AQA

A-level
FURTHER MATHEMATICS
7367/3M

Paper 3 Mechanics

Mark scheme

June 2019

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196A7367/3M/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
'their'	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
sf	significant figure(s)
dp	decimal place(s)

Examiners should consistently apply the following general marking principles

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Q	Marking Instructions	AO	Marks	Typical Solution
1	Circles correct answer	AO1.1b	B1	0.44 J
	Total		**1**	

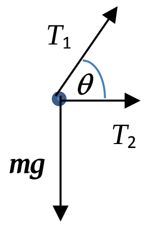
Q	Marking Instructions	AO	Marks	Typical Solution
2	Circles correct answer	AO1.1b	B1	$\frac{12\pi}{5}$
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
3	States correct dimensions of energy Condone use of units instead of dimensions.	AO1.2	B1	$[E] = ML^2T^{-2}$ $ML^2T^{-2} = [m]^a[r]^b[\omega]^c$ $= M^aL^bT^{-c}$ $a = 1$ $b = 2$ $c = 2$
	Forms equation to find the values of the unknown constants PI by two correct values.	AO1.1a	M1	
	States correct values for all three constants NMS 3/3	AO1.1b	A1	
	Total		**3**	

Q	Marking Instructions	AO	Marks	Typical Solution
4(a)	Forms an equation based on equating GPE and KE. Do not allow methods based on constant acceleration equations.	AO3.3	M1	$\frac{1}{2} \times 0.5v^2 = 0.5 \times 10 \times 1.2(1 - \cos 20°)$ $v = \sqrt{1.447} = 1.203 = 1 \text{ m s}^{-1}$ (1 sf)
	Obtains a correct equation.	AO1.1b	A1	
	Obtains the correct speed, giving answer to 1 sf. Condone AWRT1.2	AO1.1b	A1	
(b)	Recognises that the maximum tension will be at the lowest point and forms an equation to find the tension at this point, using their speed from **(a)**.	AO3.4	M1	$T - 0.5 \times 10 = 0.5 \times \frac{1.447}{1.2}$ $T = 5.60 = 6 \text{ N}$ (1 sf)
	Obtains a correct equation.	AO1.1b	A1	
	Obtains the correct tension, giving answer to 1 sf. Condone AWRT 5.6	AO1.1b	A1	
(c)	States that the sphere will not touch the chin again.	AO3.5a	B1	No because air resistance will be present and the sphere, in reality, will not return to its original position.
	Explains the reason for their answer with reference to air resistance. Follow through if they say the ball does touch her chin again.	AO2.4	E1F	
	Total		**8**	

Q	Marking Instructions	AO	Marks	Typical Solution
5(a)	States that the cone is uniform. Accept other suitable descriptions such as consistent density.	AO3.3	B1	Cone is uniform.
(b)	Finds the correct equation of the line	AO1.1b	B1	$y = 4 - \frac{1}{2}x$ Let density $= \rho$ $\text{Mass} = \frac{1}{3}\pi \times 4^2 \times 8\rho = \frac{128\pi\rho}{3}$ $\frac{128\pi\rho}{3} \times \bar{x} = \pi\rho\int_0^8 x\left(4 - \frac{1}{2}x\right)^2 dx$ $\frac{128}{3} \times \bar{x} = \frac{256}{3}$ $\bar{x} = 2$
	Finds the correct mass of the cone. PI by correct completed integral for mass. Condone missing ρ Condone cancelled π if a fraction is used from the outset.	AO1.1b	B1	
	Forms an equation to find the position of the centre of mass using their line and mass. Condone missing ρ Condone cancelled π if a fraction is used from the outset.	AO3.3	M1	
	Completes integration and obtains correct equation for $\bar{x}$.	AO1.1b	A1	
	Completes a rigorous argument and obtains the required value from correct reasoning. Must use correct notation throughout. Density must be included.	AO2.1	R1	
(c)(i)	Forms an equation to find the angle. Accept $\tan\alpha = \frac{2}{4}$ oe. PI by angle of 26.6° or 27°	AO3.4	M1	$\tan\alpha = \frac{4}{2}$ $\alpha = 63°$
	Obtains the correct angle.	AO1.1b	A1	
(c)(ii)	Uses expressions for R and F in a friction equation or inequality.	AO3.3	M1	If on the point of sliding at this angle: $F = \mu R$ $mg\sin\alpha = \mu mg\cos\alpha$ $\mu = \tan\alpha = 2$ $\therefore \mu > 2$
	Finds the correct coefficient of friction. Only follow through $\tan\alpha = \frac{1}{2}$ PI by an inequality or equation involving μ and 2.	AO1.1b	A1F	
	Deduces the correct inequality for the coefficient of friction. NMS 3/3	AO2.2a	A1	
	Total		**11**	

Q	Marking Instructions	AO	Marks	Typical Solution
6(a)	Forms an equation to find the coefficient of restitution. 7 must be seen with $\sin/\cos 40°$ and 5 must be seen with $\sin/\cos 26°$	AO3.3	M1	$7\sin 40° \times e = 5\sin 26°$ $e = \frac{5\sin 26°}{7\sin 40°} = 0.49$
	Obtains a correct equation.	AO3.3	A1	
	Obtains the correct value for the coefficient of restitution. Answer must be given to 2 sf.	AO1.1b	A1	
(b)	Obtains correct components parallel to the wall before and after the collision.	AO1.1a	M1	$7\cos 40° = 5.36$ $5\cos 26° = 4.49$ $5.36 \neq 4.49$ As the components of the velocity parallel to the wall are not equal, then the wall cannot be smooth.
	Deduces that the components are not equal or reduced, by making a valid comparison using numerical values or stating that $7\cos 40° \neq 5\cos 26°$	AO2.2a	A1	
	Completes rigorous mathematical argument and concludes that the wall is not smooth. Numerical values must be stated.	AO2.1	R1	
	Total		**6**	

Q	Marking Instructions	AO	Marks	Typical Solution
7	Finds values for $\sin\theta$ and $\cos\theta$. PI by θ = 36.9° or 53.1°	AO3.3	B1	 $\sin\theta = \frac{60}{75} = \frac{4}{5}$ $\cos\theta = \frac{3}{5}$ $r = 0.75\cos\theta = 0.45$ $T_1 \sin\theta = 2.5g$ $T_1 = \frac{25g}{8}$ $T_2 + T_1\cos\theta = 2.5\times0.45\times8^2$ $T_2 = 72 - \frac{15g}{8}$ $72 - \frac{15g}{8} = \frac{\lambda}{0.3}\times(0.45-0.3)$ $\lambda = 144 - \frac{15g}{4}$
	Finds the radius of the circle using their trig values or Pythagoras. Accept 45 cm.	AO1.1b	B1F	
	Forms equation to find the tension in the inextensible string. Allow $\sin\theta$ or $\cos\theta$.	AO3.3	M1	
	Obtains the correct tension for the inextensible string.	AO1.1b	A1	
	Resolves towards the centre of the circle to form an equation involving both of their tensions and $mr\omega^2$ with their radius. Allow $T_2\cos\theta$ or $T_2\sin\theta$	AO3.3	M1	
	Obtains a correct equation.	AO3.4	A1	
	Obtains the correct tension for the elastic string.	AO1.1b	A1	
	Uses Hooke's Law to form an equation to find the modulus of elasticity with their T_2	AO3.4	M1	
	Obtains the correct modulus of elasticity.	AO1.1b	A1	
	Total		**9**	

Q	Marking Instructions	AO	Marks	Typical Solution
8(a)	Uses Pythagoras to find the initial extension or the height to which Hannah rises.	AO3.3	M1	Initial Extension $=\sqrt{50^2+20^2}-30$ $=23.85$ m
	Calculates the initial EPE using their extension.	AO3.4	M1	Initial EPE $= 2\times\frac{1}{2}\times\frac{3150}{30}\times 23.85^2$ $= 59735$ J
	Obtains correct initial EPE. Accept AWRT 59700	AO1.1b	A1	GPE Gained $=84\times 9.8\times\left(50-\sqrt{30^2-20^2}\right)$ $=84\times 9.8\times 27.639...$ $=22753$ J
	Obtains correct GPE gained when ropes become slack. Accept AWRT 22750.	AO3.4	A1	
	Uses conservation of energy to form an equation to find the speed.	AO2.1	M1	$59735-22753=\frac{1}{2}\times 84v^2$ $36982=\frac{1}{2}\times 84v^2$
	Completes a rigorous argument and obtains correct speed giving answer to 2 sf.	AO2.1	R1	$v=\sqrt{880}$ $=29.6.....$ $=30\text{ m s}^{-1}$ (2 sf)

(b)	Obtains the first of two quantities that can be used as a basis for a comparison to reach a conclusion. For example: 1. The height at which the rope will become taut. 2. The GPE (59600) when the ropes become taut. 3. Maximum height reached if the ropes remain slack.	AO1.1a	M1	Rope becomes taut at height h $h = 50 + \sqrt{30^2 - 20^2}$ $= 72.4$ metres $59735 \ = 84 \times 9.8h$ $h = 72.6$ metres The ropes just become taut at 72.6 metres above the bottom of the gorge, just below Hannah's highest point, at which point she is moving upwards.
	Obtains the correct value for this quantity.	AO1.1b	A1	
	Obtains or states the second of two that can be used as a basis for a comparison to reach a conclusion. For example: 1. Maximum height reached if the ropes remain slack. 2. Initial EPE. 3. The length of the ropes at the maximum height of 72.6 metres.	AO2.1	M1	
	Obtains the correct value for this quantity.	AO1.1b	A1	
	Infers, by comparing two correct values, that the ropes become taut just below Hannah's highest point, at which point she is moving upwards. Or Infers, by comparing two correct values, that due to air resistance it is unlikely that the rope will become taut until she is moving down.	AO2.2b	E1	
	Total		**11**	