Mark Scheme (Results)

Summer 2018

Pearson Edexcel GCE AS Mathematics

Pure Mathematics (8MA0/01)

Summer 2018
Publications Code 8MA0_01_1806_MS

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the last candidate in exactly the same way as they mark the first.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification/indicative content will not be exhaustive.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, a senior examiner must be consulted before a mark is awarded.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Instructions for Marking

1. The total number of marks for the paper is 100

2. These mark schemes use the following types of marks:

- M marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- B marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol √ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- o.e. – or equivalent (and appropriate)
- d or dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper or ag- answer given

4. All M marks are follow through.

A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but answers that don't logically make sense e.g. if an answer given for a probability is >1 or <0, should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response.

If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \ldots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \ldots$

2. Formula

Attempt to use correct formula (with values for *a*, *b* and *c*).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \ldots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given is that the formula should be quoted first.

Normal marking procedure is then as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

AS Mathematics

Paper 8MA0 01 June 2018 Mark Scheme

Question	Scheme	Marks	AOs
1	$\int\left(\frac{2}{3}x^3-6\sqrt{x}+1\right)\mathrm{d}x$		
	Attempts to integrate awarded for any correct power	M1	1.1a
	$\int\left(\frac{2}{3}x^3-6\sqrt{x}+1\right)\mathrm{d}x=\frac{2}{3}\times\frac{x^4}{4}+...+x$	A1	1.1b
	$=...-6\frac{x^{\frac{3}{2}}}{3/2}+....$	A1	1.1b
	$=\ \frac{1}{6}x^4-4x^{\frac{3}{2}}+x+c$	A1	1.1b
			(4 marks)

Notes

M1: Allow for raising power by one. $x^n\rightarrow x^{n+1}$
Award for any correct power including sight of $1x$

A1: Correct two '**non fractional power'** terms (may be un-simplified at this stage)

A1: Correct '**fractional power'** term (may be un-simplified at this stage)

A1: Completely correct, simplified and including constant of integration seen on one line.
Simplification is expected for full marks.

Accept correct exact equivalent expressions such as $\frac{x^4}{6}-4x\sqrt{x}+1x^1+c$

Accept $\frac{x^4-24x^{\frac{3}{2}}+6x}{6}+c$

Remember to isw after a correct answer.

Condone poor notation. Eg answer given as $\int\frac{1}{6}x^4-4x^{\frac{3}{2}}+x+c$

Question	Scheme	Marks	AOs
2(i)	$x^2-8x+17=(x-4)^2-16+17$	M1	3.1a
	$=(x-4)^2+1$ with comment (see notes)	A1	1.1b
	As $(x-4)^2\geqslant 0 \Rightarrow (x-4)^2+1\geqslant 1$ hence $x^2-8x+17>0$ for all x	A1	2.4
		(3)	
(ii)	For an explanation that it may not always be true Tests say $x=-5$ $(-5+3)^2=4$ whereas $(-5)^2=25$	M1	2.3
	States sometimes true and gives reasons Eg. when $x=5$ $(5+3)^2=64$ whereas $(5)^2=25$ True When $x=-5$ $(-5+3)^2=4$ whereas $(-5)^2=25$ Not true	A1	2.4
		(2)	
			(5 marks)

Notes

(i) Method One: Completing the Square

M1: For an attempt to complete the square. Accept $(x-4)^2$ **...**

A1: For $(x-4)^2+1$ with either $(x-4)^2\geqslant 0$**,** $(x-4)^2+1\geqslant 1$ or min at (4,1). Accept the inequality statements in words. Condone $(x-4)^2>0$ or a squared number is always positive for this mark.

A1: A fully written out solution, with correct statements and no incorrect statements. There must be a valid reason and a conclusion

……

$x^2-8x+17$

$=(x-4)^2+1\geqslant 1$ as $(x-4)^2\geqslant 0$

Hence $(x-4)^2+1>0$

scores M1 A1 A1

……

$x^2-8x+17>0$

$(x-4)^2+1>0$

This is true because $(x-4)^2\geqslant 0$ and when you add 1 it is going to be positive

scores M1 A1 A1

……

$x^2-8x+17>0$

$(x-4)^2+1>0$

which is true because a squared number is positive

scores M1 A1 A0

incorrect and incomplete

……

$x^2-8x+17=(x-4)^2+1$

Minimum is (4,1) so $x^2-8x+17>0$

scores M1 A1 A0

correct but not explained

……

$x^2-8x+17=(x-4)^2+1$

Minimum is (4,1) so as $1>0 \Rightarrow x^2-8x+17>0$

scores M1 A1 A1

correct and explained

……

$x^2 - 8x + 17 > 0$

$(x-4)^2 + 1 > 0$

scores M1 A0 (no explanation) A0

……

Method Two: Use of a discriminant

M1: Attempts to find the discriminant $b^2 - 4ac$ with a correct *a*, *b* and *c* which may be within a quadratic formula. You may condone missing brackets.

A1: Correct value of $b^2 - 4ac = -4$ **and** states or shows curve is U shaped (or intercept is (0,17)) or equivalent such as $+\text{ve}\ x^2$ etc

A1: Explains that as $b^2 - 4ac < 0$, there are no roots, and curve is U shaped then $x^2 - 8x + 17 > 0$

……

Method Three: Differentiation

M1: Attempting to differentiate and finding the turning point. This would involve attempting to find $\frac{\mathrm{d}y}{\mathrm{d}x}$, then setting it equal to 0 and solving to find the *x* value and the *y* value.

A1: For differentiating $\frac{\mathrm{d}y}{\mathrm{d}x} = 2x - 8 \Rightarrow (4,1)$ is the **turning point**

A1: Shows that $(4,1)$ is the minimum point (second derivative or U shaped), hence $x^2 - 8x + 17 > 0$

……

Method 4: Sketch graph using calculator

M1: Attempting to sketch $y = x^2 - 8x + 17$, U shape with minimum in quadrant one

A1: As above with minimum at (4,1) marked

A1: Required to state that quadratics only have one turning point and as "1" is above the *x*-axis then $x^2 - 8x + 17 > 0$

(ii)

Numerical approach

Do not allow any marks if the student just mentions "positive" and "negative" numbers. Specific examples should be seen calculated if a numerical approach is chosen.

M1: Attempts a value (where it is not true) and shows/implies that it is not true for that value. For example, for -4 : $(-4+3)^2 > (-4)^2$ and indicates not true (states not true, ✗)

or writing $(-4+3)^2 < (-4)^2$ is sufficient to imply that it is not true

A1: Shows/implies that it can be true for a value **AND** states sometimes true. For example for +4 : $(4+3)^2 > 4^2$ and indicates true ✓

or writing $(4+3)^2 > 4^2$ is sufficient to imply this is true following $(-4+3)^2 < (-4)^2$ condone incorrect statements following the above such as 'it is only true for positive numbers' as long as they state "sometimes true" and show both cases.

Algebraic approach

M1: Sets the problem up algebraically Eg. $(x+3)^2 > x^2 \Rightarrow x > k$ Any inequality is fine. You may condone one error for the method mark. Accept $(x+3)^2 > x^2 \Rightarrow 6x + 9 > 0$ oe

A1: States sometimes true a**nd** states/implies true for $x > -\frac{3}{2}$ or states/implies not true for $x \leq -\frac{3}{2}$ In both cases you should expect to see the statement "sometimes true" to score the A1

Question	Scheme	Marks	AOs
3(a)	Attempts $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ or similar	M1	1.1b
	$\overrightarrow{AB} = -9\mathbf{i} + 3\mathbf{j}$	A1	1.1b
		(2)	
(b)	Finds length using 'Pythagoras' $\lvert AB \rvert = \sqrt{(-9)^2 + (3)^2}$	M1	1.1b
	$\lvert AB \rvert = 3\sqrt{10}$	A1ft	1.1b
		(2)	
			(4 marks)

Notes

(a)

M1: Attempts subtraction either way around.

This may be implied by one correct component $\overrightarrow{AB} = \pm 9\mathbf{i} \pm 3\mathbf{j}$

There must be some attempt to write in vector form.

A1: cao (allow column vector notation but not the coordinate)

Correct notation should be used. Accept $-9\text{i} + 3\text{j}$ or $\begin{pmatrix} -9 \\ 3 \end{pmatrix}$ but not $\begin{pmatrix} -9\text{i} \\ 3\text{j} \end{pmatrix}$

(b)

M1: Correct use of Pythagoras theorem or modulus formula using their answer to (a)

Note that $\lvert AB \rvert = \sqrt{(9)^2 + (3)^2}$ is also correct.

Condone missing brackets in the expression $\lvert AB \rvert = \sqrt{-9^2 + (3)^2}$

Also allow a restart usually accompanied by a diagram.

A1ft: $\lvert AB \rvert = 3\sqrt{10}$ ft from their answer to (a) as long as it has both an **i** and **j** component.

It must be simplified, if appropriate. Note that $\pm 3\sqrt{10}$ would be M1 A0

Note that, in cases where there is no working, the correct answer implies M1A1 in each part of this question

Question	Scheme	Marks	AOs
4	States gradient of $4y-3x=10$ is $\frac{3}{4}$ oe or rewrites as $y=\frac{3}{4}x+...$	B1	1.1b
	Attempts to find gradient of line joining $(5,-1)$ and $(-1,8)$	M1	1.1b
	$=\frac{-1-8}{5-(-1)}=-\frac{3}{2}$	A1	1.1b
	States neither with suitable reasons	A1	2.4
		(4)	
		(4 marks)	

Notes

B1: States that the gradient of line l_1 is $\frac{3}{4}$ or writes l_1 in the form $y=\frac{3}{4}x+...$

M1: Attempts to find the gradient of line l_2 using $\frac{\Delta y}{\Delta x}$ Condone one sign error Eg allow $\frac{9}{6}$

A1: For the gradient of l_2 $=\frac{-1-8}{5-(-1)}=-\frac{3}{2}$ or the equation of l_2 $y=-\frac{3}{2}x+...$

Allow for any equivalent such as $-\frac{9}{6}$ or -1.5

A1: CSO (on gradients)

Explains that they are neither parallel as the gradients not equal nor perpendicular as $\frac{3}{4}\times-\frac{3}{2}\neq-1$ oe

Allow a statement in words "they are not negative reciprocals " for a reason for not perpendicular and "they are not equal" for a reason for not being parallel

<table>
<tr><th>Question</th><th colspan="2">Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>5 (a)</td><td colspan="2">Identifies one of the two errors
"You cannot use the subtraction law without dealing with the 2 first"
" They undo the logs incorrectly. It should be $x = 2^3 = 8$ "</td><td>B1</td><td>2.3</td></tr>
<tr><td></td><td colspan="2">Identifies both errors. See above.</td><td>B1</td><td>2.3</td></tr>
<tr><td></td><td colspan="2"></td><td>(2)</td><td></td></tr>
<tr><td>(b)</td><td>$\log_2\left(\frac{x^2}{\sqrt{x}}\right) = 3$</td><td>$\frac{3}{2}\log_2(x) = 3$</td><td>M1</td><td>1.1b</td></tr>
<tr><td></td><td>$x^{\frac{3}{2}} = 2^3$ or $\frac{x^2}{\sqrt{x}} = 2^3$</td><td>$x = 2^2$</td><td>M1</td><td>1.1b</td></tr>
<tr><td></td><td>$x = \left(2^3\right)^{\frac{2}{3}} = 4$</td><td>$x = 4$</td><td>A1</td><td>1.1b</td></tr>
<tr><td></td><td colspan="2"></td><td>(3)</td><td></td></tr>
<tr><td colspan="5">(5 marks)</td></tr>
</table>

(a)

B1: States one of the two errors.

Error One: Either in words states 'They cannot use the subtraction law without dealing with the 2 first' or writes ' that line 2 should be $\log_2\left(\frac{x^2}{\sqrt{x}}\right)$ $(=3)$' If they rewrite line two it must be correct. Allow 'the coefficient of each log term is different so we cannot use the subtraction law' Allow responses such as 'it must be $\log x^2$ before subtracting the logs'

Do not accept an incomplete response such as "the student ignored the 2". **There must be some reference to the subtraction law as well.**

Error Two: Either in words states 'They undo the log incorrectly' or writes that 'if $\log_2 x = 3$ then $x = 2^3 = 8$' If it is rewritten it must be correct. Eg $x = \log_2 9$ is B0

B1: States both of the two errors. (See above)

(b)

M1: Uses a correct method of combining the two log terms. Either uses both the power law and the subtraction law to reach a form $\log_2\left(\frac{x^2}{\sqrt{x}}\right) = 3$ oe. Or uses both the power law and subtraction to reach $\frac{3}{2}\log_2(x) = 3$

M1: Uses correct work to "undo" the log. Eg moves from $\log_2\left(Ax^n\right) = b \Rightarrow Ax^n = 2^b$

This is independent of the previous mark so allow following earlier error.

A1: cso $x = 4$ achieved with at least one intermediate step shown. Extra solutions would be A0

SC: If the "answer" rather than the "solution" is given score 1,0,0.

Question	Scheme	Marks	AOs
6 (a)	Attempts $P=100-6.25(15-9)^2$	M1	3.4
	$=-125$ $\therefore$ not sensible as the company would make a loss	A1	2.4
		(2)	
(b)	Uses $P>80\Rightarrow(x-9)^2<3.2$ or $P=80\Rightarrow(x-9)^2=3.2$	M1	3.1b
	$\Rightarrow 9-\sqrt{3.2}<x<9+\sqrt{3.2}$	dM1	1.1b
	Minimum Price = £7.22	A1	3.2a
		(3)	
(c)	States (i) maximum profit =£ 100 000 and (ii) selling price £9	B1 B1	3.2a 2.2a
		(2)	
			(7 marks)

(a)

M1: Substitutes $x=15$ into $P=100-6.25(x-9)^2$ and attempts to calculate. This is implied by an answer of -125. Some candidates may have attempted to multiply out the brackets before they substitute in the $x = 15$. This is acceptable as long as the function obtained is quadratic. There must be a calculation seen or implied by the value of -125.

A1: Finds $P=-125$ or states that $P<0$ **and** explains that (this is not sensible as) the company would make a loss.

Condone $P=-125$ followed by an explanation that it is not sensible as the company would make a loss of £125 rather than £125 000. An explanation that it is not sensible as "the profit cannot be negative" , "the profit is negative" or "the company will not make any money" , "they might make a loss" is incomplete/incorrect. You may ignore any misconceptions or reference to the price of the toy being too cheap for this mark.

Alt: **M1:** Sets $P = 0$ and finds $x=5,13$ A1: States $15>13$ and states makes a loss

(b)

M1: Uses $P...80$ where ... is any inequality or "$=$"in $P=100-6.25(x-9)^2$ and proceeds to $(x-9)^2...k$ where k >0 and ... is any inequality or "$=$"

Eg. Condone $P<80$ in $P=100-6.25(x-9)^2\Rightarrow(x-9)^2<k$ where k >0 If the candidate attempts to multiply out then allow when they achieve a form $ax^2+bx+c=0$

dM1: Award for solving to find the two positive values for x. Allow decimal answers

FYI correct answers are $\Rightarrow 9-\sqrt{3.2}<x<9+\sqrt{3.2}$ Accept $\Rightarrow x=9\pm\sqrt{3.2}$

Condone incorrect inequality work $100-6.25(x-9)^2>80\Rightarrow(x-9)^2>3.2\Rightarrow x>9\pm\sqrt{3.2}$

Alternatively award if the candidate selects the lower of their two positive values $9-\sqrt{3.2}$

A1: Deduces that the minimum Price = £7.22 (£7.21 is not acceptable)

(c)

(i) B1: Maximum Profit = £ 100 000 with units. Accept 100 thousand pound(s).

(ii) B1: Selling price = £9 with units

SC 1: Missing units in (b) and (c) only penalise once in these parts, withhold the final mark.

SC 2: If the answers to (c) are both correct, but in the wrong order score SC B1 B0

If (i) and (ii) are not written out score in the order given.

Question	Scheme	Marks	AOs
7 (a)	Uses $15=\frac{1}{2}\times5\times10\times\sin\theta$	M1	1.1b
	$\sin\theta=\frac{3}{5}$ oe	A1	1.1b
	Uses $\cos^2\theta=1-\sin^2\theta$	M1	2.1
	$\cos\theta=\pm\frac{4}{5}$	A1	1.1b
		(4)	
(b)	Uses $BC^2=10^2+5^2-2\times10\times5\times"-\frac{4}{5}"$	M1	3.1a
	$BC=\sqrt{205}$	A1	1.1b
		(2)	
		(6 marks)	

Notes

(a)

M1: Uses the formula Area $=\frac{1}{2}ab\sin C$ in an attempt to find the value of $\sin\theta$ or θ

A1: $\sin\theta=\frac{3}{5}$ oe This may be implied by $\theta=$ awrt 36.9° or awrt 0.644 (radians)

M1: Uses their value of $\sin\theta$ to find two values of $\cos\theta$ This may be scored via the formula $\cos^2\theta=1-\sin^2\theta$ or by a triangle method. Also allow the use of a graphical calculator or candidates may just write down the **two values**. The values must be symmetrical $\pm k$

A1: $\cos\theta=\pm\frac{4}{5}$ or ±0.8 Condone these values appearing from $\pm0.79....$

(b)

M1: Uses a suitable method of finding the longest side. For example chooses the negative value (or the obtuse angle) and proceeds to find *BC* using the cosine rule. Alternatively works out *BC* using both values and chooses the larger value. If stated the cosine rule should be correct (with a minus sign). Note if the sign is +ve and the acute angle is chosen the correct value will be seen. It is however M0 A0

A1: $BC=\sqrt{205}$

Question	Scheme	Marks	AOs
8 (a)(i)	$C=\frac{1500}{v}+\frac{2v}{11}+60\Rightarrow\frac{\mathrm{d}C}{\mathrm{d}v}=-\frac{1500}{v^2}+\frac{2}{11}$	M1 A1	3.1b 1.1b
	Sets $\frac{\mathrm{d}C}{\mathrm{d}v}=0\Rightarrow v^2=8250$	M1	1.1b
	$\Rightarrow v=\sqrt{8250}\Rightarrow v=90.8\,(\mathrm{km\,h^{-1}})$	A1	1.1b
(ii)	For substituting their $v=90.8$ in $C=\frac{1500}{v}+\frac{2v}{11}+60$	M1	3.4
	Minimum cost =awrt (£) 93	A1 ft	1.1b
		(6)	
(b)	Finds $\frac{\mathrm{d}^2C}{\mathrm{d}v^2}=+\frac{3000}{v^3}$ at $v=90.8$	M1	1.1b
	$\frac{\mathrm{d}^2C}{\mathrm{d}v^2}=(+0.004)>0$ hence minimum (cost)	A1 ft	2.4
		(2)	
(c)	It would be impossible to drive at this speed over the whole journey	B1	3.5b
		(1)	
			(9 marks)

Notes

(a)(i)

M1: Attempts to differentiate (deals with the powers of v correctly).

Look for an expression for $\frac{\mathrm{d}C}{\mathrm{d}v}$ in the form $\frac{A}{v^2}+B$

A1: $\left(\frac{\mathrm{d}C}{\mathrm{d}v}\right)=-\frac{1500}{v^2}+\frac{2}{11}$

A number of students are solving part (a) numerically or graphically. Allow these students to pick up the M1 A1 here from part (b) when they attempt the second derivative.

M1: Sets $\frac{\mathrm{d}C}{\mathrm{d}v}=0$ (which may be implied) and proceeds to an equation of the type $v^n=k, k>0$

Allow here equations of the type $\frac{1}{v^n}=k, k>0$

A1: $v=\sqrt{8250}$ or $5\sqrt{330}$ awrt 90.8 ($\mathrm{km\,h^{-1}}$) .

As this is a speed withhold this mark for answers such as $v=\pm\sqrt{8250}$

* Condone $\frac{\mathrm{d}C}{\mathrm{d}v}$ appearing as $\frac{\mathrm{d}y}{\mathrm{d}x}$ or perhaps not appearing at all. Just look for the rhs.

(a)(ii)

M1: For a correct method of finding $C =$ from their solution to $\frac{dC}{dv} = 0$.

Do not accept attempts using negative values of *v*.
Award if you see $v = .., C = ...$ where the *v* used is their solution to (a)(i).

A1ft: Minimum cost = awrt (£) 93. Condone the omission of units
Follow through on sensible values of *v*. $60 < v < 110$

v	C
60	95.9
65	94.9
70	94.2
75	93.6
80	93.3
85	93.1
90	93.0
95	93.1
100	93.2
105	93.4
110	93.6

(b)

M1: Finds $\frac{d^2C}{dv^2}$ (following through on their $\frac{dC}{dv}$ which must be of equivalent difficulty) and attempts to find its value / sign at their *v*

Allow a substitution of their answer to (a) (i) in their $\frac{d^2C}{dv^2}$

Allow an explanation into the sign of $\frac{d^2C}{dv^2}$ from its terms (as $v > 0$)

A1ft: $\frac{d^2C}{dv^2} = +0.004 > 0$ hence minimum (cost). Alternatively $\frac{d^2C}{dv^2} = +\frac{3000}{v^3} > 0$ as $v > 0$

Requires a correct calculation or expression, a correct statement and a correct conclusion.

Follow through on their *v* ($v > 0$) and their $\frac{d^2C}{dv^2}$

* Condone $\frac{d^2C}{dv^2}$ appearing as $\frac{d^2y}{dx^2}$ or not appearing at all for the M1 but for the A1 the correct notation must be used (accept notation C'').

(c)

B1: Gives a limitation of the given model, for example

- It would be impossible to drive at this speed over the whole journey
- The traffic would mean that you cannot drive at a constant speed

Any statement that implies that the speed could not be constant is acceptable.

Question	Scheme	Marks	AOs
9(a)	$(\mathrm{g}(-2)) = 4\times-8-12\times4-15\times-2+50$	M1	1.1b
	$\mathrm{g}(-2)=0 \Rightarrow (x+2)$ is a factor	A1	2.4
		(2)	
(b)	$4x^3-12x^2-15x+50=(x+2)(4x^2-20x+25)$	M1 A1	1.1b 1.1b
	$=(x+2)(2x-5)^2$	M1 A1	1.1b 1.1b
		(4)	
(c)	(i) $x\leqslant -2,\ x=2.5$	M1 A1ft	1.1b 1.1b
	(ii) $x=-1,\ x=1.25$	B1ft	2.2a
		(3)	
			(9 marks)

(a)

M1: Attempts $\mathrm{g}(-2)$ Some sight of (-2) embedded or calculation is required.

So expect to see $4\times(-2)^3-12\times(-2)^2-15\times(-2)+50$ embedded

Or $-32-48+30+50$ condoning slips for the M1

Any attempt to divide or factorise is M0. (See demand in question)

A1: $\mathrm{g}(-2)=0\Rightarrow(x+2)$ is a factor.

Requires a correct statement and conclusion. Both "$\mathrm{g}(-2)=0$" and "$(x+2)$ is a factor" must be seen in the solution. This may be seen in a preamble before finding $\mathrm{g}(-2)=0$ but in these cases there must be a minimal statement ie QED, "proved", tick etc.

Also accept, in one coherent line/sentence, explanations such as, 'as g (x) =0 when $x=-2$, $(x+2)$ is a factor.'

(b)

M1: Attempts to divide $\mathrm{g}(x)$ by $(x+2)$ May be seen and awarded from part (a)

If inspection is used expect to see $4x^3-12x^2-15x+50=(x+2)(4x^2\text{..........}\pm25)$

If algebraic / long division is used expect to see

$$\begin{array}{r} 4x^2\pm20x \\ x+2\overline{)\;4x^3-12x^2-15x+50} \end{array}$$

A1: Correct quadratic factor is $(4x^2-20x+25)$ may be seen and awarded from part (a)

M1: Attempts to factorise their $(4x^2-20x+25)$ usual rule $(ax+b)(cx+d), ac=\pm4, bd=\pm25$

A1: $(x+2)(2x-5)^2$ oe seen on a single line. $(x+2)(-2x+5)^2$ is also correct.

Allow recovery for all marks for $\mathrm{g}(x)=(x+2)(x-2.5)^2=(x+2)(2x-5)^2$

(c)(i)

M1: For identifying that the solution will be where the curve is on or below the axis. Award for either $x\leqslant-2$ or $x=2.5$ Follow through on their $\mathrm{g}(x)=(x+2)(ax+b)^2$ only where $ab<0$ (that is a positive root). Condone $x<-2$ See SC below for $\mathrm{g}(x)=(x+2)(2x+5)^2$

A1ft: BOTH $x \leqslant -2$, $x = 2.5$ Follow through on their $-\frac{b}{a}$ of their $\mathrm{g}(x) = (x+2)(ax+b)^2$

May see $\{x \leqslant -2 \cup x = 2.5\}$ which is fine.

(c) (ii)

B1ft: For deducing that the solutions of $\mathrm{g}(2x) = 0$ will be where $x = -1$ and $x = 1.25$

Condone the coordinates appearing $(-1, 0)$ and $(1.25, 0)$

Follow through on their 1.25 of their $\mathrm{g}(x) = (x+2)(ax+b)^2$

……………………………………………………………………………………………………..

SC: If a candidate reaches $\mathrm{g}(x) = (x+2)(2x+5)^2$, clearly incorrect because of Figure 2, we will award

In (i) M1 A0 for $x \leqslant -2$ or $x < -2$

In (ii) B1 for $x = -1$ and $x = -1.25$

…………………………………………………………………………………………………

Alt (b)	$4x^3 - 12x^2 - 15x + 50 = (x+2)(ax+b)^2$ $= a^2x^3 + (2ba + 2a^2)x^2 + (b^2 + 4ab)x + 2b^2$		
	Compares terms to get either *a* or *b*	M1	1.1b
	Either $a = 2$ or $b = -5$	A1	1.1b
	Multiplies out expression $(x+2)(\pm 2x \pm 5)^2$ and compares to $4x^3 - 12x^2 - 15x + 50$	M1	
	All terms must be compared or else expression must be multiplied out and establishes that $4x^3 - 12x^2 - 15x + 50 = (x+2)(2x-5)^2$	A1	1.1b
		(4)	

Question	Scheme	Marks	AOs
10	Considers $\dfrac{(x+h)^3-x^3}{h}$	B1	2.1
	Expands $(x+h)^3 = x^3+3x^2h+3xh^2+h^3$	M1	1.1b
	so gradient (of chord) $=\dfrac{3x^2h+3xh^2+h^3}{h}=3x^2+3xh+h^2$	A1	1.1b
	States as $h\to 0$, $3x^2+3xh+h^2\to 3x^2$ so derivative $=3x^2$ *	A1*	2.5
		(4 marks)	

B1: Gives the correct fraction for the gradient of the chord either $\dfrac{(x+h)^3-x^3}{h}$ or $\dfrac{(x+\delta x)^3-x^3}{\delta x}$

It may also be awarded for $\dfrac{(x+h)^3-x^3}{x+h-x}$ oe. It may be seen in an expanded form

It does not have to be linked to the gradient of the chord

M1: Attempts to expand $(x+h)^3$ or $(x+\delta x)^3$ Look for two correct terms, most likely $x^3+...+h^3$
This is independent of the B1

A1: Achieves gradient (of chord) is $3x^2+3xh+h^2$ or exact un simplified equivalent such as $3x^2+2xh+xh+h^2$. Again, there is no requirement to state that this expression is the gradient of the chord

A1*: CSO. Requires correct algebra and making a link between the gradient of the chord and the gradient of the curve. See below how the link can be made. The words "gradient of the chord" do not need to be mentioned but derivative, $\mathrm{f}'(x), \dfrac{\mathrm{d}y}{\mathrm{d}x}, y'$ should be. Condone invisible brackets for the expansion of $(x+h)^3$ as long as it is only seen at the side as intermediate working.

Requires either

- $\mathrm{f}'(x) \underset{\lim h\to 0}{=} \dfrac{(x+h)^3-x^3}{h}=3x^2+3xh+h^2=3x^2$
- Gradient of chord $=3x^2+3xh+h^2$ As $h\to 0$ Gradient of chord tends to the gradient of curve so derivative is $3x^2$
- $\mathrm{f}'(x) \underset{\lim h\to 0}{=} 3x^2+3xh+h^2=3x^2$
- Gradient of **chord** $=3x^2+3xh+h^2$ when $h\to 0$ gradient of **curve** $=3x^2$
- Do not allow $h=0$ alone without limit being considered somewhere:
  so don't accept $h=0\Rightarrow \mathrm{f}'(x)=3x^2+3x\times 0+0^2=3x^2$

……

Alternative: B1: Considers $\dfrac{(x+h)^3-(x-h)^3}{2h}$ M1: As above A1: $\dfrac{6x^2h^2+2h^3}{2h}=3x^2+h^2$

Question	Scheme	Marks	AOs
11(a)	$\left(2-\frac{x}{16}\right)^9=2^9+\binom{9}{1}2^8.\left(-\frac{x}{16}\right)+\binom{9}{2}2^7.\left(-\frac{x}{16}\right)^2+...$	M1	1.1b
	$\left(2-\frac{x}{16}\right)^9 = 512+...$	B1	1.1b
	$\left(2-\frac{x}{16}\right)^9 = ...\ -144x\ +...$	A1	1.1b
	$\left(2-\frac{x}{16}\right)^9 = ...\ +...\ +\ 18x^2\ (+...)$	A1	1.1b
		(4)	
(b)	Sets $'512'a=128 \Rightarrow a=...$	M1	1.1b
	$(a=)\frac{1}{4}$ oe	A1 ft	1.1b
		(2)	
(c)	Sets $'512'b+'-144'a=36 \Rightarrow b=...$	M1	2.2a
	$(b=)\frac{9}{64}$ oe	A1	1.1b
		(2)	
			(8 marks)
11(a) alt	$\left(2-\frac{x}{16}\right)^9=2^9\left(1-\frac{x}{32}\right)^9=2^9\left(1+\binom{9}{1}\left(-\frac{x}{32}\right)+\binom{9}{2}\left(-\frac{x}{32}\right)^2+...\right)$	M1	1.1b
	$= 512+...$	B1	1.1b
	$= ...\ -144x\ +...$	A1	1.1b
	$= ...\ +...\ +\ 18x^2\ (+...)$	A1	1.1b

Notes

(a)

M1: Attempts the binomial expansion. May be awarded on either term two and/or term three Scored for a correct binomial coefficient combined with a correct power of 2 and a correct power of $\left(\pm\frac{x}{16}\right)$ Condone $\binom{9}{2}2^7.\left(-\frac{x^2}{16}\right)$ for term three.

Allow any form of the binomial coefficient. Eg $\binom{9}{2}={}^9C_2=\frac{9!}{7!2!}=36$

In the alternative it is for attempting to take out a factor of 2 (may allow 2^n outside bracket) and having a correct binomial coefficient combined with a correct power of $\left(\pm\frac{x}{32}\right)$

B1: For 512

A1: For $-144x$

A1: For $+\ 18x^2$ Allow even following $\left(+\frac{x}{16}\right)^2$

Listing is acceptable for all 4 marks

(b)

M1: For setting their $512a = 128$ and proceeding to find a value for *a.* Alternatively they could substitute $x = 0$ into both sides of the identity and proceed to find a value for *a*.

A1 ft: $a = \frac{1}{4}$ oe Follow through on $\frac{128}{\text{their } 512}$

(c)

M1: Condone $512b \pm 144 \times a = 36$ following through on their 512, their -144 and using their value of $"a"$ to find a value for $"b"$

A1: $b = \frac{9}{64}$ oe

Question	Scheme	Marks	AOs
12 (a)	$4\cos\theta - 1 = 2\sin\theta\tan\theta \Rightarrow 4\cos\theta - 1 = 2\sin\theta \times \frac{\sin\theta}{\cos\theta}$	M1	1.2

	$\Rightarrow 4\cos^2\theta - \cos\theta = 2\sin^2\theta$ oe	A1	1.1b
	$\Rightarrow 4\cos^2\theta - \cos\theta = 2\left(1-\cos^2\theta\right)$	M1	1.1b
	$6\cos^2\theta - \cos\theta - 2 = 0$ *	A1*	2.1
		(4)	
(b)	For attempting to solve given quadratic	M1	1.1b
	$\left(\cos 3x\right) = \frac{2}{3}, -\frac{1}{2}$	B1	1.1b
	$x = \frac{1}{3}\arccos\left(\frac{2}{3}\right)$ or $\frac{1}{3}\arccos\left(-\frac{1}{2}\right)$	M1	1.1b
	$x = 40°, 80°$, awrt $16.1°$	A1	2.2a
		(4)	
			(8 marks)

Notes

(a)

M1: Recall and use the identity $\tan\theta = \frac{\sin\theta}{\cos\theta}$ **Note that it cannot just be stated.**

A1: $4\cos^2\theta - \cos\theta = 2\sin^2\theta$ oe.

This is scored for a correct line that does not contain any fractional terms.

It may be awarded later in the solution after the identity $1-\cos^2\theta = \sin^2\theta$ has been used Eg for $\cos\theta(4\cos\theta - 1) = 2\left(1-\cos^2\theta\right)$ or equivalent

M1: Attempts to use the correct identity $1-\cos^2\theta = \sin^2\theta$ to form an equation in just $\cos\theta$

A1*: Proceeds to correct answer through rigorous and clear reasoning. No errors in notation or bracketing. For example $\sin^2\theta = \sin\theta^2$ is an error in notation

(b)

M1: For attempting to solve the given quadratic "$6y^2 - y - 2 = 0$ " where y could be $\cos 3x, \cos x,$ or even just y. When factorsing look for $(ay+b)(cy+d)$ where $ac = \pm 6$ and $bd = \pm 2$

This may be implied by the correct roots (even award for $\left(y \pm \frac{2}{3}\right)\left(y \pm \frac{1}{2}\right)$) , an attempt at factorising, an attempt at the quadratic formula, an attempt at completing the square and even ± the correct roots.

B1: For the roots $\frac{2}{3}, -\frac{1}{2}$ oe

M1: Finds at least one solution for x from $\cos 3x$ **within the given range** for their $\frac{2}{3}, -\frac{1}{2}$

A1: $x = 40°, 80°$, awrt $16.1°$ **only** Withhold this mark if there are **any** other values even if they are outside the range. Condone 40 and 80 appearing as 40.0 and 80.0

Question	Scheme	Marks	AOs
13(a)	For a correct equation in p or q $p = 10^{4.8}$ or $q = 10^{0.05}$	M1	1.1b

	For $p = \text{awrt } 63100$ or $q = \text{awrt } 1.122$	A1	1.1b
	For correct equations in p and q $p = 10^{4.8}$ and $q = 10^{0.05}$	dM1	3.1a
	For $p = \text{awrt } 63100$ and $q = \text{awrt } 1.122$	A1	1.1b
		(4)	
(b)	(i) The value of the painting on 1st January 1980	B1	3.4
	(ii) The proportional increase in value each year	B1	3.4
		(2)	
(c)	Uses $V = 63100 \times 1.122^{30}$ or $\log V = 0.05 \times 30 + 4.8$ leading to $V =$	M1	3.4
	$= \text{awrt } (£)2000000$	A1	1.1b
		(2)	
			(8 marks)

Notes

(a)

M1: For a correct equation in p or q This is usually $p = 10^{4.8}$ or $q = 10^{0.05}$ but may be $\log q = 0.05$ or $\log p = 4.8$

A1: For $p = \text{awrt } 63100$ or $q = \text{awrt } 1.122$

M1: For linking the two equations and forming correct equations in p and q. This is usually $p = 10^{4.8}$ and $q = 10^{0.05}$ but may be $\log q = 0.05$ and $\log p = 4.8$

A1: For $p = \text{awrt } 63100$ and $q = \text{awrt } 1.122$ Both these values implies M1 M1

……………………………………………………………………………………………

ALT I(a)

M1: Substitutes $t = 0$ and states that $\log p = 4.8$

A1: $p = \text{awrt } 63100$

M1: Uses their found value of p and another value of t to find form an equation in q

A1: $p = \text{awrt } 63100$ and $q = \text{awrt } 1.122$

……………………………………………………………………………………………

(b)(i)

B1: The value of the painting on 1st January 1980 (is £63 100)

Accept the original value/cost of the painting or the initial value/cost of the painting

(b)(ii)

B1: The proportional increase in value each year. Eg Accept an explanation that explains that the value of the painting will rise 12.2% a year. (Follow through on their value of q.)
Accept "the rate" by which the value is rising/price is changing. "1.122 is the decimal multiplier representing the year on year increase in value"
Do not accept "the amount" by which it is rising or "how much" it is rising by

If they are not labelled (b)(i) and (b)(ii) mark in the order given but accept any way around as long as clearly labelled " *p* is........... " and "*q* is ...."

(c)

M1: For substituting $t = 30$ into $V = pq^t$ using their values for p and q or substituting t =30 into $\log_{10} V = 0.05t + 4.8$ and proceeds to V

A1: For awrt either £1.99 million or £2.00 million. Condone the omission of the £ sign.
Remember to isw after a correct answer

Question	Scheme	Marks	AOs
14 (a)	Attempts to complete the square $(x \pm 3)^2 + (y \pm 5)^2 = \ldots$	M1	1.1b

	(i) Centre $(3,-5)$	A1	1.1b
	(ii) Radius 5	A1	1.1b
		(3)	
(b)	Uses a sketch or otherwise to deduce $k=0$ is a critical value	B1	2.2a
	Substitute $y=kx$ in $x^2+y^2-6x+10y+9=0$	M1	3.1a
	Collects terms to form correct 3TQ $\left(1+k^2\right)x^2+\left(10k-6\right)x+9=0$	A1	1.1b
	Attempts $b^2-4ac...0$ for their *a*, *b* and *c* leading to values for *k* $"\left(10k-6\right)^2-36\left(1+k^2\right)...0" \to k=...,...$ $\left(0 \text{ and } \frac{15}{8}\right)$	M1	1.1b
	Uses $b^2-4ac>0$ and chooses the outside region (see note) for **their** critical values (Both *a* and *b* must have been expressions in *k*)	dM1	3.1a
	Deduces $k<0, k>\frac{15}{8}$ oe	A1	2.2a
		(6)	
			(9 marks)

Notes

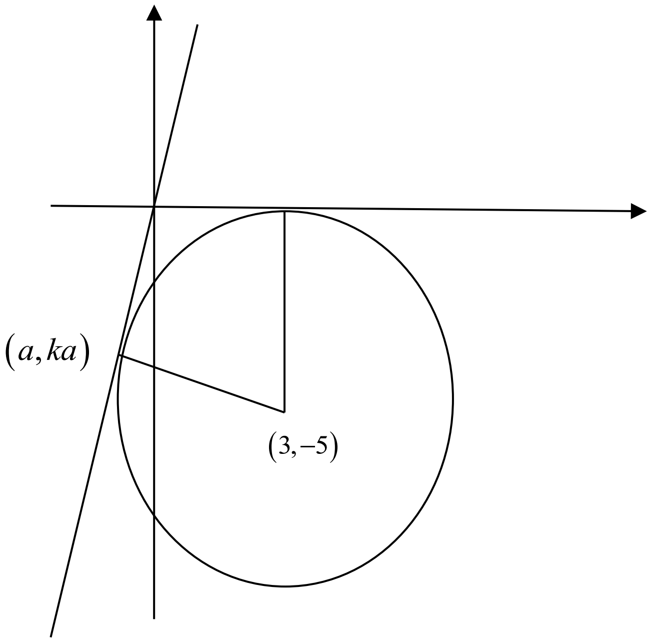
(a)
M1: Attempts $\left(x\pm3\right)^2+\left(y\pm5\right)^2=..$
This mark may be implied by candidates writing down a centre of $(\pm3,\pm5)$ or $r^2=25$
(i) A1: Centre $(3,-5)$
(ii) A1: Radius 5. Do not accept $\sqrt{25}$
Answers only (no working) scores all three marks

(b)
B1: Uses a sketch or their subsequent quadratic to deduce that $k=0$ is a critical value.
You may award for the correct $k<0$ but award if $k\leqslant 0$ or even with greater than symbols
M1: Substitutes $y=kx$ in $x^2+y^2-6x+10y+9=0$ or their $\left(x\pm3\right)^2+\left(y\pm5\right)^2=...$ to form an equation in just *x* and *k* . It is possible to substitute $x=\frac{y}{k}$ into their circle equation to form an equation in just *y* and *k*.
A1: Correct 3TQ $\left(1+k^2\right)x^2+\left(10k-6\right)x+9=0$ with the terms in *x* collected. The "= 0" can be implied by subsequent work. This may be awarded from an equation such as $x^2+k^2x^2+\left(10k-6\right)x+9=0$ so long as the correct values of *a*, *b* and *c* are used in $b^2-4ac...0$.
FYI The equation in *y* and *k* is $\left(1+k^2\right)y^2+\left(10k^2-6k\right)y+9k^2=0$ oe

M1: Attempts to find two critical values for *k* using $b^2-4ac...0$ or $b^2...4ac$ where ... could be "=" or any inequality.
dM1: Finds the outside region using their critical values. Allow the boundary to be included. It is dependent upon all previous M marks and both *a* and *b* must have been expressions in *k*.
Note that it is possible that the correct region could be the inside region if the coefficient of k^2 in $4ac$ is larger than the coefficient of k^2 in b^2 Eg.
$b^2-4ac=\left(k-6\right)^2-4\times\left(1+k^2\right)\times9>0 \Rightarrow -35k^2-12k>0 \Rightarrow k\left(35k+12\right)<0$

A1: Deduces $k<0, k>\frac{15}{8}$. This must be in terms of *k*. Allow exact equivalents such as $k<0 \bigcup k>1.875$ but not allow $0>k>\frac{15}{8}$ or the above with AND, & or $\bigcap$ between the two inequalities

Alternative using a geometric approach with a triangle with vertices at $(0,0)$, and $(3,-5)$



Alt (b)	Uses a sketch or otherwise to deduce $k=0$ is a critical value	B1	2.2a
	Distance from (a,ka) to $(0,0)$ is $3 \Rightarrow a^2(1+k^2)=9$	M1	3.1a
	Tangent and radius are perpendicular $\Rightarrow k\times\frac{ka+5}{a-3}=-1 \Rightarrow a(1+k^2)=3-5k$	M1	3.1a
	Solve simultaneously, (dependent upon both M's)	dM1	1.1b
	$k=\frac{15}{8}$	A1	1.1b
	Deduces $k<0, k>\frac{15}{8}$	A1	2.2a
		(6)	

Question	Scheme	Marks	AOs
15.	For the complete strategy of finding where the normal cuts the *x*-axis. Key points that must be seen are • Attempt at differentiation	M1	3.1a

	• Attempt at using a changed gradient to find equation of normal • Correct attempt to find where normal cuts the x - axis		
	$y=\frac{32}{x^2}+3x-8 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{64}{x^3}+3$	M1 A1	1.1b 1.1b
	For a correct method of attempting to find Either the equation of the normal: this requires substituting $x=4$ in their $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{64}{x^3}+3=(2)$, then using the perpendicular gradient rule to find the equation of normal $y-6=\text{"}-\frac{1}{2}\text{"}(x-4)$ Or where the equation of the normal at (4,6) cuts the x - axis. As above but may not see equation of normal. Eg $0-6=\text{"}-\frac{1}{2}\text{"}(x-4)\Rightarrow x=\ldots$ or an attempt using just gradients $\text{"}-\frac{1}{2}\text{"}=\frac{6}{a-4}\Rightarrow a=\ldots$	dM1	2.1
	Normal cuts the x-axis at $x=16$	A1	1.1b
	For the complete strategy of finding the values of the two key areas. Points that must be seen are • There must be an attempt to find the area under the curve by integrating between 2 and 4 • There must be an attempt to find the area of a triangle using $\frac{1}{2}\times('16'-4)\times 6$ or $\int_4^{\text{"16"}}\text{"}\left(-\frac{1}{2}x+8\right)\text{"}\mathrm{d}x$	M1	3.1a
	$\int \frac{32}{x^2}+3x-8\,\mathrm{d}x=-\frac{32}{x}+\frac{3}{2}x^2-8x$	M1 A1	1.1b 1.1b
	Area under curve = $=\left[-\frac{32}{x}+\frac{3}{2}x^2-8x\right]_2^4=(-16)-(-26)=(10)$	dM1	1.1b
	Total area =10 + 36 =46 *	A1*	2.1
		(10)	

(10 marks)

(a)

The first 5 marks are for finding the normal to the curve cuts the x - axis

M1: For the complete strategy of finding where the normal cuts the x- axis. See scheme

M1: Differentiates with at least one index reduced by one

A1: $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{64}{x^3}+3$

dM1: Method of finding

either the equation of the normal at (4, 6) .

or where the equation of the normal at (4, 6) cuts the x - axis

See scheme. It is dependent upon having gained the M mark for differentiation.

A1: Normal cuts the x-axis at $x = 16$

The next 5 marks are for finding the area *R*

M1: For the complete strategy of finding the values of two key areas. See scheme

M1: Integrates $\int \frac{32}{x^2} + 3x - 8 \, dx$ raising the power of at least one index

A1: $\int \frac{32}{x^2} + 3x - 8 \, dx = -\frac{32}{x} + \frac{3}{2}x^2 - 8x$ which may be unsimplified

dM1: Area $= \left[-\frac{32}{x} + \frac{3}{2}x^2 - 8x \right]_2^4 = (-16) - (-26) = (10)$

It is dependent upon having scored the M mark for integration, for substituting in both 4 and 2 and subtracting either way around. The above line shows the minimum allowed working for a correct answer.

A1*: Shows that the area under curve = 46. No errors or omissions are allowed

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A number of candidates are equating the line and the curve (or subtracting the line from the curve) The last 5 marks are scored as follows.

M1: For the complete strategy of finding the values of the two key areas. Points that must be seen are

- There must be an attempt to find the area BETWEEN the line and the curve either way around by integrating between 2 and 4
- There must be an attempt to find the area of a triangle using $\frac{1}{2} \times ('16' - 2) \times \left(-\frac{1}{2} \times 2 + 8 \right)$ or via integration $\int_2^{16} \left("-\frac{1}{2}x + 8" \right) dx$

M1: Integrates $\int \left("-\frac{1}{2}x + 8" \right) - \left(\frac{32}{x^2} + 3x - 8 \right) dx$ either way around and raises the power of at least one index by one

A1: $\pm \left(-\frac{32}{x} + \frac{7}{4}x^2 - 16x \right)$ must be correct

dM1: Area = $\int_2^4 \left("-\frac{1}{2}x + 8" \right) - \left(\frac{32}{x^2} + 3x - 8 \right) dx =$ ......either way around

A1: Area = $49 - 3 = 46$

NB: Watch for candidates who calculate the area under the curve between 2 and 4 = 10 and subtract this from the large triangle = 56. They will lose both the strategy mark and the answer mark.

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