Pearson Edexcel

Mark Scheme (Results)

Summer 2019

Pearson Edexcel GCE Further Mathematics
AS Further Pure 2 Paper 8FM0_22

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS
General Instructions for Marking

1. The total number of marks for the paper is 40.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⌈ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.
If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

General Principles for Further Pure Mathematics Marking

(*But note that specific mark schemes may sometimes override these general principles*)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \ldots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \ldots$

2. Formula

Attempt to use the correct formula (with values for *a*, *b* and *c*)

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \ldots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $(x^n \to x^{n-1})$

2. Integration

Power of at least one term increased by 1. $(x^n \to x^{n+1})$

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question	Scheme	Marks	AOs
1(a)	$\det\begin{pmatrix}3-\lambda & 2\\ 2 & 2-\lambda\end{pmatrix}=(3-\lambda)(2-\lambda)-4(=0)$	M1	1.1b
	$\lambda^2-5\lambda+2=0$	A1	1.1b
		(2)	
(b)	$\mathbf{A}^2-5\mathbf{A}+2\mathbf{I}=0$	B1ft	1.1b
	Multiplies through by $\mathbf{A}^{-1}$ $\mathbf{A}-5\mathbf{I}+2\mathbf{A}^{-1}=0$ and rearranges to get $\mathbf{A}^{-1}=...$ **OR** Rearranges to make **I** the subject, takes out a factor of **A** and rearranges to get $\mathbf{A}^{-1}=...$ $\mathbf{I}=\frac{(5\mathbf{A}-\mathbf{A}^2)}{2}=\mathbf{A}\frac{(5\mathbf{I}-\mathbf{A})}{2}\Rightarrow\mathbf{A}^{-1}=...$ **OR** Rearranges to make **I** the subject and multiplies through by $\mathbf{A}^{-1}$ $\mathbf{I}=\frac{5}{2}\mathbf{A}-\frac{1}{2}\mathbf{A}^2\Rightarrow\mathbf{A}^{-1}=\frac{5}{2}\mathbf{A}\mathbf{A}^{-1}-\frac{1}{2}\mathbf{A}^2\mathbf{A}^{-1}$	M1	3.1a
	Identifies $\mathbf{A}^{-1}=-\frac{1}{2}\mathbf{A}+\frac{5}{2}\mathbf{I}$	A1	1.1b
		(3)	
			(5 marks)

Notes

(a)
M1: Complete method to find the characteristic equation, condone missing = 0
A1: Obtains a correct three term quadratic equation – may use any variable.

(b)
B1ft: Uses Cayley Hamilton Theorem to produce equation replacing λ with **A** and constant term with constant multiple of the identity matrix **I**

M1: A complete method using part (a) to find $\mathbf{A}^{-1}$

Multiplies through by $\mathbf{A}^{-1}$ and rearranges to get $\mathbf{A}^{-1}=...$

Or rearranges to make **I** the subject, takes out a factor of **A**, and rearranges to get $\mathbf{A}^{-1}=...$

Or rearranges to make **I** the subject and multiplies through by $\mathbf{A}^{-1}$ to get $\mathbf{A}^{-1}=...$

A1: Correct expression for $\mathbf{A}^{-1}$, must be using their answer to part (a).

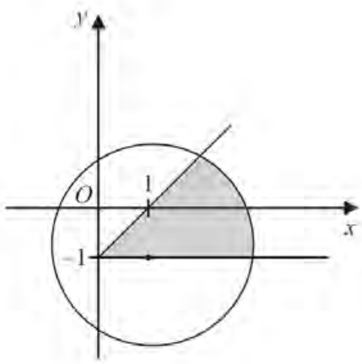
Question	Scheme		Marks	AOs
2(i)	For any correct value for a = 4, 6 or 12		M1	1.1b
	For all three correct values for a and no extras a = 4, 6 & 12		A1	1.1b
			(2)	
(ii)	$x^2 - 1$ is divisible by p **OR** $x^2 - 1 \equiv 0 \bmod p$ **OR** $p / (x^2 - 1)$		B1	1.1b
	$\therefore (x-1)(x+1)$ is divisible by p and **since p is prime** either $(x-1)$ is divisible by p or $(x+1)$ is divisible by p **OR** $\therefore (x-1)(x+1) \equiv 0 \bmod p$ and **since p is prime** either $x - 1 \equiv 0 \bmod p$ or $x + 1 \equiv 0 \bmod p$ **OR** $\therefore p/(x-1)(x+1)$ and **since p is prime** either $p/(x-1)$ or $p/(x+1)$		M1	2.1
	$\therefore \quad x \equiv 1 \bmod p \quad$ or $\quad x \equiv -1 \bmod p$ *		A1*	1.1b
			(3)	
(iii)	For selecting and performing a divisibility test for dividing by 11		M1	1.1b
	1 – 3 + 9 – 4 + 0 – 2 + 2 – 0 = 3 or 1 – 3 + 9 – 4 + 0 – 2 + 2 – 0 + 0 – 0 = 3 3 is not divisible by 11 or $11 \nmid 3$	Sum odd = 12 Sum even = 9 Difference = 3 which is not 0 or divisible by 11		
	Fully correct method with reason (must have correct sum ± 3) and conclusion £13 940 220 is not divisible by 11 Therefore, it is **not is it possible** to share this **money equally** between the 11 **charities**		A1	3.2a
			(2)	
				(7 marks)

Notes

(i)
M1: For an understanding of mod notation and finding a correct value for a = 4, 6 or 12
A1: For all three correct values for a and no extras a = 4, 6 & 12

(ii) see scheme

(iii)
M1: For applying a divisibility test for dividing by 11 to £13 940 220 or 139 402 2000p
A1: Fully correct method and concludes not divisible by 11 and interprets conclusion in context

Question	Scheme	Marks	AOs
3(a)	$(x-1)^2+(y-8)^2=9\left[(x-1)^2+y^2\right]$ Or $\sqrt{(x-1)^2+(y-8)^2}=3\sqrt{(x-1)^2+y^2}$	M1	2.1
	$8x^2-16x+8y^2+16y-56=0$	A1	1.1b
	$x^2-2x+y^2+2y-7=0$ so $(x-1)^2+(y+1)^2=9$ and finds the centre and radius	M1	1.1b
	Therefore, a circle with centre (1, −1) and radius = 3	A1	2.2a
		(4)	
(b)	Distance $=\sqrt{(3-1)^2+(-3--1)^2}=...$ or finds $(d^2=)(3-1)^2+(-3--1)^2=...$	M1	1.1b
	Distance $=\sqrt{8}=2.828<3 \therefore\ z=3-3\text{i}$ **satisfies the inequality** Or $8<9\ \therefore\ z=3-3\text{i}$ **satisfies the inequality**	A1	2.2a
		(2)	
(c)	 Circle with their centre and radius	M1	1.1b
	Circle with centre in the fourth quadrant	A1	1.1b
	Half line drawn from $(0,-1)$ and passes through the x-axis within the circle	M1	1.1b
	Correct region shaded	A1	2.2a
		(4)	
		(10 marks)	

Notes
(a) **M1:** Obtains an equation in terms of *x* and *y* using the given information. Condone $(x-1)^2+(y-8)^2=3\left[(x-1)^2+y^2\right]$ for this mark. **A1:** Expands and simplifies the algebra, collecting terms and obtains a correct equation. **M1:** Completes the square for their equation to find the centre and radius. **A1**: Deduces that it is a circle (may be seen anywhere in their solution) with centre (1, −1) and radius = 3
(b) **M1:** Finds the distance between $(3,-3)$ and their centre or d^2 (note: correct centre is $(1,-1)$ **A1:** Compares distance with 3 or compares d^2 with 9 and deduces that the inequality is satisfied – must be using correct centre and radius.
(c) **M1:** Circle for their centre and radius. **A1:** Correct circle with centre in the fourth quadrant and passing through all four quadrants. Condone dotted circle. **M1:** Half line drawn from $(0,-1)$ and passing the *x*-axis within the circle. Condone dotted line. **A1:** Correct region shaded with both half-line and circle correct and not dotted. **<u>Special case:</u>** M1A1M1A0 if no coordinates stated throughout and it is clear that the half-line intersects the coordinate axes level with the correct centre of the circle.

Question	Scheme	Marks	AOs
4(a)	$p*q = p*p*p*p = s*s = r$ OR $s*s = r \Rightarrow p*p*p*p = r \Rightarrow p*q = r$	B1	2.1
	$s*p = p*p*p = q$ OR as $p*p*p = q$ and $p*p = s \Rightarrow s*p = q$	B1	2.1
		(2)	
(b)	(see table below)	M1 A1	1.1b 1.1b
		(2)	
(c)	$p*q*r*s = e$	B1	1.1b
		(1)	
(d)	The order of a subgroup is a factor of the order of the group (Lagrange's Theorem)	M1	1.2
	As 3 is not a factor of 5, the student's statement is wrong	A1	2.3
		(2)	
		(7 marks)	

Table for (b):

*	**e**	**p**	**q**	**r**	**s**
e	e	p	q	r	s
p	p	s	r	e	q
q	q	r	p	s	e
r	r	e	s	q	p
s	s	q	e	p	r

Notes

(a)
B1: Correct proof to achieve the printed statement
B1: Correct proof to achieve the printed statement

(b) Marked B1 B1 on ePen
M1: Finds at least 13 correct entries – usually the highlighted
A1: Completely correct table

(c)
B1: See scheme

(d)
M1: Some indication that the order of a subgroup must be a factor of the order of the group. May say that 3 is not a factor of 5 or equivalent
A1: Fully correct unambiguous statement that refers Lagrange's theorem and either

- 3 is not a factor of 5
- 3 does not divide 5
- 5 is not divisible by 3

and comments that the student's statement is incorrect. No contradictory statements

Question	Scheme	Marks	AOs
5(a)	U_{n-1} is the amount in the saving account $n - 1$ years after Jim's 11th birthday. This is increased by 2% each year, so is multiplied by 1.02 to give $1.02U_{n-1}$	B1	3.3
	Jim's parents invest £500 for each subsequent birthday so 500 is added	B1	3.4
	$U_0 = 1000$ as this is the amount invested on Jim's 11th birthday	B1	1.1b
		(3)	
(b)	To use this model, one of, for example The interest rate stays the same each year Jim does not withdraw any money from the savings account Jim only saves the birthday money +£500 in this saving account, he does not invest any other money.	B1	3.5b
		(1)	
(c)	A complete method to solve the recurrence relation using $U_n =$ CF + PS = $c(1.02)^n + \lambda$	M1	3.1a
	PS $= \lambda \quad \Rightarrow \lambda = 1.02\lambda + 500$ leading to $\lambda = ...$	M1	1.1b
	$\lambda = -25\,000$	A1	1.1b
	Uses $U_0 = 1000$ and their value for λ to find the value of $1000 = c(1.02)^0 - 25\ 000$ $c = ...(26\ 000)$	M1	1.1b
	$U_n = 26\ 000(1.02)^n - 25\ 000 \qquad (n \geq 0)$	A1	1.1b
		(5)	
	Alternative 1 Realises that $U_n =$ term of a GP + sum of a GP both with $r = 1.02$	M1	3.1a
	Sum of a GP $= \dfrac{500(1-1.02^n)}{1-1.02}$ or $\dfrac{500(1.02^n - 1)}{1.02-1}$	M1 A1	1.1b 1.1b
	Term of a GP $= 1000(1.02)^n$ or $1000(1.02)^{n-1}$	M1	1.1b
	$U_n = 1000(1.02)^n - 25\ 000(1-1.02^n)$ or $U_n = 1000(1.02)^n + 25\ 000(1.02^n - 1)$	A1	1.1b
		(5)	
(d)	Uses $U_n = 26\ 000(1.02)^n - 25\ 000$, with either $n = 7$ or 8	M1	3.4
	$U_7 = 4865.83 > 4500$ therefore, Jim will have enough money in his savings account to buy a car costing £ 4500.	A1ft	2.2a
		(2)	
			(11 marks)

Notes
(a) **B1:** Need to explain that 2% interest rate linked to multiplication by scale factor 1.02 **B1:** Need to explain that 500 is added due to receiving £500 each year **B1:** Needs to explain that $U_0 = 1000$ is the initial amount invested
(b) **B1:** See main scheme
(c) **M1:** A complete method to solve the recurrence relation using $U_n = \text{CF + PS} = c(1.02)^n + \lambda$ **M1**: Uses $\text{PS} = \lambda \quad \Rightarrow \lambda = 1.02\lambda + 500$ to find a value for λ **A1:** $\lambda = -25\ 000$ **M1:** Uses U_0 and their value for λ to find a value of c **A1:** Fully correctly defined sequence $U_n = 26000(1.02)^n - 25\ 000, \quad (n \geq 0)$
Alternative 1 **M1:** A correct form for U_n term of a GP + Sum of a GP both with r = 1.02 **M1**: For the sum of a GP with a = 500, r = 1.02 and uses n or $n-1$ **A1:** Correct the sum of a GP with a = 500, r = 1.02 and n **M1:** For the term of a GP with a = 1000, r = 1.02 and uses n or $n-1$ **A1:** Fully correctly defined sequence U_n
(d) **M1:** Uses their U_n with either n = 7 or 8 **A1ft:** Finds U_7 compares with 4 500 and comes to an appropriate conclusion. Follow through on their value of U_7