AQA

A-LEVEL
MATHEMATICS
7357/3

Paper 3

Mark scheme
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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
'their'	Indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
sf	significant figure(s)
dp	decimal place(s)

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Examiners should consistently apply the following general marking principles

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to students showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the student to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

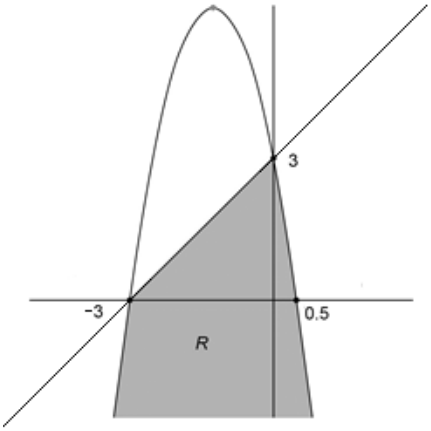
Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

Q	Marking instructions	AO	Mark	Typical solution
1	Ticks the correct response	1.2	B1	$\{x \in \mathbb{R} : -1 \leq x \leq 1\}$
	Total		**1**	

Q	Marking instructions	AO	Mark	Typical solution
2	Circles the correct response	1.1b	B1	1650
	Total		**1**	

Q	Marking instructions	AO	Mark	Typical solution
3	Circles the correct response	2.2a	B1	$u_n = 2 - 0.9^{n-1}$
				$u_n = 2 - 0.9^{n-1}$
	Total		**1**	

Q	Marking instructions	AO	Mark	Typical solution
4	Draws quadratic curve in the correct orientation eg vertex above x-axis and two intersections on the x-axis	1.1a	M1	
	Labels all correct points of intersection for the correct quadratic curve with vertex clearly in the 2nd quadrant Must see −3, 0.5 and 3	1.1b	A1	
	Draws **correct straight line** passing through (−3, 0) and (0, 3) or straight line which intersects their quadratic curve on the negative x-axis and positive y-axis **and shades** corresponding region for their quadratic curve **FT** their quadratic All lines must be **solid** Condone missing label R	2.2a	A1F	
	Total		**3**	

Q	Marking instructions	AO	Mark	Typical solution
5	Uses appropriate method to find radius eg complete the square by using 3^2 or 4^2 on LHS or RHS **PI** by correct radius 17 or 289	3.1a	M1	$(x-3)^2-9+(y-4)^2-16=264$ $(x-3)^2+(y-4)^2=289$ $\frac{1}{2}\times17^2\times0.9=130.05$ $\frac{1}{2}\times17^2\sin0.9=113.19$ Area of segment = 16.9
	Deduces correct radius or radius squared or fully correct completed square form seen	2.2a	A1	
	Uses appropriate method to find area of sector using radius 17 or their stated value of radius or value of radius clearly shown on diagram	1.1a	M1	
	Uses appropriate method to find area of triangle using radius 17 or their stated radius	1.1a	M1	
	Obtains area correct to at least 3 significant figures **AWRT** 16.9	1.1b	A1	
	Total		**5**	

Q	Marking instructions	AO	Mark	Typical solution
6(a)	States an appropriate even Pythagorean triple	2.2a	B1	$a=6$ $b=8$ $c=10$
6(b)	Begins an appropriate method of proof assuming at least two sides are odd eg states 'assume a, b odd' or defines a, b (or c) algebraically with different unknowns	3.1a	B1	Assume a and b are odd so a = 2m + 1 and b = 2n + 1 $(2m+1)^2$ + $(2n+1)^2$ = $4m^2$ + $4m$ + 1 + $4n^2$ + $4n$ + 1 = $2(2m^2+2m+2n^2+2n+1)$ which is even, so c^2 is even, so c is even. Therefore it is not possible for all three to be odd.
	Uses Pythagoras' theorem with at least two odd sides either in words or algebraically	1.1a	M1	
	Completes rigorous argument to prove the required result **CSO**	2.1	R1	
	Total		**4**	

Q	Marking instructions	AO	Mark	Typical solution
7(a)	Forms $4x+3 \equiv A(x-1)+B$	1.1b	B1	$\frac{4x+3}{(x-1)^2} \equiv \frac{A}{x-1}+\frac{B}{(x-1)^2}$ $4x+3 \equiv A(x-1)+B$
	Uses substitution or comparison of coefficients to find their A or B (must have degree of LHS = degree of RHS)	1.1a	M1	Let x = 1 hence B = 7 Let x = 0 then 3 = B – A and hence A = 4
	Obtains correct A and B	1.1b	A1	A = 4 and B = 7
7(b)	Integrates their expression, at least one term correct	3.1a	M1	$\int_3^4 \left(\frac{4}{x-1}+\frac{7}{(x-1)^2}\right)dx$
	Integrates their expression fully correctly Must be of the form $A\ln(x-1)-\frac{B}{x-1}$ **OE** **FT** their A and B	1.1b	A1F	$=\left[4\ln(x-1)-\frac{7}{x-1}\right]_3^4$ $=\left[4\ln 3-\frac{7}{3}\right]-\left[4\ln 2-\frac{7}{2}\right]$ $=4\ln\frac{3}{2}+\frac{7}{6}$
	Substitutes limits correctly into their integrated expression	1.1a	M1	$=\ln\frac{3^4}{2^4}+\frac{7}{6}$
	Uses at least one law of logs correctly	1.1a	M1	$=\ln\frac{81}{16}+\frac{7}{6}$
	Completes argument to obtain **correct exact answer** in correct form or stating $p=\frac{7}{6}$ and $q=\frac{81}{16}$ No subsequent incorrect working	2.1	R1	
	Total		**8**	

Q	Marking instructions	AO	Mark	Typical solution
8(a)	Uses model with $t = 0$ and $\theta = 75$ to form an equation	3.4	M1	$75 = 5\left(4 + \lambda e^{0}\right)$
	Obtains correct λ	1.1b	A1	$\lambda = 11$
	Uses model with $t = 2$, $\theta = 68$ and their λ to form an equation	3.4	M1	$68 = 5\left(4 + 11e^{-2k}\right)$
	Solves their equation correctly to find k	1.1a	M1	$k = 0.068066$
	Obtains correct k AWRT 0.07 **OE**	1.1b	A1	$\theta = 5\left(4 + 11e^{-0.068066\times 15}\right)$
	Uses model with their λ and their k and $t = 15$	3.4	M1	$= 39.8^{\circ}C$
	Obtains correct temperature Condone missing units **AWRT** 39.8	1.1b	A1	
8 (b)(i)	States correct room temperature Condone missing units **CAO**	3.4	B1	$20^{\circ}C$
	Explains that the temperature predicted by the model will approach room temperature as t increases. **OE**	2.4	E1	As t gets large the temperature predicted by the model will get close to room temperature
8 (b)(ii)	Uses the model with their k and their room temperature+1 to form equation for t	3.4	M1	$5\left(4 + 11e^{-0.068066t}\right) = 21$
	Obtains the correct value of t **AWRT 59** **ISW**	1.1b	A1	$t = 58.87$
8(c)	Room temperature change/higher/lower Cooling rate change/higher/lower or identifies a factor that may be different in a different place.	3.5a	E1	The new room temperature might change
	Total		**12**	

Q	Marking instructions	AO	Mark	Typical solution
9(a)	Demonstrates by substitution that $x=0$ or $y=0$ leads to value on the LHS = 0	2.4	E1	When x = 0 $0^2y^2 + 0y^4 = 0$ When y = 0 $x^20^2 + x0^4 = 0$ This is a contradiction because $x^2y^2 + xy^4 = 12$ so the curve does not intersect either axis
	Completes rigorous argument to show required result	2.1	R1	
9 (b)(i)	Uses implicit differentiation	3.1a	M1	$2xy^2 + 2x^2y\frac{dy}{dx} + y^4 + 4xy^3\frac{dy}{dx} = 0$ $\frac{dy}{dx} = -\frac{2xy^2+y^4}{2x^2y+4xy^3}$ $= -\frac{y(2xy+y^3)}{y(2x^2+4xy^2)}$ $= -\frac{2xy+y^3}{2x^2+4xy^2}$,
	Product rule used LHS (at least one pair of terms correct)	1.1a	M1	
	Differentiates equation of curve fully correctly	1.1b	A1	
	Collects their $\frac{dy}{dx}$ terms in an equation and factorises	3.1a	M1	
	Completes convincing argument to obtain required result by **factorising** then simplifying y **AG**	2.1	R1	
9 (b)(ii)	Begins argument by setting $\frac{dy}{dx}=0$ to form an equation for x and y **PI** by $2xy+y^3=0$	2.1	M1	For stationary points $\frac{dy}{dx}=0$ $\Rightarrow 2xy+y^3=0$ $\Rightarrow y^2=-2x$ $\Rightarrow x^2y^2+x(-2x)y^2=12$ $\Rightarrow -x^2y^2=12$ Since $-x^2y^2 < 0$ there can be no stationary points.
	Obtains $y^2=-2x$ or $y=\sqrt{-2x}$ or $x=\frac{-y^2}{2}$	1.1b	A1	
	Substitutes $y^2=-2x$ or $x=\frac{-y^2}{2}$ into equation for curve	1.1a	M1	
	Completes convincing argument to deduce the required result	2.2a	R1	
9 (b)(iii)	Substitutes y = 1 into equation of curve to obtain correct quadratic **ACF**	3.1a	M1	$y=1\Rightarrow x^2+x-12=0$ $\Rightarrow x=3 \quad (x>0)$ $\Rightarrow \frac{dy}{dx}=-\frac{7}{30}$ $y-1=-\frac{7}{30}(x-3)$
	Deduces x = 3 **PI** by substituting their x in their dy/dx	2.2a	R1	
	Substitutes their x and y = 1 in their dy/dx	1.1a	M1	
	Obtains correct equation of tangent **ACF** **ISW**	1.1b	A1	
	Total		**15**	

Q	Marking Instructions	AO	Mark	Typical Solution
10	Ticks correct box	1.2	B1	Strong negative
	Total		**1**	

Q	Marking Instructions	AO	Mark	Typical Solution
11	Circles correct answer	1.2	B1	Quota
	Total		**1**	

Q	Marking Instructions	AO	Mark	Typical Solution
12(a)	Calculates correct value of mean (accept 161)	1.1b	B1	$\overline{x}$ =160.6 sd = 6.8 160.6 – 2 × 6.8 = 147 146 < 147 Hence Ann is an outlier
	Calculates correct value of standard deviation (accept 7.2 or better)	1.1b	B1	
	Uses their $\overline{x}$ *and* their $s.d$ in $\overline{x} - 2 \times s.d$ (accept 146.2)	1.1b	M1	
	Compares 146 with their calculation and correctly concludes that Ann's height is an outlier **FT** their $\overline{x}$ and their s.d	2.1	R1F	
12(b)	States correctly that the mean would **increase** with a valid reason **or** **increases** to 162.2 Accept the mean would increase as the lower/lowest value has been removed or other valid reason	2.2b	B1	The mean would increase because Ann's height is less than the mean Standard deviation would decrease because Ann's height is an outlier
	States correctly that the standard deviation would **decrease** with a valid reason **or** **decreases** to 5.03 Accept the standard deviation would decrease because the data is less spread out or other valid reason	2.2b	B1	
	Total		**6**	

Q	Marking Instructions	AO	Mark	Typical Solution
13 (a)(i)	Obtains correct mean	1.1b	B1	6
13 (a)(ii)	Obtains correct variance	1.1b	B1	4.8
13 (b)(i)	Uses the Binomial formula with n = 30, p = 0.2 or $P(X \leq 10) - P(X \leq 9)$ **PI** by correct answer	1.1a	M1	$P(X = 10) = \binom{30}{10} 0.2^{10} 0.8^{20}$ $= 0.0355$
	Obtains correct probability **AWFW** [0.035, 0.036]	1.1b	A1	
13 (b)(ii)	Calculates either $P(X \leq 4) = 0.255$ or $P(X \leq 5) = 0.4275$ using the Binomial distribution	3.1b	M1	$P(X \leq 4) = 0.255$ $P(X \geq 5) = 1 - P(X \leq 4)$ $= 1 - 0.255$ $= 0.745$
	States $P(X \geq 5) = 1 - P(X \leq 4)$ or subtracts their stated value of $P(X \leq 4)$ from 1	1.1b	M1	
	Obtains correct probability **AWFW** [0.74, 0.75]	1.1b	A1	
13 (c)(i)	Raises their 0.745 to power of 5	3.1b	M1	$0.745^5 = 0.229$
	Obtains their correct probability **FT** their 0.745 **AWRT** their 0.229	1.1b	A1F	
13 (c)(ii)	Gives a valid reason that probability/likelihood/chances may change/increase/decrease as a result of external factor change over 5 day period or Patrick improves	3.5b	E1	Probability may change as Patrick improves
	Total		**10**	

Q	Marking Instructions	AO	Mark	Typical Solution
14 (a)(i)	Finds correct probability **OE**	1.1b	B1	$\frac{10}{120}$
14 (a)(ii)	Finds total number for 'depression'	1.1a	M1	9 + 2 + 1 = 12
	Calculates correct probability **OE**	1.1b	A1	$\frac{12}{120}$
14 (a)(iii)	Uses conditional probability to calculate $\mathrm{P}(\text{stress}\|\text{low exercise})$ to obtain $\frac{38}{39\leq x\leq 119}$	1.1a	M1	$\frac{38}{50}$
	Obtains correct probability **ACF**	3.1b	A1	
14(b)	Shows that 14+38 or 52 or $\frac{14}{50}+\frac{38}{50}$	3.1b	M1	14 + 38 = 52 52 > 50 so events are not mutually exclusive
	Compares 14+38 with 50 or compares $\frac{14}{50}+\frac{38}{50}$ with 1 and concludes events are not mutually exclusive	2.4	R1	
	Total		**7**	

Q	Marking Instructions	AO	Mark	Typical Solution
15	Identifies critical value = 0.549	1.1b	B1	0.567 > 0.549 There is sufficient evidence that the larger the rainfall, the greater the yield.
	Compares 0.567 correctly to their critical value chosen from the table	3.5a	M1	
	Makes correct inference eg there is sufficient/significant evidence that the larger the rainfall, the greater the yield/positive correlation **between** the two **FT** their critical value chosen from the table	2.2b	R1F	
	Total		**3**	

Q	Marking Instructions	AO	Mark	Typical Solution
16(a)	States correct first reason involving the y-axis Accept there is no scale on the *y*-axis **or** graph does not start at 0	2.3	E1	Scale on y-axis does not start at zero.
	States correct second reason involving salt purchased and consumed implying data not comparable	2.3	E1	Data is for salt purchased as separate food stuff, not consumed
16(b)	States both hypotheses correctly for two-tailed test Accept H_0: **population** mean is 78.9	2.5	B1	$H_0: \mu = 78.9$ $H_1: \mu \neq 78.9$
	Formulates the test statistic or uses the correct distribution of the sample mean **PI** by correct test statistic value or probability or acceptance region Condone $78.9 - 80.4$ If region used, condone any z = (- 4, 4)	1.1a	M1	$\text{Test statistic} = \frac{80.4 - 78.9}{25.0/\sqrt{918}}$ $= 1.82$ Critical z value 1.96 1.82 < 1.96
	Obtains the correct value of the test statistic 1.82 or obtains the correct probability 0.0345 or 0.0691 obtains acceptance region of [77.3, 80.5]	1.1b	A1	Accept H_0 - there is insufficient evidence to suggest that the mean amount of sugar purchased has changed
	Compares their 1.82 with 1.96 or compares their 0.0345 with 0.025 compares their 0.0691 with 0.05 or compares 80.4 with their region [77.3, 80.5]	1.1a	M1	
	Infers H_0 accepted **CSO** Must refer to H_0	2.2b	A1	
	Correctly concludes in context that there is **insufficient evidence** to suggest that the **mean** amount of **sugar purchased** has **changed** **CSO**	3.2a	E1	
16(c)	Explains role of significance level in rejecting null hypothesis in error Accept Type I error	2.3	E1	There is a 10% chance of rejecting null hypothesis in error
	Explains that there is 10 % chance for this to occur Reference to 10 % chance the conclusion is incorrect scores E0E1	2.3	E1	
	Total		**10**	

Q	Marking Instructions	AO	Mark	Typical Solution
17(a)	Obtains either z-value from inverse normal distribution Condone sign error **AWFW** [−1.29, −1.28] or [−0.85, −0.84]	3.1b	B1	$P\left(Z < \frac{30-\mu}{\sigma}\right) = 0.1$ $P\left(Z > \frac{32.5-\mu}{\sigma}\right) = 0.8$ z= −1.2816 z = −0.8416 $\frac{30-\mu}{\sigma} = -1.2816$ $\frac{32.5-\mu}{\sigma} = -0.8416$ $2.5 = 0.44\sigma$ $\sigma = 5.68$ $\mu = 37.3$
	Forms one equation with unknown μ and σ using standardised result and z-value (for 0.1) Accept z = (−4, 4) except ±0.1, ±0.2, ±0.8, ±0.9 Condone $\mu - 30$ Must use 30	1.1a	M1	
	Forms next equation with unknown μ and σ using standardised result and z-value (for 0.8) Accept z = (−4, 4) except ±0.1, ±0.2, ±0.8, ±0.9 Condone $\mu - 32.5$ Must use 32.5	1.1a	M1	
	Obtains both equations **correctly**	1.1b	A1	
	Solves their two simultaneous equations in the form of μ and σ	1.1a	M1	
	Obtains correct value of σ **AWFW** (5.2, 5.9) **ISW**	1.1b	A1	
	Obtains correct value of μ **AWFW** (37.1, 37.5) **ISW**	1.1b	A1	
17 (b)(i)	States correct probability	1.2	B1	1
17 (b)(ii)	Uses their μ and their σ to find $P(X < 35)$ **PI** by correct value of probability using their μ and their σ or correctly calculated z-value using their μ and their σ	1.1a	M1	$P(X < 35) = 0.344$
	Obtains correct probability to 2 decimal places or better **FT** their μ and their σ If μ = (37.1, 37.5) and σ = (5.2, 5.9) used, answer will be (0.31, 0.37)	1.1b	A1F	
17(c)	Identifies the Binomial distribution model with $n = 13,\ p = \text{their } 0.344$ **PI** by correct value of probability using their p	3.1b	M1	Y= no. of brownies less than 35g in a batch of 13 $Y \sim B(13, 0.344)$ $P(\mathrm{Y} \leq 3) = 0.294$
	Obtains correct probability to 2 decimal places or better **FT** their p If p = (0.31, 0.37) answer will be [0.23, 0.39]	1.1b	A1F	
	Total		**12**	