AS
FURTHER MATHEMATICS
7366/2D

Paper 2 Discrete

Mark scheme

June 2019

Version: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
'their'	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
sf	significant figure(s)
dp	decimal place(s)

Examiners should consistently apply the following general marking principles

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

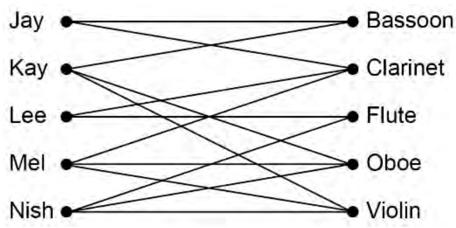
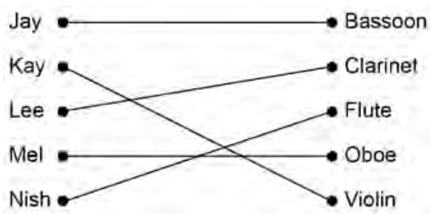
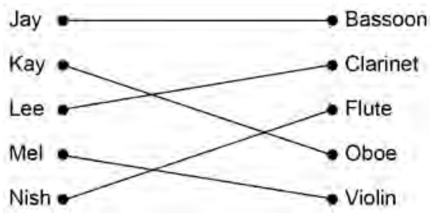
AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Q	Marking Instructions	AO	Marks	Typical Solution
1	Circles correct answer	AO1.1b	B1	31
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
2	Circles correct answer	AO1.1b	B1	$d = 10$
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
3	Defines two variables to represent the **number of** apple cakes and the **number of** banana cakes	AO3.1b	B1	x = number of apple cakes y = number of banana cakes Maximise $P = x + y$ subject to
	Obtains the correct non-trivial constraint for eggs or flour (condone strict inequality)	AO1.1a	M1	$3x + 2y \leq 36$ $100x + 150y \leq 1500$
	Obtains both correct non-trivial constraints for eggs and flour (condone strict inequality)	AO1.1b	A1	$x \geq 0$, $y \geq 0$ x, y are integer
	Formulates the linear programming problem correctly with statement of maximising $x + y$ and all constraints fully correct (Condone x, y are integer missing)	AO2.5	A1	
	Total		**4**	

Q	Marking Instructions	AO	Marks	Typical Solution
4(a)	Defines bipartite graph with reference to two sets of vertices and that vertices of the same set are not connected	AO1.2	B1	A bipartite graph is one in which the vertices can be split into two sets where no edge connects vertices in the same set
4(b)(i)	Sets up 10 correctly labelled vertices (Accept abbreviations, e.g. J for Jay) Draws correct bipartite graph showing all connections	AO1.1a AO1.1b	M1 A1	
4(b)(ii)	Uses graph theory terminology to explain that the vertex representing Nish is not connected to the vertex representing the bassoon or Uses graph theory terminology to explain that the vertex representing Nish is only connected to the vertices representing the flute, oboe and violin	AO2.4	E1	The vertex for Nish is not connected to the vertex for bassoon
4(b)(iii)	Draws at least one correct subgraph Draws both correct subgraphs and no others	AO3.1b AO1.1b	M1 A1	 
	Total		**6**	

<table>
<tr><th>Q</th><th>Marking Instructions</th><th>AO</th><th>Marks</th><th>Typical Solution</th></tr>
<tr><td>5(a)</td><td>Partially completes Cayley table (3 correct rows or 3 correct columns)

Fully completes Cayley table correctly</td><td>AO1.1a

AO1.1b</td><td>M1

A1</td><td>
<table>
<tr><th>$\times_4$</th><th>0</th><th>1</th><th>2</th><th>3</th></tr>
<tr><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>1</td><td>0</td><td>1</td><td>2</td><td>3</td></tr>
<tr><td>2</td><td>0</td><td>2</td><td>0</td><td>2</td></tr>
<tr><td>3</td><td>0</td><td>3</td><td>2</td><td>1</td></tr>
</table>
</td></tr>
<tr><td>5(b)(i)</td><td>Completes table correctly</td><td>AO1.1b</td><td>B1</td><td>
<table>
<tr><th>$\bullet$</th><th>a</th><th>b</th><th>c</th><th>d</th></tr>
<tr><td>a</td><td>b</td><td>a</td><td>a</td><td>c</td></tr>
<tr><td>b</td><td>a</td><td>c</td><td>d</td><td>c</td></tr>
<tr><td>c</td><td>a</td><td>d</td><td>d</td><td>d</td></tr>
<tr><td>d</td><td>c</td><td>c</td><td>d</td><td>d</td></tr>
</table>
</td></tr>
<tr><td>5(b)(ii)</td><td>Sets up a test for associativity with at least two different elements of S

Constructs complete mathematical argument to justify non-associativity</td><td>AO1.1a

AO2.1</td><td>M1

R1</td><td>$a \bullet (b \bullet c) = a \bullet d = c$

$(a \bullet b) \bullet c = a \bullet c = a$

$a \bullet (b \bullet c) \neq (a \bullet b) \bullet c$
Therefore $\bullet$ is not associative</td></tr>
<tr><td></td><td>Total</td><td></td><td>5</td><td></td></tr>
</table>

Q	Marking Instructions	AO	Marks	Typical Solution
6(a)	Sets up a model by identifying the problem as a route inspection problem by noting that *B*, *E*, *G* and *H* are odd-degree nodes (PI)	AO3.3	M1	Odd nodes are *B, E, G, H*
	Uses the model to find six distances between the odd-degree nodes with at least four correct shortest distances (PI)	AO3.4	M1	Shortest Distances *B–E*: 195 *B–G*: 105 *B–H*: 110 *E–H*: 250 *E–G*: 230 *G–H*: 50
	Finds all three correct minimum pairs of shortest distances	AO1.1b	A1	Pairings (*B–E*)(*G–H*) = 195 + 50 = 245* (*B–G*)(*E–H*) = 105 + 250 = 355 (*B–H*)(*E–G*) = 110 + 230 = 340
	Uses their shortest pairing to determine the total distance Ashley will cover during the journey	AO1.1a	M1	245 m is the shortest length to be repeated. 1465 + 245 = 1710 m 1710 m = 1.710 km
	Determines the correct least possible time using their total distance with use of 1465 metres	AO3.2a	A1F	1.710 ÷ 7 = 0.244 hours
6(b)(i)	Identifies the weights of the two arcs representing the shortest paths from entrance	AO1.1b	B1	The two shortest arcs from the entrance are *AB:* 45 *AG:* 60
	Finds the correct lower bound	AO1.1b	B1	Lower bound = 510 + 45 + 60 = 615 metres
6(b)(ii)	Uses the nearest neighbour algorithm beginning at *A* to find the correct Hamiltonian cycle (PI)	AO1.1a	M1	*A–B–C–D–E–F–I–H–G–A*
	Determines correctly the upper bound	AO1.1b	A1	Upper bound = 45 + 50 + 60 + 100 + 90 + 75 + 85 + 50 + 60 = 615 metres

6(c)(i)	Justifies the optimal distance for Brook's journey Calculates correctly a time from their lower bound or their upper bound for Brook's journey Shows that the journeys for Ashley and Brook have approximately the same time by comparing the journey times in consistent units	AO2.1 AO3.2a AO3.2a	R1 M1 A1	The upper bound and lower bound are both 615 metres, so the optimal distance is 615 metres $0.615 \div 5 + 8 \times \frac{1}{60}$ $= 0.256$ hours The times for Ashley and Brook are both about ¼ of an hour
6(c)(ii)	Recognises a limitation of model and states a plausible assumption in context	AO3.5b	B1	Ashley and Brook work at a constant rate with no breaks or The speed at which Ashley and Brook move on the paths remains constant whether or not the grass has been cut
	Total		**13**	

<table>
<tr><th>Q</th><th>Marking Instructions</th><th>AO</th><th>Marks</th><th>Typical Solution</th></tr>
<tr><td>7(a)(i)</td><td>Writes down a transposed matrix of 3 rows and 4 columns with any twelve elements

Writes down fully correct pay-off matrix for Bex</td><td>AO1.1a

AO1.1b</td><td>M1

A1</td><td>
<table>
<tr><td></td><td></td><td colspan="4">Ali</td></tr>
<tr><td></td><td>Strategy</td><td>A_1</td><td>A_2</td><td>A_3</td><td>A_4</td></tr>
<tr><td rowspan="3">Bex</td><td>B_1</td><td>–2</td><td>4</td><td>0</td><td>3</td></tr>
<tr><td>B_2</td><td>1</td><td>2</td><td>–1</td><td>–2</td></tr>
<tr><td>B_3</td><td>–3</td><td>–2</td><td>–1</td><td>2</td></tr>
</table>
</td></tr>
<tr><td>7(a)(ii)</td><td>Explains, using game theory terminology, that strategy $\mathbf{B}_1$ dominates strategy $\mathbf{B}_3$

Explains, using game theory terminology, how strategy $\mathbf{A}_2$ is dominated</td><td>AO2.4

AO2.4</td><td>E1

E1</td><td>$-3 < -2$, $-2 < 4$, $-1 < 0$, $2 < 3$ hence strategy $\mathbf{B}_1$ dominates strategy $\mathbf{B}_3$

$-2 < 4$, $1 < 2$, $-3 < -2$, hence strategy $\mathbf{A}_1$ dominates strategy $\mathbf{A}_2$</td></tr>
<tr><td>7(b)</td><td>Introduces and defines a probability variable (PI)

Uses the model to find one (unsimplified) expected gain for Bex

Finds correctly all three (unsimplified) expected gains for Bex

Draws a graph with at least one vertical axis and their 3 expected gains (allow one slip)

Identifies correctly the optimal point of intersection from the graph and finds the correct value of the probability variable

Deduces their value of the game for Ali</td><td>AO3.3

AO3.4

AO1.1b

AO1.1a

AO1.1b

AO2.2a</td><td>B1

M1

A1

M1

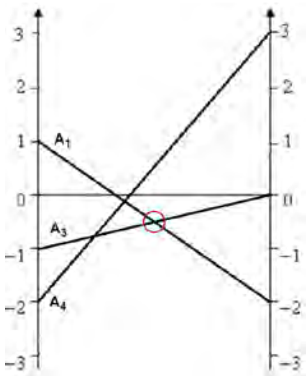
A1

B1F</td><td>Let Bex choose strategy $\mathbf{B}_1$ with probability p and strategy $\mathbf{B}_2$ with probability $1 - p$

If Ali plays:
$\mathbf{A}_1$: expected gain for Bex
$= -2p + 1(1 - p) = -3p + 1$

$\mathbf{A}_3$: expected gain for Bex
$= -1(1 - p) = p - 1$

$\mathbf{A}_4$: expected gain for Bex
$= 3p - 2(1 - p) = 5p - 2$



$p - 1 = -3p + 1$
$p = \frac{1}{2}$

Value of game for Bex
$= \frac{1}{2} - 1 = -\frac{1}{2}$

Hence, value of game for Ali $= \frac{1}{2}$</td></tr>
<tr><td></td><td align="right">Total</td><td></td><td>10</td><td></td></tr>
<tr><td></td><td align="right">Total</td><td></td><td>40</td><td></td></tr>
</table>