Mark Scheme (Results)

Summer 2018

Pearson Edexcel GCE Further Mathematics

AS Further Core Pure Mathematics Paper 8FM0_01

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Summer 2018
Publications Code 8FM0_01_1806_MS

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the last candidate in exactly the same way as they mark the first.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification/indicative content will not be exhaustive.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 80.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⌈ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.
If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

Question	Scheme	Marks	AOs
1(a)	$\mathbf{M}^{-1}=\frac{1}{69}\begin{pmatrix}1 & 13 & 5\\ -11 & -5 & 14\\ -26 & 7 & 8\end{pmatrix}$	B1 B1	1.1b 1.1b
		(2)	
(b)	$\frac{1}{69}\begin{pmatrix}1 & 13 & 5\\ -11 & -5 & 14\\ -26 & 7 & 8\end{pmatrix}\begin{pmatrix}-4\\ 9\\ 5\end{pmatrix}=\ldots$	M1	1.1b
	$x=2, y=1, z=3$ or $(2,1,3)$ or $2\mathbf{i}+\mathbf{j}+3\mathbf{k}$ or $\begin{pmatrix}2\\ 1\\ 3\end{pmatrix}$	A1	1.1b
		(2)	
(c)	The **point** where three **planes** meet	B1ft	2.2a
		(1)	
		(5 marks)	

Notes

(a)
B1: Evidence that the determinant is ± 69 (may be implied by their matrix e.g. where entries are not in exact form: $\pm\begin{pmatrix}0.014 & 0.188 & 0.072\\ -0.159 & -0.072 & 0.203\\ -0.377 & 0.101 & 0.116\end{pmatrix}$)(Should be mostly correct)

Must be seen in part (a).
B1: Fully correct inverse with all elements in **exact** form
(b)
M1: Any complete method to find the values of x, y and z (Must be using **their inverse** if using the method in the main scheme)
A1: Correct coordinates
A solution not using the inverse requires a complete method to find values for x, y and z for the method mark.
Correct coordinates only scores both marks.
(c)
B1: Describes the correct geometrical configuration.
Must include the two ideas of **planes** and **meet in a point** with no contradictory statements.
This is dependent on having obtained a unique point in part (b)

Question	Scheme	Marks	AOs
2	$w = 2z+1 \Rightarrow z = \frac{w-1}{2}$	B1	3.1a
	$\left(\frac{w-1}{2}\right)^3 - 3\left(\frac{w-1}{2}\right)^2 + \left(\frac{w-1}{2}\right) + 5 = 0$	M1	3.1a
	$\frac{1}{8}\left(w^3 - 3w^2 + 3w - 1\right) - \frac{3}{4}\left(w^2 - 2w + 1\right) + \frac{w-1}{2} + 5 = 0$		
	$w^3 - 9w^2 + 19w + 29 = 0$	M1 A1 A1	1.1b 1.1b 1.1b
		(5)	
ALT 1	$\alpha + \beta + \gamma = 3, \alpha\beta + \beta\gamma + \alpha\gamma = 1, \alpha\beta\gamma = -5$	B1	3.1a
	New sum = $2(\alpha+\beta+\gamma) + 3 = 9$ New pair sum = $4(\alpha\beta + \beta\gamma + \gamma\alpha) + 4(\alpha+\beta+\gamma) + 3 = 19$ New product = $8\alpha\beta\gamma + 4(\alpha\beta + \beta\gamma + \gamma\alpha) + 2(\alpha+\beta+\gamma) + 1 = -29$	M1	3.1a
	$w^3 - 9w^2 + 19w + 29 = 0$	M1 A1 A1	1.1b 1.1b 1.1b
		(5)	
		(5 marks)	

Notes

B1: Selects the method of making a connection between z and w by writing $z = \frac{w-1}{2}$

M1: Applies the process of substituting their $z = \frac{w-1}{2}$ into $z^3 - 3z^2 + z + 5 = 0$

(Allow $z = 2w + 1$)

M1: Manipulates their equation into the form $w^3 + pw^2 + qw + r (= 0)$ having substituted their z in terms of w. Note that the "= 0" can be missing for this mark.

A1: At least two of p, q, r correct. Note that the "= 0" can be missing for this mark.

A1: Fully correct equation including "= 0"

The first 4 marks are available if another letter is used instead of w but the final answer must be in terms of w.

ALT1

B1: Selects the method of giving three correct equations containing α, β and γ

M1: Applies the process of finding the new sum, new pair sum, new product

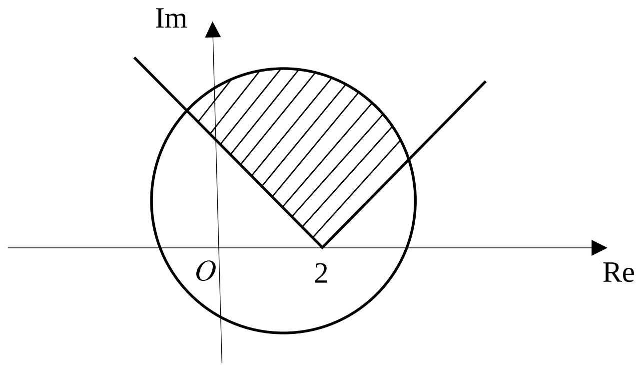
M1: Applies $w^3 - (\text{new sum})w^2 + (\text{new pair sum})w - (\text{new product})(= 0)$

or identifies p as –(new sum) q as (new pair sum) and r as –(new product)

A1: At least two of p, q, r correct.

A1: Fully correct equation including "= 0"

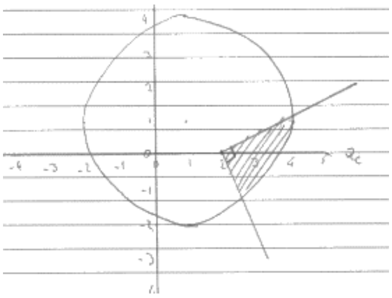
The first 4 marks are available if another letter is used instead of w but the final answer must be in terms of w.

Question	Scheme	Marks	AOs
3(a)		M1	1.1b
		M1	1.1b
		A1	2.2a
		M1	3.1a
		A1	1.1b
		(5)	
(b)	$(x-1)^2+(y-1)^2=9,\ \ y=x-2 \Rightarrow x=...,\text{ or } y=...$	M1	3.1a
	$x=2+\frac{\sqrt{14}}{2}, y=\frac{\sqrt{14}}{2}$	A1	1.1b
	$\lvert w\rvert^2=\left(2+\frac{\sqrt{14}}{2}\right)^2+\left(\frac{\sqrt{14}}{2}\right)^2$	M1	1.1b
	$=11+2\sqrt{14}$	A1	1.1b
		(4)	
			(9 marks)

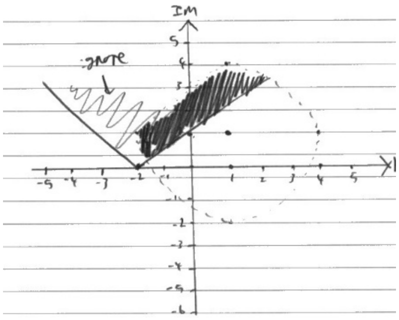
Notes

(a)
M1: Circle or arc of a circle with centre in first quadrant and with the circle in all 4 quadrants or arc of circle in quadrants 1 and 2
M1: A "V" shape i.e. with both branches above the x-axis and with the vertex on the positive real axis. Ignore any branches below the x-axis.
A1: Two half lines that meet on the positive real axis where the right branch intersects the circle or arc of a circle in the first quadrant and the left branch intersects the circle or arc of a circle in the second quadrant but not on the y-axis.
M1: Shades the region between the half-lines and within the circle
A1: Cso. A fully correct diagram including 2 marked (or implied by ticks) at the vertex on the real axis with the correct region shaded and all the previous marks scored.
(b)
M1: Identifies a suitable strategy for finding the x or y coordinate of the point of intersection.
Look for an attempt to solve equations of the form $(x\pm1)^2+(y\pm1)^2=9\text{ or }3$ and $y=\pm x\pm 2$
A1: Correct coordinates for the intersection (there may be other points but allow this mark if the correct coordinates are seen). (The correct coordinates may be implied by subsequent work.)
Allow equivalent exact forms and allow as a complex number e.g. $2+\frac{\sqrt{14}}{2}+\frac{\sqrt{14}}{2}\mathrm{i}$
M1: Correct use of Pythagoras on their coordinates (There must be no i's)
A1: Correct **exact** value by cso
Note that solving $(x-1)^2+(y-1)^2=9,\ \ y=x+2$ gives $x=\frac{\sqrt{14}}{2}, y=2+\frac{\sqrt{14}}{2}$ and hence the correct answer fortuitously so scores M1A0M1A0

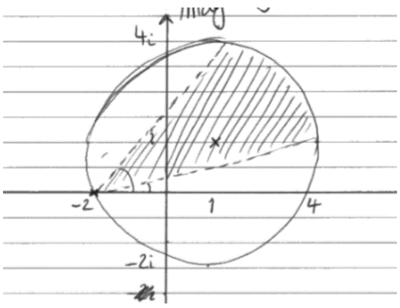
Example marking for 3(a)



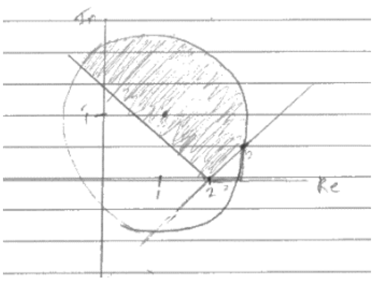
M1: Circle with centre in first quadrant
M0: The branches of the "V" must be above the x-axis
A0: Follows M0
M1: Shades the region between the half-lines and within the circle
A0: Depends on all previous marks



M1: Circle with centre in first quadrant
M0: The vertex of the "V" must be on the positive x-axis
A0: Follows M0
M1: Shades the region between the half-lines and within the circle (BOD)
A0: Depends on all previous marks



M1: Circle with centre in first quadrant
M0: The vertex of the "V" must be on the positive x-axis
A0: Follows M0
M1: Shades the region between the half-lines and within the circle
A0: Depends on all previous marks



M1: Circle with centre in first quadrant
M1: A "V" shape i.e. with both branches above the x-axis and with the vertex on the positive real axis. Ignore any branches below the x-axis.
A1: Two half lines that meet on the positive real axis where the right branch intersects the circle in the first quadrant and the left branch intersects the circle in the second quadrant.
M1: Shades the region between the half-lines and within the circle
A1: A fully correct diagram including 2 marked at the vertex on the real axis with the correct region shaded and all the previous marks scored.

Question	Scheme	Marks	AOs
4(a)	Attempts the scalar product between the direction of W and the normal to the road and uses trigonometry to find an angle.	M1	3.1a
	$\left(\begin{pmatrix}1\\2\\-3\end{pmatrix}-\begin{pmatrix}-1\\-1\\-3\end{pmatrix}\right)\bullet\begin{pmatrix}3\\-5\\-18\end{pmatrix}=-9$ or $\left(\begin{pmatrix}-1\\-1\\-3\end{pmatrix}-\begin{pmatrix}1\\2\\-3\end{pmatrix}\right)\bullet\begin{pmatrix}3\\-5\\-18\end{pmatrix}=9$	M1 A1	1.1b 1.1b
	$\sqrt{(2)^2+(3)^3+(0)^2}\sqrt{(3)^2+(-5)^3+(-18)^2}\cos\alpha="-9"$ $\theta=90-\arccos\left(\frac{9}{\sqrt{13}\sqrt{358}}\right)$ or $\theta=\arcsin\left(\frac{9}{\sqrt{13}\sqrt{358}}\right)$ Angle between pipe and road $=7.58^\circ$ (3sf) or 0.132 radians (3sf) (Allow -7.58° or -0.132 radians)	M1 A1	1.1b 3.2a
		(5)	
(b)	$W:\begin{pmatrix}-1\\-1\\-3\end{pmatrix}+t\begin{pmatrix}2\\3\\0\end{pmatrix}$ or $\begin{pmatrix}1\\2\\-3\end{pmatrix}+\lambda\begin{pmatrix}2\\3\\0\end{pmatrix}$	B1ft	1.1b
	$C\text{ to }W:\left\{\begin{pmatrix}-1\\-1\\-3\end{pmatrix}+t\begin{pmatrix}2\\3\\0\end{pmatrix}-\begin{pmatrix}-1\\-2\\0\end{pmatrix}\right\}$ or $\left\{\begin{pmatrix}1\\2\\-3\end{pmatrix}+\lambda\begin{pmatrix}2\\3\\0\end{pmatrix}-\begin{pmatrix}-1\\-2\\0\end{pmatrix}\right\}$	M1	3.4
	$\begin{pmatrix}2t\\3t+1\\-3\end{pmatrix}\bullet\begin{pmatrix}2\\3\\0\end{pmatrix}=0\Rightarrow t=...$ or $\begin{pmatrix}2+2\lambda\\4+3\lambda\\-3\end{pmatrix}\bullet\begin{pmatrix}2\\3\\0\end{pmatrix}=0\Rightarrow\lambda=...$ or $(2t)^2+(3t+1)^2+(-3)^2=...$ or $(2+2t)^2+(4+3t)^2+(-3)^2=...$	M1	3.1b
	$t=-\frac{3}{13}$ or $\lambda=-\frac{16}{13}\Rightarrow(C\text{ to }W)_{\min}$ is $-\frac{6}{13}\mathbf{i}+\frac{4}{13}\mathbf{j}-3\mathbf{k}$ or $(2t)^2+(3t+1)^2+(-3)^2=13\left(t+\frac{3}{13}\right)^2+\frac{121}{13}$ or $(2+2t)^2+(4+3t)^2+(-3)^2=13\left(\lambda+\frac{16}{13}\right)^2+\frac{121}{13}$ or $\frac{\mathrm{d}\left((2t)^2+(3t+1)^2+(-3)^2\right)}{\mathrm{d}t}=0\Rightarrow t=-\frac{3}{13}\Rightarrow C\text{ to }W$ is $-\frac{6}{13}\mathbf{i}+\frac{4}{13}\mathbf{j}-3\mathbf{k}$ Or $\frac{\mathrm{d}\left((2+2t)^2+(4+3t)^2+(-3)^2\right)}{\mathrm{d}t}=0\Rightarrow t=-\frac{16}{13}\Rightarrow(C\text{ to }W)_{\min}$ is $-\frac{6}{13}\mathbf{i}+\frac{4}{13}\mathbf{j}-3\mathbf{k}$	A1	1.1b
	$d=\sqrt{\left(-\frac{6}{13}\right)^2+\left(\frac{4}{13}\right)^2+(-3)^2}$ or $d=\sqrt{\frac{121}{13}}$	**dd**M1	1.1b

	Shortest length of pipe needed is 305 or 305 cm or 3.05 m	A1	3.2a
		(6)	
			(11 marks)

Notes

(a)

M1: Realises the scalar product between the direction of W and the normal to the road is needed and so applies it and uses trigonometry to find an angle

M1: Calculates the scalar product between $\pm\left(\begin{pmatrix}1\\2\\-3\end{pmatrix}-\begin{pmatrix}-1\\-1\\-3\end{pmatrix}\right)$ and $\pm\begin{pmatrix}3\\-5\\-18\end{pmatrix}$ (Allow sign slips as long as the intention is clear)

A1: $\begin{pmatrix}2\\3\\0\end{pmatrix}\bullet\begin{pmatrix}3\\-5\\-18\end{pmatrix}=-9$ or $\begin{pmatrix}-2\\-3\\0\end{pmatrix}\bullet\begin{pmatrix}3\\-5\\-18\end{pmatrix}=9$ or $\begin{pmatrix}2\\3\\0\end{pmatrix}\bullet\begin{pmatrix}-3\\5\\18\end{pmatrix}=9$ or $\begin{pmatrix}-2\\-3\\0\end{pmatrix}\bullet\begin{pmatrix}-3\\5\\18\end{pmatrix}=-9$

M1: A fully complete and correct method for obtaining the acute angle

A1: Awrt 7.58° or awrt 0.132 radians **(must see units)**. Do not isw and withhold this mark if extra answers are given.

(b)

B1ft: Forms the correct parametric form for the pipe W. Follow through their direction vector for W from part (a).

M1: Identifies the need to and forms a vector connecting C to W using a parametric form for W

M1: Uses the model to form the scalar product of C to W and the direction of W to find the value of their parameter **or** finds the distance C to W or $(C \text{ to } W)^2$ in terms of their parameter

A1: Correct vector or correct completion of the square

ddM1: Correct use of Pythagoras on their vector CW or appropriate method to find the shortest distance between the point and the pipe. **Dependent on both previous method marks.**

A1: Correct length for the required section of pipe is 305 or 305 cm or 3.05 m

Alternatives for part (b):

4(b) Way 2	$\mathbf{AC}=\begin{pmatrix}0\\1\\-3\end{pmatrix},\quad \mathbf{AB}=\begin{pmatrix}2\\3\\0\end{pmatrix}$	B1ft	1.1b
	$\mathbf{AC.AB}=\begin{pmatrix}0\\1\\-3\end{pmatrix}.\begin{pmatrix}2\\3\\0\end{pmatrix}=3$	M1	3.4
	$\Rightarrow \cos CAB=\dfrac{3}{\sqrt{10}\sqrt{13}}\Rightarrow CAB=...$	M1	3.1b
	$CAB=74.74....^\circ$	A1	1.1b
	$d=\sqrt{10}\sin 74.74....^\circ$	**dd**M1	1.1b
	Shortest length of pipe needed is 305 or 305 cm or 3.05 m	A1	3.2a
		(6)	

	Notes		
	(b) B1ft: Forms the correct vectors. Follow through their direction vector for *W* from part (a). M1: Identifies the need to and forms the scalar product between **AC** and **AB** M1: Uses the model to form the scalar product and uses this to find the angle *CAB* A1: Correct angle ddM1: Correct method using their values or appropriate method to find the shortest distance between the point and the pipe. **Dependent on both previous method marks.** A1: Correct length for the required section of pipe is 305 or 305 cm or 3.05 m		

4(b) Way 3	$\mathbf{AC} = \begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix}, \quad \mathbf{AB} = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}$	B1ft	1.1b
	$\mathbf{AC} \times \mathbf{AB} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -1 & 3 \\ 2 & 3 & 0 \end{vmatrix} = \begin{pmatrix} -9 \\ 6 \\ 2 \end{pmatrix}$	M1	3.4
	$\lvert\mathbf{AC} \times \mathbf{AB}\rvert = \sqrt{9^2 + 6^2 + 2^2} = ...$	M1	3.1b
	$= 11$	A1	1.1b
	$d = \frac{11}{\lvert\mathbf{AB}\rvert} = \frac{11}{\sqrt{2^2 + 3^2}} = ...$	**dd**M1	1.1b
	Shortest length of pipe needed is 305 or 305 cm or 3.05 m	A1	3.2a
		(6)	
	Notes		
	(b) B1ft: Forms the correct vectors. Follow through their direction vector for *W* from part (a). M1: Identifies the need to and forms the vector product between **AC** and **AB** M1: Uses the model to find the magnitude of their vector product A1: Correct value ddM1: Correct method using their values or appropriate method to find the shortest distance between the point and the pipe. **Dependent on both previous method marks.** A1: Correct length for the required section of pipe is 305 or 305 cm or 3.05 m		

Question	Scheme	Marks	AOs
5(a)	Rotation	B1	1.1b
	120 degrees (anticlockwise) or $\frac{2\pi}{3}$ radians (anticlockwise) Or 240 degrees clockwise or $\frac{4\pi}{3}$ radians clockwise	B1	2.5
	About (from) the origin. Allow (0, 0) or O for origin.	B1	1.2
		(3)	
(b)	$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	B1	1.1b
		(1)	
(c)	$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix} = \begin{pmatrix} \dots & \dots \\ \dots & \dots \end{pmatrix}$	M1	1.1b
	$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix} = \begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$	A1ft	1.1b
		(2)	
(d)	$\begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}\begin{pmatrix} 1 \\ k \end{pmatrix} = \begin{pmatrix} 1 \\ k \end{pmatrix} = \dots$ or $\begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} = \dots$ Note: $\begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}\begin{pmatrix} 1 \\ k \end{pmatrix} = \begin{pmatrix} -\frac{\sqrt{3}}{2}+\frac{1}{2}k \\ \frac{1}{2}+\frac{\sqrt{3}}{2}k \end{pmatrix} = \begin{pmatrix} 1 \\ k \end{pmatrix}$ can score M1 (for the matrix equation) but needs an equation to be "extracted" to score the next A1	M1	3.1a
	$-\frac{\sqrt{3}}{2}+\frac{1}{2}k=1$ or $\frac{1}{2}+\frac{\sqrt{3}}{2}k=k$ or $x=-\frac{\sqrt{3}}{2}x+\frac{1}{2}y$ or $y=\frac{1}{2}x+\frac{\sqrt{3}}{2}y$ (Note that candidates may then substitute x = 1 which is acceptable)	A1ft	1.1b
	$-\frac{\sqrt{3}}{2}+\frac{1}{2}k=1$ or $x=-\frac{\sqrt{3}}{2}x+\frac{1}{2}y \Rightarrow k=2+\sqrt{3}\left(\text{or } \frac{1}{2-\sqrt{3}}\right)$	A1	1.1b
	$\frac{1}{2}+\frac{\sqrt{3}}{2}k=k$ or $y=\frac{1}{2}x+\frac{\sqrt{3}}{2}y \Rightarrow k=2+\sqrt{3}\left(\text{or } \frac{1}{2-\sqrt{3}}\right)$	B1	1.1b
		(4)	
			(10 marks)

Notes
(a) B1: Identifies the transformation as a rotation B1: Correct angle. Allow equivalents in degrees or radians. B1: Identifies the origin as the centre of rotation These marks can only be awarded as the elements of a **single transformation** (b) B1: Shows the correct matrix in the correct form (c) M1: Multiplies the matrices in the correct order (evidence of multiplication can be taken from 3 correct or 3 correct ft elements) A1ft: Correct matrix (follow through their matrix from part (b)) A correct matrix or a correct follow through matrix implies both marks. (d) M1: Translates the problem into a matrix multiplication to obtain at least one equation in k or in x and y A1ft: Obtains one correct equation (follow through their matrix from part (c)) A1: Correct value for k in any form B1: Checks their answer by independently solving both equations **correctly** to obtain $2+\sqrt{3}$ both times or substitutes $2+\sqrt{3}$ into the other equation to confirm its validity

Question	Scheme	Marks	AOs
6(a)	$(3r-2)^2 = 9r^2 - 12r + 4$	B1	1.1b
	$\sum_{r=1}^{n}(9r^2-12r+4) = 9\times\frac{1}{6}n(n+1)(2n+1) - 12\times\frac{1}{2}n(n+1)+...$	M1	2.1
	$= 9\times\frac{1}{6}n(n+1)(2n+1) - 12\times\frac{1}{2}n(n+1)+4n$	A1	1.1b
	$= \frac{1}{2}n\left[3(n+1)(2n+1)-12(n+1)+8\right]$	dM1	1.1b
	$= \frac{1}{2}n\left[6n^2-3n-1\right]$*	A1*	1.1b
		(5)	
(b)	$\sum_{r=5}^{n}(3r-2)^2 = \frac{1}{2}n(6n^2-3n-1) - \frac{1}{2}(4)(6(4)^2-3\times4-1)$	M1	3.1a
	$\sum_{r=1}^{28} r\cos\left(\frac{r\pi}{2}\right) = 0-2+0+4+0-6+0+8+0-10+0+12+...$	M1	3.1a
	$3n^3 - \frac{3}{2}n^2 - \frac{1}{2}n - 166 + 103\times14 = 3n^3$ $\Rightarrow 3n^2+n-2552=0$	A1	1.1b
	$\Rightarrow 3n^2+n-2552=0 \Rightarrow n = ...$	M1	1.1b
	$n = 29$	A1	2.3
		(5)	
			(10 marks)

Notes

(a) Do not allow proof by induction (but the B1 could score for $(3r-2)^2 = 9r^2 - 12r + 4$ if seen)

B1: Correct expansion
M1: **Substitutes** at least one of the standard formulae into their expanded expression
A1: Fully correct expression
dM1: Attempts to factorise $\frac{1}{2}n$ having used at least one standard formula correctly. Dependent on the first M mark and dependent on there being an n in all terms.
A1*: Obtains the printed result with no errors seen
(b)
M1: Uses the result from part (a) by substituting n = 4 and subtracts from the result in (a) in order to find the first sum in terms of n.
M1: Identifies the periodic nature of the second sum by calculating terms. This may be implied by a sum of 14.
A1: Uses their sum and the given result to form the correct 3 term quadratic
M1: Solves their three term quadratic to obtain at least one value for n
A1: Obtains n = 29 only or obtains n = 29 and $n = -\frac{88}{3}$ and rejects the $-\frac{88}{3}$

Question	Scheme	Marks	AOs
7	Complex roots are e.g. $\alpha \pm \beta\text{i}$ or $\left(z^3+z^2+pz+q\right)\div\left(z-3\right)=z^2+4z+p+12$ or $\text{f}\left(3\right)=0\Rightarrow 3^3+3^2+3p+q=0$ or One of: $3+z_2+z_3=-1,\ \ 3z_2z_3=-q,\ \ 3z_2+3z_3+z_2z_3=p$	B1	3.1a
	Sum of roots $\alpha+\beta i+\alpha-\beta i+3=-1\Rightarrow\alpha=...$ or $\alpha+\beta i+\alpha-\beta i=-4\Rightarrow\alpha=...$	M1	1.1b
	$\alpha=-2$	A1	1.1b
	So $\frac{1}{2}\times 2\beta\times 5=35\Rightarrow\beta=7$	M1	1.1b
	$q=-3\left(\text{"}-2+7\text{i"}\right)\left(\text{"}-2-7\text{i"}\right)=...$ or $p=3\left(\text{"}-2+7\text{i"}\right)+3\left(\text{"}-2-7\text{i"}\right)+\left(\text{"}-2+7\text{i"}\right)\left(\text{"}-2-7\text{i"}\right)$ or $\left(z-3\right)\left(z-\left(\text{"}-2+7\text{i"}\right)\right)\left(z-\left(\text{"}-2-7\text{i"}\right)\right)=...$	M1	3.1a
	$q=-159$ or $p=41$	A1	1.1b
	$3p+q=-36\Rightarrow p=\frac{-36-q}{3}=41$ and $q=-159$	A1	1.1b
		(7)	
	Alternative		
	$\left(z^3+z^2+pz+q\right)\div\left(z-3\right)=z^2+4z+p+12$	B1	3.1a
	$z^2+4z+p+12=0\Rightarrow z=\frac{-4\pm\sqrt{4^2-4\left(p+12\right)}}{2}\left(=-2\pm\text{i}\sqrt{p+8}\right)$	M1	1.1b
	$\alpha=-2$	A1	1.1b
	$\beta=\sqrt{p+8}$	M1	1.1b
	$\frac{1}{2}\times\left(3+2\right)\times 2\sqrt{p+8}=35\Rightarrow p=...$	M1	3.1a
	$p=41$	A1	1.1b
	$3p+q=-36\Rightarrow q=-159$	A1	1.1b
		(7)	
			(7 marks)

Notes
B1: Recognises that the other roots must form a conjugate pair **or** obtains $z^2+4z+p+12$ (or $z^2+4z-\frac{q}{3}$) as the quadratic factor **or** writes down a correct equation for p and q **or** writes down a correct equation involving "z_2" and "z_3"
M1: Uses the sum of the roots of the cubic or the sum of the roots of their quadratic to find a value for "α"
A1: Correct value for "α"
M1: Uses their value for "α" and the given area to find a value for "β". Must be using the area and triangle dimensions correctly e.g. $\frac{1}{2}\times\beta\times5=35\Rightarrow\beta=14$ scores M0
M1: Uses an appropriate method to find p or q
A1: A correct value for p or q
A1: Correct values for p and q
Alternative
B1: Obtains $z^2+4z+p+12$ (or $z^2+4z-\frac{q}{3}$) as the quadratic factor
M1: Solves their quadratic factor by completing the square or using the quadratic formula
A1: Correct value for "α"
M1: Uses their imaginary part to find "β" in terms of p
M1: Draws together the fact that the imaginary parts of their complex conjugate pair and the real root form the sides of the required triangle and forms an equation in terms of p, sets equal to 35 and solves for p
A1: A correct value for p or q
A1: Correct values for p and q

Question	Scheme	Marks	AOs
8(i)	$n=1, \begin{pmatrix}5 & -8\\ 2 & -3\end{pmatrix}^1 = \begin{pmatrix}5 & -8\\ 2 & -3\end{pmatrix}, \quad \begin{pmatrix}4\times1+1 & -8(1)\\ 2\times1 & 1-4(1)\end{pmatrix} = \begin{pmatrix}5 & -8\\ 2 & -3\end{pmatrix}$ So the result is true for $n = 1$	B1	2.2a
	Assume true for $n = k$ so $\begin{pmatrix}5 & -8\\ 2 & -3\end{pmatrix}^k = \begin{pmatrix}4k+1 & -8k\\ 2k & 1-4k\end{pmatrix}$	M1	2.4
	$\begin{pmatrix}5 & -8\\ 2 & -3\end{pmatrix}^{k+1} = \begin{pmatrix}4k+1 & -8k\\ 2k & 1-4k\end{pmatrix}\begin{pmatrix}5 & -8\\ 2 & -3\end{pmatrix}$ or $\begin{pmatrix}5 & -8\\ 2 & -3\end{pmatrix}^{k+1} = \begin{pmatrix}5 & -8\\ 2 & -3\end{pmatrix}\begin{pmatrix}4k+1 & -8k\\ 2k & 1-4k\end{pmatrix}$	M1	1.1b
	$\begin{pmatrix}4k+1 & -8k\\ 2k & 1-4k\end{pmatrix}\begin{pmatrix}5 & -8\\ 2 & -3\end{pmatrix} = \begin{pmatrix}5(4k+1)-16k & -8(4k+1)+24k\\ 10k+2(1-4k) & -16k-3(1-4k)\end{pmatrix}$ or $\begin{pmatrix}5 & -8\\ 2 & -3\end{pmatrix}\begin{pmatrix}4k+1 & -8k\\ 2k & 1-4k\end{pmatrix} = \begin{pmatrix}5(4k+1)-16k & -40k-8(1-4k)\\ 2(1+4k)-6k & -16k-3(1-4k)\end{pmatrix}$	A1	1.1b
	$= \begin{pmatrix}4(k+1)+1 & -8(k+1)\\ 2(k+1) & 1-4(k+1)\end{pmatrix}$	A1	2.1
	<u>If true for $n = k$</u> then <u>true for $n = k + 1$</u>, <u>true for $n = 1$</u> so <u>true for all (positive integers) n</u> (Allow "for all values")	A1	2.4
		(6)	
(ii) **Way 1**	$f(k + 1) – f(k)$		
	When $n = 1$, $4^{n+1} + 5^{2n-1} = 16 + 5 = 21$ so the statement is true for $n = 1$	B1	2.2a
	Assume true for $n = k$ so $4^{k+1} + 5^{2k-1}$ is divisible by 21	M1	2.4
	$f(k+1) - f(k) = 4^{k+2} + 5^{2k+1} - 4^{k+1} - 5^{2k-1}$	M1	2.1
	$= 4\times4^{k+1} + 25\times5^{2k-1} - 4^{k+1} - 5^{2k-1}$		
	$= 3f(k) + 21\times5^{2k-1}$ or e.g. $= 24f(k) - 21\times4^{k+1}$	A1	1.1b
	$f(k+1) = 4f(k) + 21\times5^{2k-1}$ or e.g. $f(k+1) = 25f(k) - 21\times4^{k+1}$	A1	1.1b
	<u>If true for $n = k$</u> then <u>true for $n = k + 1$</u>, <u>true for $n = 1$</u> so <u>true for all (positive integers) n</u> (Allow "for all values")	A1	2.4
		(6)	

(ii) **Way 2**	f(k + 1)		
	When n = 1, $4^{n+1}+5^{2n-1}=16+5=21$ so the statement is true for n = 1	B1	2.2a
	Assume true for n = k so $4^{k+1}+5^{2k-1}$ is divisible by 21	M1	2.4
	$\mathrm{f}\left(k+1\right)=4^{k+1+1}+5^{2(k+1)-1}$	M1	2.1
	$\mathrm{f}\left(k+1\right)=4\times4^{k+1}+5^{2k+1}=4\times4^{k+1}+4\times5^{2k-1}+25\times5^{2k-1}-4\times5^{2k-1}$ $\mathrm{f}\left(k+1\right)=4\mathrm{f}\left(k\right)+21\times5^{2k-1}$	A1 A1	1.1b 1.1b
	If true for $n = k$ then true for $n = k + 1$, true for $n = 1$ so true for all (positive integers) n (Allow "for all values")	A1	2.4
		(6)	
(ii) **Way 3**	f(k + 1) – mf(k)		
	When n = 1, $4^{n+1}+5^{2n-1}=16+5=21$ so the statement is true for n = 1	B1	2.2a
	Assume true for n = k so $4^{k+1}+5^{2k-1}$ is divisible by 21	M1	2.4
	$\mathrm{f}\left(k+1\right)-m\mathrm{f}\left(k\right)=4^{k+2}+5^{2k+1}-m\left(4^{k+1}+5^{2k-1}\right)$	M1	2.1
	$=\left(4-m\right)4^{k+1}+5^{2k+1}-m\times5^{2k-1}$ $=\left(4-m\right)\left(4^{k+1}+5^{2k-1}\right)+21\times5^{2k-1}$	A1	1.1b
	$=\left(4-m\right)\left(4^{k+1}+5^{2k-1}\right)+21\times5^{2k-1}+m\mathrm{f}\left(k\right)$	A1	1.1b
	If true for $n = k$ then true for $n = k + 1$, true for $n = 1$ so true for all (positive integers) n (Allow "for all values")	A1	2.4
		(6)	
(ii) **Way 4**	f(k) = 21M		
	When n = 1, $4^{n+1}+5^{2n-1}=16+5=21$ so the statement is true for n = 1	B1	2.2a
	Assume true for n = k so $4^{k+1}+5^{2k-1}=21M$	M1	2.4
	$\mathrm{f}\left(k+1\right)=4^{k+1+1}+5^{2(k+1)-1}$	M1	2.1
	$\mathrm{f}\left(k+1\right)=4\times4^{k+1}+5^{2k+1}=4\left(21M-5^{2k-1}\right)+5^{2k+1}$ $\mathrm{f}\left(k+1\right)=84M+21\times5^{2k-1}$	A1 A1	1.1b 1.1b
	If true for $n = k$ then true for $n = k + 1$, true for $n = 1$ so true for all (positive integers) n (Allow "for all values")	A1	2.4
		(6)	
			(12 marks)

Notes
(i) B1: Shows that the result holds for n = 1. Must see **substitution** into the rhs. The minimum would be: $\begin{pmatrix} 4+1 & -8 \\ 2 & 1-4 \end{pmatrix} = \begin{pmatrix} 5 & -8 \\ 2 & -3 \end{pmatrix}$. M1: Makes a statement that assumes the result is true for some value of n (Assume (true for) $n = k$ is sufficient – note that this may be recovered in their conclusion if they say e.g. if true for $n = k$ then … etc.) M1: Sets up a correct multiplication statement either way round A1: Achieves a correct un-simplified matrix A1: Reaches a correct simplified matrix with no errors **<u>and the correct un-simplified matrix seen previously.</u>** Note that the simplified result may be proved by equivalence. A1: Correct conclusion. This mark is dependent on all previous marks apart from the B mark. It is gained by conveying the ideas of **all** four underlined points **either** at the end of their solution **or** as a narrative in their solution. (ii) **Way 1** B1: Shows that f(1) = 21 M1: Makes a statement that assumes the result is true for some value of n (Assume (true for) $n = k$ is sufficient – note that this may be recovered in their conclusion if they say e.g. if true for $n = k$ then … etc.) M1: Attempts f(k + 1) – f(k) or equivalent work A1: Achieves a correct expression for f(k + 1) – f(k) in terms of f(k) A1: Reaches a correct expression for f(k + 1) in terms of f(k) A1: Correct conclusion. This mark is dependent on all previous marks apart from the B mark. It is gained by conveying the ideas of **all** four underlined points **either** at the end of their solution **or** as a narrative in their solution. **Way 2** B1: Shows that f(1) = 21 M1: Makes a statement that assumes the result is true for some value of n (Assume (true for) $n = k$ is sufficient – note that this may be recovered in their conclusion if they say e.g. if true for $n = k$ then … etc.) M1: Attempts f(k + 1) A1: Correctly obtains 4f(k) **or** $21\times5^{2k-1}$ A1: Reaches a correct expression for f(k + 1) in terms of f(k) A1: Correct conclusion. This mark is dependent on all previous marks apart from the B mark. It is gained by conveying the ideas of **all** four underlined points **either** at the end of their solution **or** as a narrative in their solution. **Way 3** B1: Shows that f(1) = 21 M1: Makes a statement that assumes the result is true for some value of n (Assume (true for) $n = k$ is sufficient – note that this may be recovered in their conclusion if they say e.g. if true for $n = k$ then … etc.) M1: Attempts f(k + 1) – mf(k) A1: Achieves a correct expression for f(k + 1) – mf(k) in terms of f(k) A1: Reaches a correct expression for f(k + 1) in terms of f(k) A1: Correct conclusion. This mark is dependent on all previous marks apart from the B mark. It is gained by conveying the ideas of **all** four underlined points **either** at the end of their solution **or** as a narrative in their solution.

Way 4
B1: Shows that f(1) = 21
M1: Makes a statement that assumes the result is true for some value of n (Assume (true for) $n = k$ is sufficient – note that this may be recovered in their conclusion if they say e.g. if true for $n = k$ then … etc.)
M1: Attempts f(k + 1)
A1: Correctly obtains 84M **or** $21\times5^{2k-1}$
A1: Reaches a correct expression for f(k + 1) in terms of M and 5^{2k-1}
A1: Correct conclusion. This mark is dependent on all previous marks apart from the B mark. It is gained by conveying the ideas of **all** four underlined points **either** at the end of their solution **or** as a narrative in their solution.

Question	Scheme	Marks	AOs
9(a)	$(4, 14),\ (1, 18) \Rightarrow 14 = a(4)^2 + b,\ 18 = a(1)^2 + b \Rightarrow a = ..., b = ...$	M1	3.3
	$a = -\frac{4}{15}, b = \frac{274}{15}$	A1	1.1b
		(2)	
(b)	$\pi \times 4^2 \times 14$ and $\pi \times 1^2 \times 10$	B1	1.1b
	$\pi \int x^2\,dy = \frac{\pi}{4}\int (274 - 15y)\,dy$	B1ft	1.1a
	$= \frac{\pi}{4}\int_{14}^{18} (274 - 15y)\,dy$	M1	3.3
	$= \frac{\pi}{4}\left[274y - \frac{15y^2}{2}\right]_{14}^{18}$	M1 A1	1.1b 1.1b
	$V = 234\pi + \frac{\pi}{4}\left[274(18) - \frac{15(18)^2}{2} - \left(274(14) - \frac{15(14)^2}{2}\right)\right]$	ddM1	3.4
	$V = 268\pi \approx 842\,\text{cm}^3$	A1	2.2b
		(7)	
(c)	Any one of e.g. The measurements may not be accurate The equation of the curve may not be a suitable model The bottom of the bottle may not be flat The thickness of the glass may not have been considered The glass may not be smooth This part asks for a limitation of the model so their answer must refer to e.g. : • The measuring of the dimensions • The model used for the curve • The simplified model (the thickness of glass, the simplified shape, smoothness of the glass etc.)	B1	3.5b
		(1)	
(d)	There are 2 criteria for this mark: • A comparison of their value to 750 e.g. larger, smaller, about the same or a difference **demonstrated** e.g. 810 – 750 = … but not **just** a percentage error or **just** a difference with no calculation • A conclusion that is consistent with their values e.g. this is not a good model, this is a good model etc. If they reach an answer that is less than 750, they need to conclude that it is not a good model If they reach an answer that is greater than 750 then look for a sensible comment that is consistent with their value	B1ft	3.5a
		(1)	
		(11 marks)	

Notes
(a) M1: Chooses (4, 14) and (1, 18) and substitutes into the equation modelling the curve to obtain at least one correct equation and attempts to find the values of *a* and *b*. A1: Infers from the data in the model, the values of *a* and *b* (b) B1: Correct expressions for the 2 cylindrical parts. May be seen as a sum or as separate cylinders. B1ft: Uses the model to obtain $\pi\int\left(\frac{y-\text{their } b}{\text{their } a}\right)\mathrm{d}y$ (Note that the π may be recovered later) M1: Chooses limits appropriate to the model i.e. 14 and 18 M1: Integrates to obtain an expression of the form $\alpha y+\beta y^2$ A1: Uses their model correctly to give $274y-\frac{15y^2}{2}$ ddM1: Uses the model to find the sum of their cylinders + their integrated volume. Must be a fully correct method here and is dependent on both previous method marks. So must have attempted the volumes of the cylinders "AHBG" and "CFED" and adds these to the magnitude of their integrated volume. A1: 268π or awrt 842 (c) B1: States an acceptable limitation of the model with no contradictory statements. (This is independent of part (b)) (d) B1ft: Compares the actual volume to their answer to part (b) and makes an assessment of the model with a reason with no contradictory statements.

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