

$$\frac{d+2}{2} + 4\left(\frac{6+c}{2}\right) = 11$$

$$d+2 + 24 + 4c = 22$$

$$4c + d + 4 = 0 \quad ②$$

From ①, $c = 14 - 4d$. Substitute into ②:

$$4(14-4d) + d + 4 = 0$$

$$56 - 16d + d + 4 = 0$$

$$60 = 15d$$

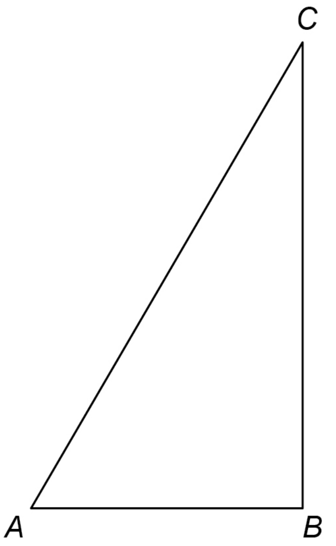
$$d = 4$$

$c = 14 - 4d = 14 - 4(4) = -2$

So, $c = -2$, $d = 4$.

Turn over ▶

6 ABC is a right-angled triangle.

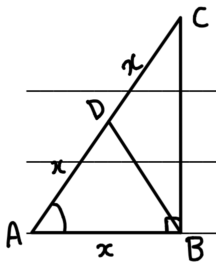


D is the point on hypotenuse AC such that $AD = AB$.

The area of ΔABD is equal to half that of ΔABC.

6 (a) Show that $\tan A = 2 \sin A$

[4 marks]



Considering areas:

$\frac{1}{2} \Delta ABC = \Delta ABD$

$\frac{1}{2} (\Delta ABD + \Delta BDC) = \Delta ABD$

$\frac{1}{2} \Delta BDC = \frac{1}{2} \Delta ABD$

$\Delta BDC = \Delta ABD$

So, the area of ΔADB = Area of ΔBDC

So, we must have $CD = AD = x$, so $AC = 2x$.

$\cos A = \frac{AB}{AC} = \frac{x}{2x} = \frac{1}{2}$, So $A = 60$.

$\tan A = \tan 60 = \sqrt{3}$

$\sin A = \sin 60 = \frac{\sqrt{3}}{2}$

Therefore, $2 \sin A = \tan A = \sqrt{3}$.

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6 (b) (i) Show that the equation given in part **(a)** has two solutions for $0° \leq A \leq 90°$

[2 marks]

$2\sin A = \tan A \Rightarrow 2\sin A = \frac{\sin A}{\cos A}$

$2\sin A\cos A - \sin A = 0 \Rightarrow \sin A(2\cos A - 1) = 0$

$\sin A = 0$ or $\cos A = \frac{1}{2}$.

If $\sin A = 0$, $A = 0°$

If $\cos A = \frac{1}{2}$, $A = 60°$

so two solutions: $0°$, $60°$.

6 (b) (ii) State the solution which is appropriate in this context.

[1 mark]

$60°$

Turn over for the next question

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7 Prove that

$$n \text{ is a prime number greater than } 5 \Rightarrow n^4 \text{ has final digit } 1$$

[5 marks]

If n is prime then it cannot be even so cannot end in 2, 4, 6, 8, 0.

It also cannot end in 5 as then it would be divisible by 5.

So a prime number must end in 1, 3, 7 or 9.

If n ends in 1, $1^4 = 1$ so n^4 ends in 1.

If n ends in 3, $3^4 = 81$ so n^4 ends in 1.

If n ends in 7, $7^4 = 2401$ so n^4 ends in 1.

If n ends in 9, $9^4 = 6561$ so n^4 ends in 1.

So for all possible endings, n^4 ends in 1.

Hence, proven by exhaustion.

Turn over for the next question

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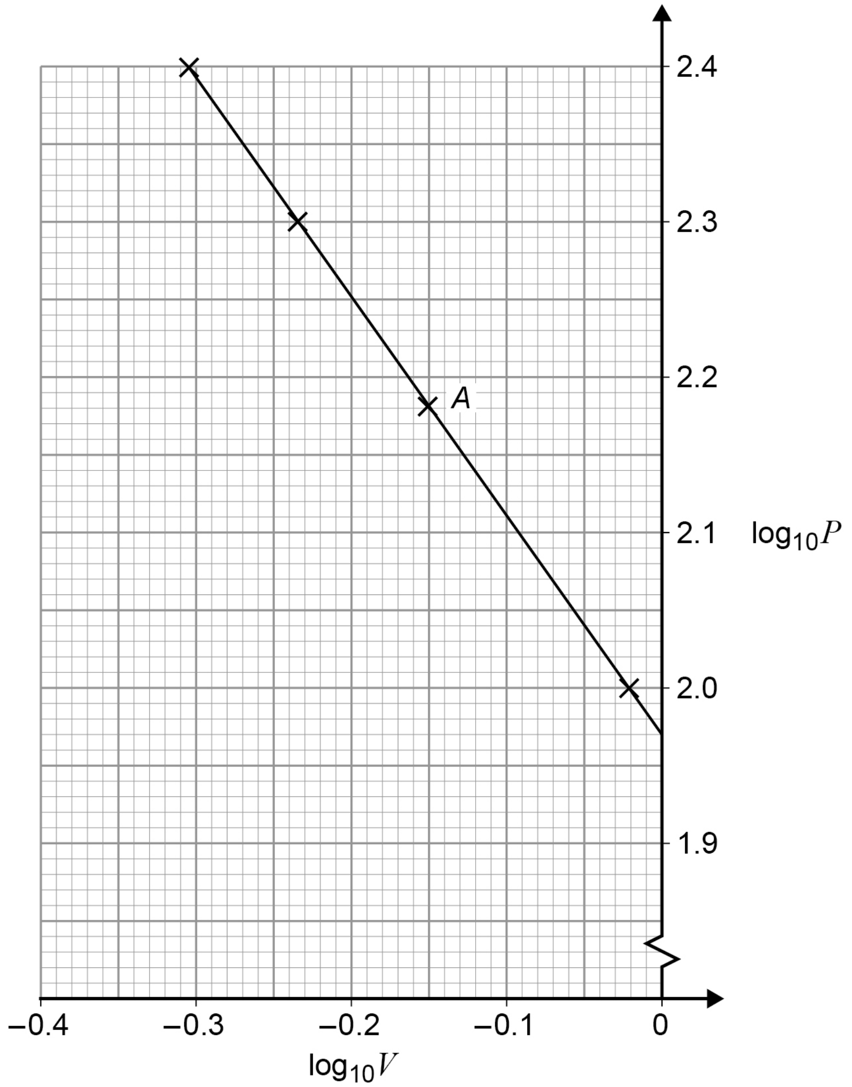
8 Maxine measures the pressure, P kilopascals, and the volume, V litres, in a fixed quantity of gas.

Maxine believes that the pressure and volume are connected by the equation

$$P = cV^d$$

where c and d are constants.

Using four experimental results, Maxine plots $\log_{10}P$ against $\log_{10}V$, as shown in the graph below.



8 (a) Find the value of P and the value of V for the data point labelled A on the graph. **[2 marks]**

$\log_{10}V = -0.15$ $\log_{10}P = 2.18$

$V = 10^{-0.15}$ $P = 10^{2.18}$

$V = 0.708$ $P = 151.4$

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8 (b) Calculate the value of each of the constants c and d.

[4 marks]

We can turn $P = cV^d$ into a linear equation by taking logs:

$\log_{10} P = \log_{10}(cV^d)$

$\log_{10} P = \log_{10} c + d\log_{10} V$

So here shows that d is the gradient and $\log_{10} c$ is the intercept.

From the graph, the gradient is -1.4 and the intercept is 1.97.

So, $d = -1.4$, $\log_{10} c = 1.97$

$c = 10^{1.97}$

$c = 93.3$

8 (c) Estimate the pressure of the gas when the volume is 2 litres.

[2 marks]

When $V = 2$, $P = 93.3 \times 2^{-1.4}$

$= 35.4$ kilopascals.

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9 Craig is investigating the gradient of chords of the curve with equation $f(x) = x - x^2$

Each chord joins the point $(3, -6)$ to the point $(3 + h, f(3 + h))$

The table shows some of Craig's results.

x	$f(x)$	h	$x + h$	$f(x + h)$	Gradient
3	-6	1	4	-12	-6
3	-6	0.1	3.1	-6.51	-5.1
3	-6	0.01	3.01	-6.0501	-5.01
3	-6	0.001			
3	-6	0.0001			

9 (a) Show how the value -5.1 has been calculated.

[1 mark]

$\frac{f(x+h) - f(x)}{h} = \frac{-6.51 - (-6)}{0.1} = -5.1$

9 (b) Complete the third row of the table above.

[2 marks]

$x + h = 3 + 0.01 = 3.01$

$f(x+h) = 3.01 - (3.01)^2 = -6.0501$

$\text{Gradient} = \frac{f(x+h) - f(x)}{h} = \frac{-6.0501 - (-6)}{0.01} = -5.01$

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9 (c) State the limit suggested by Craig's investigation for the gradient of these chords as h tends to 0

[1 mark]

-5

9 (d) Using differentiation from first principles, verify that your result in part **(c)** is correct.

[4 marks]

Gradient of chord $= \dfrac{f(3+h)-f(3)}{h}$

$= \dfrac{(3+h)-(3+h)^2-(3-3^2)}{h}$

$= \dfrac{3+h-9-6h-h^2+6}{h}$

$= \dfrac{-h^2-5h}{h}$

$= -h-5$

As $h \to 0$, $-h-5 \to -5$.

So, at $x=3$, the gradient is -5.

Do not write outside the box

10 A curve has equation $y = 2x^2 - 8x\sqrt{x} + 8x + 1$ for $x \geq 0$

10 (a) Prove that the curve has a maximum point at (1, 3)

Fully justify your answer.

[9 marks]

$y = 2x^2 - 8x^{\frac{3}{2}} + 8x + 1$

$\frac{dy}{dx} = 4x - 12x^{\frac{1}{2}} + 8$

Maximum points occur when $\frac{dy}{dx} = 0$

$\frac{dy}{dx} = 4x - 12x^{\frac{1}{2}} + 8 = 0$

$4(x - 3x^{\frac{1}{2}} + 2) = 0$

$4(x^{\frac{1}{2}} - 2)(x^{\frac{1}{2}} - 1) = 0$

so $x^{\frac{1}{2}} - 2 = 0$ or $x^{\frac{1}{2}} - 1 = 0$

$x = 4$ $x = 1$

When $x = 1$, $y = 2(1) - 8(1) + 8(1) + 1$

$= 2 - 8 + 8 + 1 = 3$

So (1,3) is a stationary point, but now we need to show it is a maximum.

For a maximum, $\frac{d^2y}{dx^2} < 0$.

$\frac{d^2y}{dx^2} = 4 - 6x^{-\frac{1}{2}} = 4 - \frac{6}{\sqrt{x}}$

At $x = 1$, $\frac{d^2y}{dx^2} = 4 - \frac{6}{1} = -2$

$-2 < 0$ so (1,3) is a maximum.

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10 (b) Find the coordinates of the other stationary point of the curve and state its nature.

[2 marks]

We found in (a) that the other stationary point was at $x=4$.

When $x=4$, $y = 2(16) - 8(4)(2) + 8(4) + 1$

$= 32 - 64 + 32 + 1 = 1$

So it is at $(4,1)$.

At $x=4$, $\frac{d^2y}{dx^2} = 4 - \frac{6}{\sqrt{4}} = 4-3=1>0$

So $(4,1)$ is a minimum.

Turn over for Section B

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Section B

Answer **all** questions in the spaces provided.

11 **In this question use $g = 9.8\,\mathrm{m\,s^{-2}}$**

A ball, initially at rest, is dropped from a height of 40 m above the ground.

Calculate the speed of the ball when it reaches the ground.

Circle your answer.

[1 mark]

$-28\,\mathrm{m\,s^{-1}}$ $\quad$ $28\,\mathrm{m\,s^{-1}}$ $\quad$ $-780\,\mathrm{m\,s^{-1}}$ $\quad$ $780\,\mathrm{m\,s^{-1}}$

$v^2 = u^2 + 2as$
$v^2 = 0^2 + 2(9.8)(40)$
$v^2 = 784$
$v = 28$

12 An object of mass 5 kg is moving in a straight line.

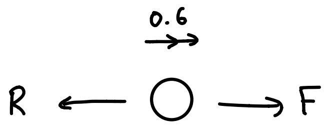
As a result of experiencing a forward force of F newtons and a resistant force of R newtons it accelerates at $0.6\,\mathrm{m\,s^{-2}}$

Which one of the following equations is correct?

Circle your answer.

[1 mark]

$F - R = 0$ $\quad$ $F - R = 5$ $\quad$ $F - R = 3$ $\quad$ $F - R = 0.6$



Using $F = ma$: $F - R = 5(0.6)$
$F - R = 3$

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13 A vehicle, which begins at rest at point P, is travelling in a straight line.

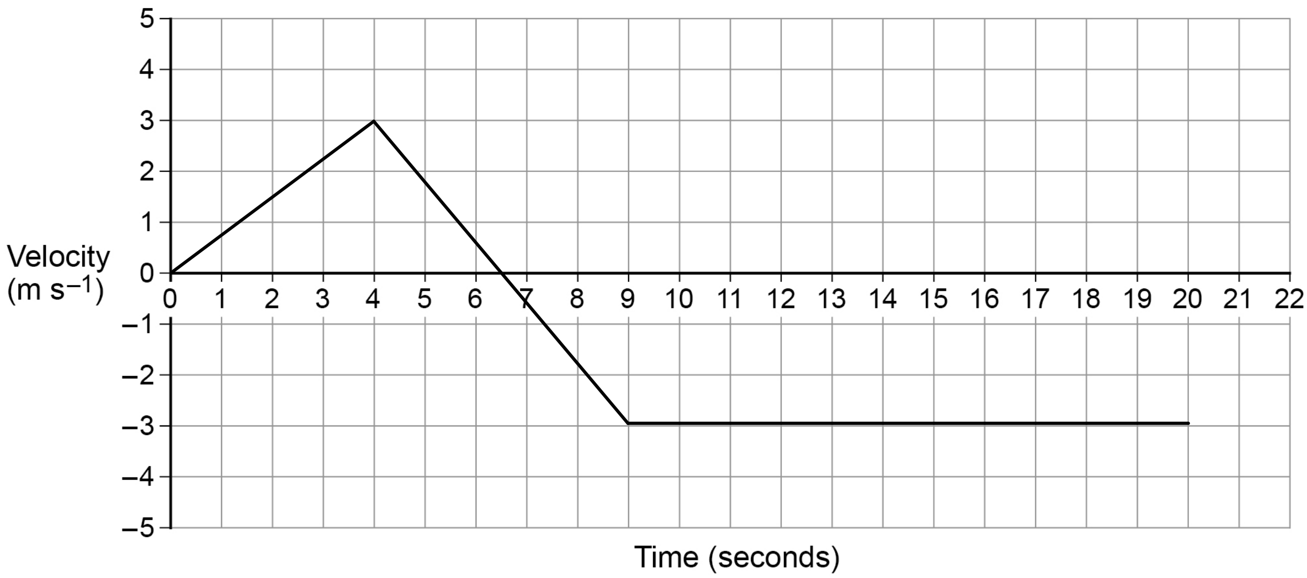
For the first 4 seconds the vehicle moves with a constant acceleration of $0.75\,\text{m s}^{-2}$

For the next 5 seconds the vehicle moves with a constant acceleration of $-1.2\,\text{m s}^{-2}$

The vehicle then immediately stops accelerating, and travels a further 33 m at constant speed.

13 (a) Draw a velocity–time graph for this journey on the grid below.

[3 marks]



13 (b) Find the distance of the car from P after 20 seconds.

[3 marks]

The distance travelled is the area underneath the graph.
When the graph is below the x axis, this counts as negative distance as the vehicle is travelling backwards.

Distance $= \frac{4 \times 3}{2} + \frac{(6.5-4)\times 3}{2} - \frac{(9-6.5)\times 3}{2} - (11 \times 3)$

$= 6 + \frac{7.5}{2} - \frac{7.5}{2} - 33 = -27$

So it travels -27m backwards.
Distance is 27m.

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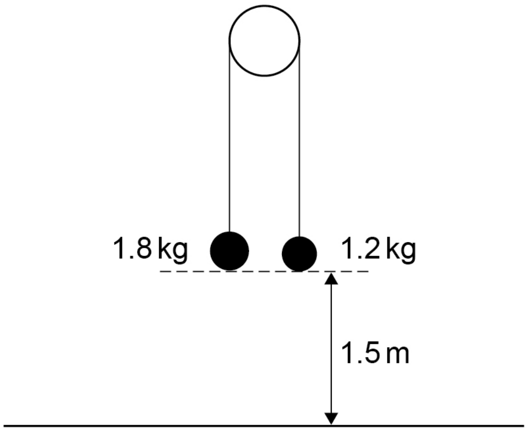
14 **In this question use $g = 9.81\,\mathrm{m\,s^{-2}}$**

Two particles, of mass 1.8 kg and 1.2 kg, are connected by a light, inextensible string over a smooth peg.



14 (a) Initially the particles are held at rest 1.5 m above horizontal ground and the string between them is taut.

The particles are released from rest.

Find the time taken for the 1.8 kg particle to reach the ground.

[5 marks]

T ↑ A ↓ 1.8g T ↑ B ↓ 1.2g

Using $F=ma$:

A: $1.8g - T = 1.8a$ ①

B: $T - 1.2g = 1.2a$ ②

① + ②: $1.8g - 1.2g = 1.8a + 1.2a$

$3a = 0.6g$

$a = 1.96$

$s = 1.5$

$u = 0$

$v = -$

$a = 1.96$

$t = t$

$s = ut + \frac{1}{2}at^2$

$1.5 = 0 + \frac{1}{2}(1.96)t^2$

$t^2 = 1.5306$

$t = 1.24$ seconds

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14 (b) State one assumption you have made in answering part **(a)**.

[1 mark]

The lighter mass does not reach the peg before the heavier mass reaches the ground.

Turn over for the next question

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15 A cyclist, Laura, is travelling in a straight line on a horizontal road at a constant speed of $25\,\text{km}\,\text{h}^{-1}$

A second cyclist, Jason, is riding closely and directly behind Laura. He is also moving with a constant speed of $25\,\text{km}\,\text{h}^{-1}$

15 (a) The driving force applied by Jason is likely to be less than the driving force applied by Laura.

Explain why.

[1 mark]

Laura is sheltering Jason so he experiences less air resistance than Laura.

15 (b) Jason has a problem and stops, but Laura continues at the same constant speed.

Laura sees an accident 40 m ahead, so she stops pedalling and applies the brakes.

She experiences a total resistance force of 40 N

Laura and her cycle have a combined mass of 64 kg

15 (b) (i) Determine whether Laura stops before reaching the accident.

Fully justify your answer.

[4 marks]

Using F=ma: $-40 = 64a$

$a = -0.625$

$25\text{ km/h} = \dfrac{25 \times 1000}{60 \times 60} = 6.944\text{ m/s}$

$s = s$	$v^2 = u^2 + 2as$
$u = 6.944$	$0 = 6.944^2 + 2(-0.625)(s)$
$v = 0$	$0 = 48.225 - 1.25s$
$a = -0.625$	$s = \dfrac{48.225}{1.25} = 38.6\text{m}$
$t = -$	

So she stops in $38.6\text{m} < 40\text{m}$, so she stops before reaching the accident.

15 (b) (ii) State one assumption you have made that could affect your answer to part **(b)(i)**.
[1 mark]

Her resistance force would decrease as she slows down.

Turn over for the next question

Do not write outside the box

16 A remote-controlled toy car is moving over a horizontal surface. It moves in a straight line through a point *A*.

The toy is initially at the point with displacement 3 metres from *A*. Its velocity, v m s^{-1}, at time t seconds is defined by

$$v = 0.06(2 + t - t^2)$$

16 (a) Find an expression for the displacement, r metres, of the toy from *A* at time t seconds.

[4 marks]

$r = \int v \, dt = \int 0.06(2 + t - t^2) \, dt$

$r = 0.06(2t + \frac{t^2}{2} - \frac{t^3}{3}) + c$

At $t=0$, $r=3$ so $c=3$.

$r = 0.06(2t + \frac{t^2}{2} - \frac{t^3}{3}) + 3$

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16 (b) **In this question use $g = 9.8\,\text{m}\,\text{s}^{-2}$**

At time $t = 2$ seconds, the toy launches a ball which travels directly upwards with initial speed $3.43\,\text{m}\,\text{s}^{-1}$

Find the time taken for the ball to reach its highest point.

[3 marks]

$s = -$

$u = 3.43$

$v = 0$

$a = -9.8$

$t = t$

$v = u + at$

$0 = 3.43 - 9.8t$

$t = \frac{3.43}{9.8}$

$t = 0.35$ seconds

END OF QUESTIONS

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