AQA

A-level
FURTHER MATHEMATICS
7367/1

Paper 1

Mark scheme

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
'their'	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
sf	significant figure(s)
dp	decimal place(s)

Examiners should consistently apply the following general marking principles

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Q	Marking Instructions	AO	Marks	Typical Solution
1	Circles correct answer	AO2.2a	B1	$\tanh^{-1} x$
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
2	Circles correct answer	AO2.2a	B1	$x\cos x$
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
3	Circles correct answer	AO1.1b	B1	-0.237
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
4	Uses correct conjugate of z and expresses equation in terms of x and y where x and y are real	AO1.1a	M1	$z = x + \mathrm{i}y$ $2(x+\mathrm{i}y) - 5\mathrm{i}(x - \mathrm{i}y) = 12$
	Equates real and imaginary parts of their equation (conjugate might be wrong).	AO1.1a	M1	Re: $2x - 5y = 12$ Im: $2y - 5x = 0 \Rightarrow y = 2.5x$ $2x - 12.5x = 12$
	Solves their equations correctly for x and y having used the correct conjugate of z	AO1.1a	M1	$x = -\frac{8}{7}$ and $y = -\frac{20}{7}$
	States a fully correct solution, must be z =…	AO1.1b	A1	$z = -\frac{8}{7} - \frac{20}{7}\mathrm{i}$
	Total		**4**	

Q	Marking Instructions	AO	Marks	Typical solution
5	Finds scalar (or vector) product of the correct vectors PI by seeing AWRT 35°	AO1.1a	M1	$\begin{bmatrix}1\\0\\1\end{bmatrix} \cdot \begin{bmatrix}1\\1\\1\end{bmatrix} = 2$
	Divides their scalar product (or magnitude of vector product) of their vectors by product of their magnitudes PI by seeing AWRT 35°	AO1.1a	M1	Moduli of vectors are $\sqrt{2}$ and $\sqrt{3}$ Let α be angle between normal & line $\cos\alpha = \frac{2}{\sqrt{6}}$
	Deduces the correct angle, correct to at least 1dp	AO2.2a	A1	Angle between plane & line $= 90 - \alpha = 54.7^{\circ}$
	Total		**3**	

Q	Marking Instructions	AO	Marks	Typical Solution
6(a)	Uses correct expressions for $\cosh x$ and $\sinh x$,and uses them to simplify LHS.	AO1.1a	M1	$\cosh^3 x = \frac{1}{8}\left(e^{3x} + 3e^{x} + 3e^{-x} + e^{-3x}\right)$
	Finds a correct, unsimplified expansion of the LHS in terms of exponentials.	AO1.1b	A1	$\sinh^3 x = \frac{1}{8}\left(e^{3x} - 3e^{x} + 3e^{-x} - e^{-3x}\right)$
	Completes a rigorous argument to obtain the correct result. Must include clear definitions for $\cosh x$ & $\sinh x$. NMS = 0/3	AO2.1	R1	$\cosh^3 x + \sinh^3 x = \frac{1}{4}e^{3x} + \frac{3}{4}e^{-x}$
6(b)	Finds $\cosh^3 x - \sinh^3 x$ in exponential form PI correct exponential expression.	AO3.1a	B1	$\cosh^6 x - \sinh^6 x = \left(\cosh^3 x + \sinh^3 x\right)\left(\cosh^3 x - \sinh^3 x\right)$ $\cosh^3 x - \sinh^3 x = \frac{3}{4}e^{x} + \frac{1}{4}e^{-3x}$
	Uses their expressions to find $\cosh^6 x - \sinh^6 x$ in exponential form.	AO3.1a	M1	
	Obtains their correct result	AO1.1b	A1F	$\cosh^6 x - \sinh^6 x = \left(\frac{1}{4}e^{3x} + \frac{3}{4}e^{-x}\right)\left(\frac{3}{4}e^{x} + \frac{1}{4}e^{-3x}\right)$
	Correctly separates out $\frac{3}{8}\cosh 4x$ or equivalent from their expression	AO2.2a	M1	$= \frac{3}{16}e^{4x} + \frac{9}{16} + \frac{1}{16} + \frac{3}{16}e^{-4x}$ $= \frac{3}{8}\left(\frac{e^{4x} + e^{-4x}}{2}\right) + \frac{5}{8}$ $= \frac{3\cosh 4x + 5}{8}$
	Completes a rigorous argument to obtain the correct result.	AO2.1	R1	
	Total		**8**	

Q	Marking Instructions	AO	Marks	Typical solution
7(a)	Finds the correct matrix for $\mathbf{R}^{-1}$ PI by correct $\mathbf{A}$	AO2.2a	B1	$\mathbf{R}^{-1} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\mathbf{A} = \mathbf{B}\mathbf{R}^{-1}$ $\mathbf{A} = \begin{bmatrix} -\cos\theta & \sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\mathbf{A} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\mathbf{A}$ is independent of θ .
	Appropriate method to find $\mathbf{A}$, such as post multiplying $\mathbf{B}$ by $\mathbf{R}^{-1}$ PI by correct $\mathbf{A}$	AO1.1a	M1	
	Completes a rigorous argument to show the required result, including finding the correct matrix for $\mathbf{A}$. Must include conclusion that $\mathbf{A}$ is independent of θ	AO2.1	R1	
7(b)	States fully correct (single) geometrical description. Eg Reflection in y/z plane.	AO3.2a	E1	Reflection in $x = 0$ plane.
	Total		**4**	

Q	Marking Instructions	AO	Marks	Typical Solution
8(a)	Obtains correct expression for z^n in terms of $\cos n\theta$ and $\sin n\theta$	AO1.1b	B1	$z^n = (\cos\theta + i\sin\theta)^n$ $= \cos n\theta + i\sin n\theta$ $\frac{1}{z^n} = (\cos\theta + i\sin\theta)^{-n}$ $= \cos(-n\theta) + i\sin(-n\theta)$ $= \cos n\theta - i\sin n\theta$ $z^n - \frac{1}{z^n} = \cos n\theta + i\sin n\theta - (\cos n\theta - i\sin n\theta)$ $= 2i\sin n\theta$
	Obtains correct expression for $\frac{1}{z^n}$ in terms of $\cos n\theta$ and $\sin n\theta$ Or expresses whole LHS as $\frac{-2\sin^2 n\theta + 2i\cos n\theta \sin n\theta}{\cos n\theta + i\sin n\theta}$	AO1.1b	B1	
	Completes a rigorous argument (with all intermediate steps) to show the required result, using properties of sine and cosine functions to obtain results in terms of $\cos n\theta$ and $\sin n\theta$	AO2.1	R1	
8(b)	Selects the correct process by expanding $\left(z - \frac{1}{z}\right)^5$	AO3.1a	M1	$\left(z - \frac{1}{z}\right)^5 = z^5 - 5z^3 + 10z - 10z^{-1} + 5z^{-3} - z^{-5}$ $(2i\sin\theta)^5 = z^5 - z^{-5} - 5(z^3 - z^{-3}) + 10(z - z^{-1})$ $32i\sin^5\theta = 2i\sin 5\theta - 5(2i\sin 3\theta) + 10(2i\sin\theta)$ $\sin^5\theta = \frac{1}{16}\sin 5\theta - \frac{5}{16}\sin 3\theta + \frac{5}{8}\sin\theta$
	Obtains three pairs of terms in the form $z^n - \frac{1}{z^n}$ (ignore LHS)	AO1.1a	M1	
	Replaces each pair with the correct $\sin n\theta$ (ignore coefficients and LHS)	AO1.1a	M1	
	Obtains correct result	AO1.1b	A1	
8(c)	Integrates their answer to part (b) correctly, provided all terms in integrand are of the form $k\sin n\theta$	AO1.1a	M1	$\int_0^{\pi/3} \sin^5\theta \, d\theta = \int_0^{\pi/3} \left(\frac{1}{16}\sin 5\theta - \frac{5}{16}\sin 3\theta + \frac{5}{8}\sin\theta\right) d\theta$ $= \left[\frac{-1}{80}\cos 5\theta + \frac{5}{48}\cos 3\theta - \frac{5}{8}\cos\theta\right]_0^{\pi/3}$ $= \left(-\frac{1}{80}\cos\frac{5\pi}{3} + \frac{5}{48}\cos\frac{3\pi}{3} - \frac{5}{8}\cos\frac{\pi}{3}\right) - \left(-\frac{1}{80}\cos 0 + \frac{5}{48}\cos 0 - \frac{5}{8}\cos 0\right)$ $= \left(-\frac{1}{80} \times \frac{1}{2} + \frac{5}{48} \times (-1) - \frac{5}{8} \times \frac{1}{2}\right) - \left(-\frac{1}{80} + \frac{5}{48} - \frac{5}{8}\right)$ $= \frac{53}{480}$
	Shows substitution clearly	AO2.4	M1	
	Completes a rigorous argument to show the required result. NMS = 0/3	AO2.1	R1	

	Total		**10**	

Q	Marking Instructions	AO	Marks	Typical solution
9(a)	Writes complex number in Eulerian form or equivalent. PI correct r & θ	AO1.1b	B1	$z^3 = 2\sqrt{2}e^{\frac{-\pi i}{3}}$ $r = \sqrt{2}$ $\theta = \frac{-\pi}{9}$ $\theta = \frac{5\pi}{9}$, $\frac{11\pi}{9}$, $\frac{17\pi}{9}$ $z = \sqrt{2}e^{\frac{5\pi i}{9}}$, $\sqrt{2}e^{\frac{11\pi i}{9}}$, $\sqrt{2}e^{\frac{17\pi i}{9}}$
	Obtains r by taking cube root of their modulus of z^3, accept AWRT 1.41 or $\left(2\sqrt{2}\right)^{1/3}$ OE	AO1.1b	B1F	
	Divides their argument by 3	AO1.1a	M1	
	Finds three correct angles $\theta = \frac{5\pi}{9}$, $\frac{11\pi}{9}\left(\text{or} \frac{-7\pi}{9}\right)$, $\frac{17\pi}{9}\left(\text{or} \frac{-\pi}{9}\right)$	AO2.2a	A1	
	Finds fully correct solution, accept decimal equivalents & $\left(2\sqrt{2}\right)^{1/3}$ OE Accept $\theta = \frac{5\pi}{9}$, $\frac{11\pi}{9}\left(\text{or} \frac{-7\pi}{9}\right)$, $\frac{17\pi}{9}\left(\text{or} \frac{-\pi}{9}\right)$	AO2.2a	A1	
9(b)	Finds the area of their triangle from part (a) - ft their r only Or Applies Matrix $\mathbf{M}$ to the three points	AO3.1a	M1	$\text{Area of } \Delta = 3 \times \frac{1}{2} \times \sqrt{2} \times \sqrt{2} \times \sin\left(\frac{2\pi}{3}\right)$ $= \frac{3\sqrt{3}}{2}$ $\lvert\mathbf{M}\rvert = 14$ $\text{Required Area} = 14 \times \frac{3\sqrt{3}}{2}$ $= 21\sqrt{3}$.
	Finds correct area of original triangle Or Finds three correct new points	AO1.1b	A1	
	Finds correct $\lvert\mathbf{M}\rvert$ and uses as area scale factor with their area of original triangle Or Works out area of new triangle	AO2.2a	M1	
	Finds correct answer from correct reasoning in exact form only	AO1.1b	A1	
	Total		**9**	

Q	Marking Instructions	AO	Marks	Typical solution
10(a)	Obtains an equation of L. Condone one error in their direction vector. Condone lack of "$\mathbf{r}=$", PI by correct $\mathbf{v}$	AO1.1a	M1	$\mathbf{r}=\begin{bmatrix}5\\-4\\6\end{bmatrix}+\mu\begin{bmatrix}1\\-2\\2\end{bmatrix}$
	Obtains a correct equation of L. Condone lack of "$\mathbf{r}=$", PI by correct $\mathbf{v}$	AO1.1b	A1	
	Obtains their correct general vector from line to C	AO3.1a	B1F	$\mathbf{v}=\begin{bmatrix}-10+\mu\\1-2\mu\\-3+2\mu\end{bmatrix}$
	Finds scalar product of their $\mathbf{v}$ and their $\overrightarrow{AB}$	AO3.1a	M1	$0=\begin{bmatrix}1\\-2\\2\end{bmatrix}\cdot\begin{bmatrix}-10+\mu\\1-2\mu\\-3+2\mu\end{bmatrix}$
	Solves to find the correct μ for their equation.	AO1.1b	A1F	$-10+\mu-2+4\mu-6+4\mu=0$ $\mu=2$
	Finds correct D	AO3.2a	A1	$D=(7,\ -8,\ 10)$
10(b)	Obtains their components of $\overrightarrow{CD}$, must have their correct magnitude, but ignore sign. Allow one error.	AO1.1a	M1	$\overrightarrow{CD}=\begin{pmatrix}-8\\-3\\1\end{pmatrix}$
	Obtains their correct CD, in exact form.	AO1.1b	A1F	$CD=\sqrt{74}$
	Total		**8**	

Q	Marking Instructions	AO	Marks	Typical Solution
11	Divides through by x	AO1.1a	M1	$\frac{\mathrm{d}y}{\mathrm{d}x}-\frac{2y}{x}=\frac{x^2}{\sqrt{4-2x-x^2}}$
	Recognises that the Integrating Factor Method can be applied and finds correct integrating factor, accept $\mathrm{e}^{-2\ln x}$	AO3.1a	M1	$\int P\mathrm{d}x=-\int\frac{2}{x}\mathrm{d}x=-2\ln x$
	Multiplies equation by their integrating factor	AO1.1a	M1	Integrating factor $=\mathrm{e}^{\int P\mathrm{d}x}=\mathrm{e}^{-2\ln x}=x^{-2}$
	Integrates LHS to obtain $\frac{y}{x^2}$	AO1.1b	A1	$\frac{1}{x^2}\frac{\mathrm{d}y}{\mathrm{d}x}-\frac{2y}{x^3}=\frac{1}{\sqrt{4-2x-x^2}}$
	Recognises the need to complete the square inside the square root.	AO3.1a	M1	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{y}{x^2}\right)=\frac{1}{\sqrt{4-2x-x^2}}$
	Correctly uses the appropriate inverse sine, inverse cosh or inverse sinh function to integrate all (or part of) their RHS.	AO3.1a	M1	
	Finds correct solution including constant of integration. ACF Accept $\frac{y}{x^2}=\ldots$	AO1.1b	A1	To find $\int\frac{1}{\sqrt{4-2x-x^2}}\mathrm{d}x$ $4-2x-x^2=5-(x+1)^2$ $\int\frac{1}{\sqrt{4-2x-x^2}}\mathrm{d}x=\int\frac{1}{\sqrt{5-(x+1)^2}}\mathrm{d}x$ $\therefore y=x^2\left\{\sin^{-1}\left(\frac{x+1}{\sqrt{5}}\right)+c\right\}$
	Total		**7**	

Q	Marking Instructions	AO	Marks	Typical solution
12(a)	Recognises the need to set the determinant = 0	AO3.1a	M1	$9k^2-9k-180=0$ $k=5$ and $k=-4$
	Obtains and solves a three-term quadratic equation in k	AO1.1a	M1	
	Obtains the correct values of k	AO1.1b	A1	
12(b)	Selects an appropriate method and substitutes their first value of k	AO3.1a	M1	For $k=5$ $\begin{bmatrix} 4 & -5 & 1 & 8 \\ 0 & 23 & -23 & 0 \\ 0 & 35 & -35 & 0 \end{bmatrix}$ Consistent Line of intersection (sheaf)
	For $k=5$ (k must be correct): Deduces that equations are consistent – must have sufficient working to justify comment.	AO2.2a	M1	
	Gives correct geometrical description with full working.	AO3.2a	A1	For $k=-4$ $3x+2y+4z=6$ $-6x-4y-8z=6$ Inconsistent Two planes parallel and distinct with third plane crossing both
	For $k=-4$ (k must be correct): Deduces that equations are inconsistent by comparing eqs 2 & 3 – must have comment.	AO2.2a	B1	
	Gives correct geometrical description.	AO3.2a	B1	
	Total		**8**	

Q	Marking Instructions	AO	Marks	Typical Solution
13(a)(i)	Recognises that result can be obtained by expanding $(\alpha+\beta+\gamma)^2$ and completes correct expansion.	AO3.1a	M1	$(\alpha+\beta+\gamma)^2 =$ $\alpha^2+\beta^2+\gamma^2+2(\alpha\beta+\beta\gamma+\gamma\alpha)$ But $\alpha+\beta+\gamma=-k$ and $\alpha\beta+\beta\gamma+\gamma\alpha=0$ So $\alpha^2+\beta^2+\gamma^2=k^2$
	Correctly uses sum of roots and sum of pairs of roots, in their expansion. Condone $\alpha+\beta+\gamma=k$	AO1.1a	B1	
	Completes a rigorous argument to show the required result. Do not condone $\alpha+\beta+\gamma=k$	AO2.1	R1	
13(a)(ii)	Expands $(\alpha\beta+\beta\gamma+\gamma\alpha)^2$	AO1.1a	M1	$(\alpha\beta+\beta\gamma+\gamma\alpha)^2=\alpha^2\beta^2+\beta^2\gamma^2+\gamma^2\alpha^2$ $+2\alpha\beta^2\gamma+2\alpha^2\beta\gamma+2\alpha\beta\gamma^2$ $=\alpha^2\beta^2+\beta^2\gamma^2+\gamma^2\alpha^2$ $+2\alpha\beta\gamma(\alpha+\beta+\gamma)$ But $\alpha+\beta+\gamma=-k$ And $\alpha\beta+\beta\gamma+\gamma\alpha=0$ And $\alpha\beta\gamma=-9$ So $\alpha^2\beta^2+\beta^2\gamma^2+\gamma^2\alpha^2=-18k$
	Rearranges to express $\alpha^2\beta^2+\beta^2\gamma^2+\gamma^2\alpha^2$ in terms of sum, product and sum of pairs of roots	AO3.1a	M1	
	Correctly states product of roots – could be seen in (a)(i)	AO1.1a	B1	
	Completes a rigorous argument to show the required result.	AO2.1	R1	
13(b)(i)	Deduces result correctly using both equations and previous working.	AO2.2a	R1	$\alpha\beta+\gamma+\beta\gamma+\alpha+\gamma\alpha+\beta=\frac{40}{9}$ But $\alpha+\beta+\gamma=-k$ And $\alpha\beta+\beta\gamma+\gamma\alpha=0$ so $k=-\frac{40}{9}$
13(b)(ii)	Forms correct equation from product of roots	AO1.1b	B1	$-\frac{s}{9}=(\alpha\beta+\gamma)(\beta\gamma+\alpha)(\gamma\alpha+\beta)$ $=\alpha^2\beta^2\gamma^2+\alpha\beta^3\gamma+\alpha^3\beta\gamma+\alpha^2\beta^2$ $+\alpha\beta\gamma^3+\beta^2\gamma^2+\gamma^2\alpha^2+\alpha\beta\gamma$ $=(\alpha\beta\gamma)^2+\alpha\beta\gamma+\alpha^2\beta^2+\beta^2\gamma^2+\gamma^2\alpha^2$ $+\alpha\beta\gamma(\alpha^2+\beta^2+\gamma^2)$ $=81-9-18k-9k^2$ $=81-9+80-9\times\frac{1600}{81}$ $=-\frac{232}{9}$ So $s=232$
	Expands product of roots. Condone one or two errors	AO1.1a	M1	
	Identifies the degree 5 terms and fully factorises them in the form $\alpha\beta\gamma(\alpha^2+\beta^2+\gamma^2)$	AO3.1a	M1	
	Collects all other terms on RHS as $(\alpha\beta\gamma)^2+\alpha\beta\gamma+\alpha^2\beta^2+\beta^2\gamma^2+\gamma^2\alpha^2$ $(\alpha\beta\gamma)^2$ PI 81 or $(-9)^2$	AO3.1a	M1	
	Correctly substitutes their values into their equation	AO1.1a	M1	
	Obtains the correct answer from correct reasoning.	AO2.1	R1	
	Total		**14**	

Q	Marking Instructions	AO	Marks	Typical solution
14(a)	Forms general force equation (at least three terms) with at least two terms correct (allow equivalent notation for derivatives – condone a and v).	AO3.1b	M1	$9m\left(\frac{20}{9}-x\right)-mg-6m\dot{x}=m\ddot{x}$
	Obtains fully correct general force equation & cancels down into 2nd order DE form (allow equivalent notation for derivatives).	AO1.1b	A1	$\ddot{x}+6\dot{x}+9x=10$
	Obtains correct solution to their Auxiliary Equation	AO1.1a	M1	$\lambda^2+6\lambda+9=0$ $\lambda=-3$ (twice)
	Obtains their correct RHS of Complementary Function	AO1.1b	A1F	CF: $x=Ae^{-3t}+Bte^{-3t}$
	Obtains their correct (non-zero) Particular Integral	AO1.1b	B1F	PI: $x=\frac{10}{9}$
	Obtains correct RHS of General Solution (ft their CF & non-zero PI, but must have two unknowns)	AO2.2a	A1F	General Solution: $x=Ae^{-3t}+Bte^{-3t}+\frac{10}{9}$
	Uses $x=0$ when $t=0$ to obtain correct A	AO1.1b	B1	$x=0, t=0 \Rightarrow A=\frac{-10}{9}$
	Sets their correct $\dot{x}=0$ when $t=0$	AO3.3	M1	$\dot{x}=-3Ae^{-3t}+Be^{-3t}-3Bte^{-3t}$
	Obtains correct B – can be unsimplified.	AO1.1b	A1	$0=-3A+B$ $B=-\frac{30}{9}$
	Obtains correct final equation – can be unsimplified.	AO2.1	R1	$x=-\frac{10}{9}e^{-3t}-\frac{10}{3}te^{-3t}+\frac{10}{9}$
14(b)	States critical damping because the Auxiliary Equation has equal roots (or equivalent)	AO1.2	B1	Critical damping, because the Auxiliary Equation has equal roots
	Total		**11**	

Q	Marking Instructions	AO	Marks	Typical Solution
15(a)	Correctly converts polar to Cartesian coordinates (PI)	AO3.1a	B1	$x = r\cos\theta$ and $y = r\sin\theta$
	Finds correct expression for at least one of $\frac{\mathrm{d}y}{\mathrm{d}\theta}$ and $\frac{\mathrm{d}x}{\mathrm{d}\theta}$	AO1.1a	M1	$\frac{\mathrm{d}y}{\mathrm{d}\theta} = \frac{\mathrm{d}r}{\mathrm{d}\theta}\sin\theta + r\cos\theta$ $\frac{\mathrm{d}x}{\mathrm{d}\theta} = \frac{\mathrm{d}r}{\mathrm{d}\theta}\cos\theta - r\sin\theta$
	Divides their $\frac{\mathrm{d}y}{\mathrm{d}\theta}$ by $\frac{\mathrm{d}x}{\mathrm{d}\theta}$	AO1.1a	M1	Gradient $= \frac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\frac{\mathrm{d}r}{\mathrm{d}\theta}\sin\theta + r\cos\theta}{\frac{\mathrm{d}r}{\mathrm{d}\theta}\cos\theta - r\sin\theta}$
	Completes proof, including statement that gradient $= \frac{\mathrm{d}y}{\mathrm{d}x}$	AO2.1	R1	
15(b)	Differentiates r correctly using standard result.	AO1.2	B1	$\frac{\mathrm{d}r}{\mathrm{d}\theta} = (\cot b)\mathrm{e}^{(\cot b)\theta}$
	Substitutes r and their $\frac{\mathrm{d}r}{\mathrm{d}\theta}$ into expression for gradient from (a)	AO3.1a	M1	$\frac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{(\cot b)\mathrm{e}^{(\cot b)\theta}\sin\theta + \mathrm{e}^{(\cot b)\theta}\cos\theta}{(\cot b)\mathrm{e}^{(\cot b)\theta}\cos\theta - \mathrm{e}^{(\cot b)\theta}\sin\theta}$
	Rearranges into a form that can be recognised as a compound angle formula.	AO1.1a	M1	$= \dfrac{\cot b\sin\theta + \cos\theta}{\cot b\cos\theta - \sin\theta}$
	Deduces that gradient of *TPQ* is equal to $\tan(\theta + b)$	AO2.2a	M1	$= \dfrac{\frac{\cos b}{\sin b}\sin\theta + \cos\theta}{\frac{\cos b}{\sin b}\cos\theta - \sin\theta}$
	Deduces that angle $STP = \theta + b$ Condone $-(\theta + b)$	AO2.2a	M1	$= \dfrac{\cos b\sin\theta + \sin b\cos\theta}{\cos b\cos\theta - \sin b\sin\theta}$
	Uses a geometric argument to explain why OPT is independent of θ	AO2.4	M1	$= \dfrac{\sin(\theta + b)}{\cos(\theta + b)} = \tan(\theta + b)$
	Completes a rigorous argument to show the required result.	AO2.1	R1	$\therefore$ angle $STP = \theta + b$ $\therefore$ angle $OPT = b$ (exterior angle of a triangle) So the angle between the line OP and the tangent TPQ does not depend on θ.
	Total		**11**	