Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Tuesday 23 June 2020

Afternoon (Time: 1 hour 30 minutes) | Paper Reference **9FM0/4C**

Further Mathematics

Advanced

Paper 4C: Further Mechanics 2

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a value of g is required, take $g = 9.8\,\text{m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

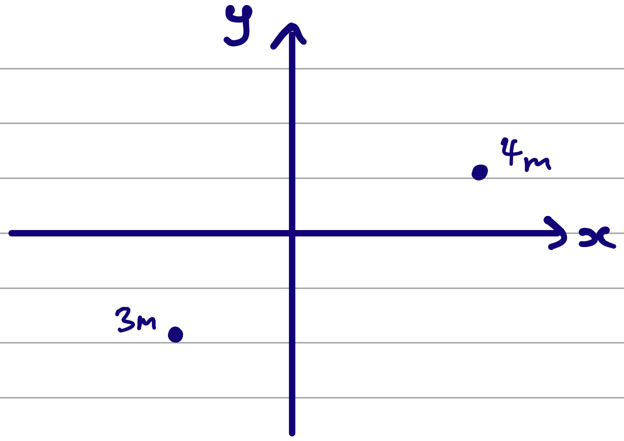
Turn over ▶

1. Three particles of masses $3m$, $4m$ and $2m$ are placed at the points $(-2, 2)$, $(3, 1)$ and (p, p) respectively.

The value of p is such that the distance of the centre of mass of the three particles from the point $(0, 0)$ is as small as possible.

Find the value of p.

(7)



For $\bar{x}$,

$$\sum mx = \bar{x}\sum m$$

$$3m \times -2 + 4m \times 3 + 2m \times p = \bar{x}(3m + 4m + 2m)$$

$$\Rightarrow \quad 6 + 2p = 9\bar{x} \longrightarrow (1)$$

For $\bar{y}$,

$$3m \times 2 + 4m \times 1 + 2m \times p = \bar{y}(3m + 4m + 2)$$

$$\Rightarrow \quad 10 + 2p = 9\bar{y} \longrightarrow (2)$$

Distance from CoM to origin $= r = \sqrt{\bar{x}^2 + \bar{y}^2}$

Relating r to p,

$$(1)^2 + (2)^2 : \quad (9\bar{x})^2 + (9\bar{y})^2 = (6+2p)^2 + (10+2p)^2$$

$$9r^2 = 36 + 24p + 4p^2 + 100 + 400p + 4p^2$$

$$= 136 + 64p + 8p^2$$

[Completing the square] $\quad = 8(p^2 + 8p + 17)$

$$= 8((p+4)^2 + 17 - 16)$$

$$9r^2 = 8(p+4)^2 + 8$$

r is minimised when $\underline{\underline{p = -4}}$

Question 1 continued

(Total for Question 1 is 7 marks)

2.

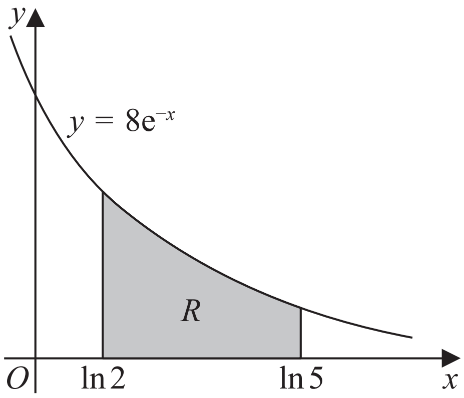


Figure 1

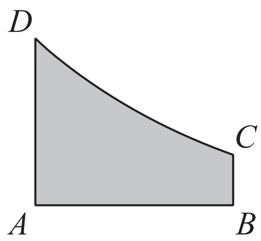


Figure 2

A uniform plane figure R, shown shaded in Figure 1, is bounded by the x-axis, the line with equation $x = \ln 5$, the curve with equation $y = 8e^{-x}$ and the line with equation $x = \ln 2$. The unit of length on each axis is one metre.

The area of R is $2.4\,\text{m}^2$

The centre of mass of R is at the point with coordinates $(\bar{x}, \bar{y})$.

(a) Use algebraic integration to show that $\bar{y} = 1.4$

(4)

Figure 2 shows a uniform lamina $ABCD$, which is the same size and shape as R. The lamina is freely suspended from C and hangs in equilibrium with CB at an angle $\theta°$ to the downward vertical.

(b) Find the value of θ

(6)

a) By definition of CoM:

$$A\bar{y} = \frac{1}{2}\int_{x_1}^{x_2} y^2 \, dx$$

$$2.4\,\bar{y} = \frac{1}{2}\int_{\ln 2}^{\ln 5} 64e^{-2x} \, dx$$

$$2.4\,\bar{y} = -16\left[e^{-2x}\right]_{\ln 2}^{\ln 5}$$

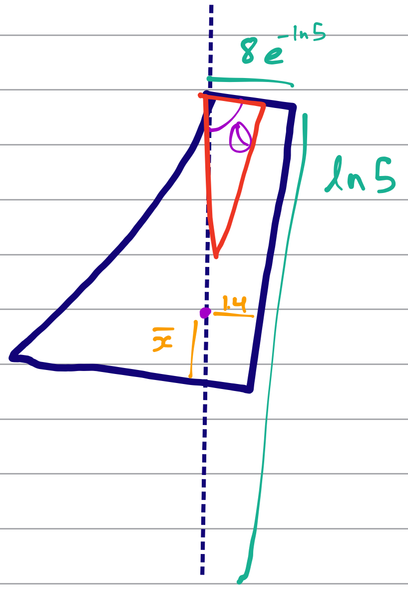
$$\bar{y} = -\frac{20}{3}\left[e^{\ln\left(\frac{1}{5^2}\right)} - e^{\ln\left(\frac{1}{2^2}\right)}\right]$$

$$= -\frac{20}{3}\left[\frac{1}{25} - \frac{1}{4}\right]$$

$$= \frac{-20}{3} \times \frac{-21}{100} = \frac{7}{5} = 1.4$$

Question 2 continued

b)



We can solve for θ using $\triangle$ [right angled triangle]

But we need to know $\bar{x}$ first.

$$2.4\bar{x} = \int_{\ln 2}^{\ln 5} 8xe^{-x}\,dx$$

Using integration by parts:

$$2.4\bar{x} = \left[-8xe^{-x} - 8e^{-x}\right]_{\ln 2}^{\ln 5}$$

$$2.4\bar{x} = -8\cdot\ln 5\cdot e^{\ln\left(\frac{1}{5}\right)} - 8e^{\ln(1/5)} + 8\cdot\ln 2\cdot e^{\ln(1/2)} + 8e^{\ln(1/2)}$$

$$2.4\bar{x} = 8\left(\frac{\ln 2}{2} + \frac{1}{2} - \frac{\ln 5}{5} - \frac{1}{5}\right)$$

$$\bar{x} = \frac{10}{3}\left(\frac{\ln 2}{2} - \frac{\ln 5}{5} + \frac{12}{5}\right)$$

$$\bar{x} \approx 1.08$$

From the right $\triangle$,

$$\tan\theta = \frac{\ln 5 - \bar{x}}{8e^{-\ln 5} - 1.4}$$

$$\theta = \arctan(2.63\ldots) \approx \underline{\underline{69^\circ}}$$

Question 2 continued

Question 2 continued

(Total for Question 2 is 10 marks)

3. A particle P of mass 0.5 kg is moving along the positive x-axis in the direction of x increasing. At time t seconds $(t \geqslant 0)$, P is x metres from the origin O and the speed of P is $v\,\mathrm{m\,s^{-1}}$. The resultant force acting on P is directed towards O and has magnitude kv^2 N, where k is a positive constant.

When $x = 1, v = 4$ and when $x = 2, v = 2$

(a) Show that $v = ab^x$, where a and b are constants to be found.

(6)

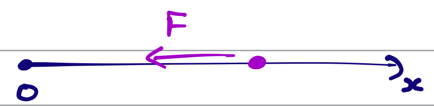
The time taken for the speed of P to decrease from $4\,\mathrm{m\,s^{-1}}$ to $2\,\mathrm{m\,s^{-1}}$ is T seconds.

(b) Show that $T = \dfrac{1}{4\ln 2}$

(4)

a) Using Newton's 2^{nd} Law:

$$F_{resultant} = ma$$



$$-kv^2 = 0.5a \quad [-\text{ve force as it is in } -x \text{ direction}]$$

$$-kv^2 = 0.5\, v\frac{dv}{dx}$$

$$\Rightarrow \int -k\, dx = \frac{1}{2}\int \frac{1}{v}\, dv$$

$$-kx + c = \frac{1}{2}\ln(v) \rightarrow \text{General Solution}$$

Using our boundary conditions:

$x = 1,\ v = 4$: $\quad \frac{1}{2}\ln 4 = -k + C \longrightarrow ①$

$x = 2,\ v = 2$: $\quad \frac{1}{2}\ln 2 = -2k + C \longrightarrow ②$

① − ② : $\quad \frac{1}{2}(\ln(4) - \ln(2)) = k$

$$\Rightarrow k = \ln(\sqrt{2})$$

Plug k into ① to find C:

$$\ln(\sqrt{4}) = -\ln(\sqrt{2}) + c$$

$$C = \ln(2) + \ln(\sqrt{2}) = \ln(2\sqrt{2}) = \frac{1}{2}\ln 8$$

DO NOT WRITE IN THIS AREA

Question 3 continued

Specific solution for this problem:

$$\ln(v) = -x\ln(2) + \ln(8)$$
$$\ln(v) = \ln\left(\frac{1}{2^x}\right) + \ln(8)$$
$$\ln(v) = \ln\left(\frac{8}{2^x}\right)$$
$$\Rightarrow v = 8\left(\frac{1}{2}\right)^x \equiv ab^x$$
$$\therefore a = 8,\ b = \frac{1}{2}$$

b) Expressing Newton's 2^{nd} law in terms of a time derivative:

$$-kv^2 = 0.5a$$
$$\Rightarrow -kv^2 = 0.5\frac{dv}{dt}$$
$$-\ln(\sqrt{2})v^2 = \frac{1}{2}\frac{dv}{dt}$$
$$\int_0^T -\ln(2)\,dt = \int_4^2 \frac{1}{v^2}\,dv$$
$$\Big[-t\ln(2)\Big]_0^T = \Big[-\frac{1}{v}\Big]_4^2$$
$$-T\ln(2) = -\frac{1}{2} + \frac{1}{4} = -\frac{1}{4}$$
$$T = \frac{1}{4\ln 2}$$

Question 3 continued

Question 3 continued

(Total for Question 3 is 10 marks)

4.

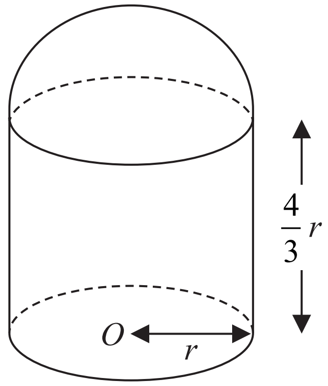


Figure 3

A uniform solid cylinder of base radius r and height $\frac{4}{3}r$ has the same density as a uniform solid hemisphere of radius r. The plane face of the hemisphere is joined to a plane face of the cylinder to form the composite solid S shown in Figure 3.
The point O is the centre of the plane face of S.

(a) Show that the distance from O to the centre of mass of S is $\frac{73}{72}r$

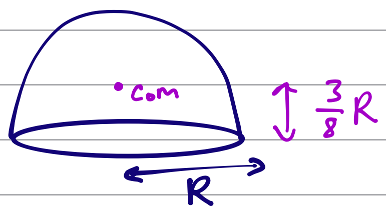
(4)

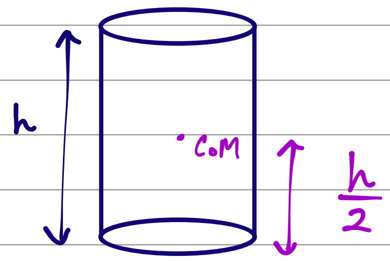
The solid S is placed with its plane face on a rough horizontal plane. The coefficient of friction between S and the plane is μ. A horizontal force P is applied to the highest point of S. The magnitude of P is gradually increased.

(b) Find the range of values of μ for which S will slide before it starts to tilt.

(5)

(a) Using the standard results for uniform bodies:





For the compound solid, the centre of mass (located d units along the axis of symmetry) is found by weighting these distances by their respective masses.

$$M_{cylinder} \times \left(\frac{1}{2} \times \frac{4}{3}r\right) + M_{hemisphere} \times \left(\frac{4}{3}r + \frac{3}{8}r\right) = M_{total} \times d$$

Question 4 continued

$$M_{cylinder} = \pi r^2 \left(\frac{4}{3}r\right) \times \rho = \frac{4}{3}\pi\rho r^3$$

$$M_{hemisphere} = \frac{2}{3}\pi r^3 \times \rho = \frac{2}{3}\pi\rho r^3$$

$$M_{total} = \frac{4}{3}\pi\rho r^3 + \frac{2}{3}\pi\rho r^3 = 2\pi\rho r^3$$

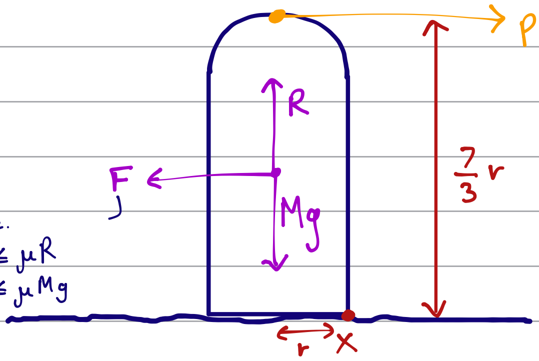
$\therefore$ By cancelling out ρ since everything has the same mass density:

$$\frac{4}{3}\pi r^3 \times \frac{2}{3}r + \frac{2}{3}\pi r^3 \times \frac{41}{24}r = 2\pi r^3 d$$

$$\frac{8}{9}r + \frac{41}{36}r = 2d$$

$$d = \frac{73}{72}r$$

(b)



Friction Force.
$F \leq \mu R$
$\Rightarrow F \leq \mu Mg$

By resolving $\rightarrow$, we can see that S slides when:

$$\underbrace{P > \mu Mg} \rightarrow ①$$

By taking moments about X,

$$\frac{7}{3}rP = rMg$$

Tilting (i.e $\sum \circlearrowright > \sum \circlearrowleft$)

$$\frac{7}{3}Pr > rMg$$

$$\left.P > \frac{3Mg}{7}\right\} \rightarrow ②$$

Question 4 continued

By comparing 1 and 2, for S to slide but NOT tilt:

$$\mu Mg < P < \frac{3}{7}Mg$$

$$\mu < P < \frac{3}{7}$$

$$\Rightarrow 0 < \mu < \frac{3}{7}$$

Question 4 continued

(Total for Question 4 is 9 marks)

5.

$$x = \sqrt{0.6^2 - 0.4^2}$$
$$= \frac{\sqrt{20}}{10}$$
$$\therefore \sin\theta = \frac{\sqrt{20}/10}{0.6} = \frac{\sqrt{20}}{6}$$
$$\cos\theta = \frac{0.4}{0.6} = \frac{2}{3}$$

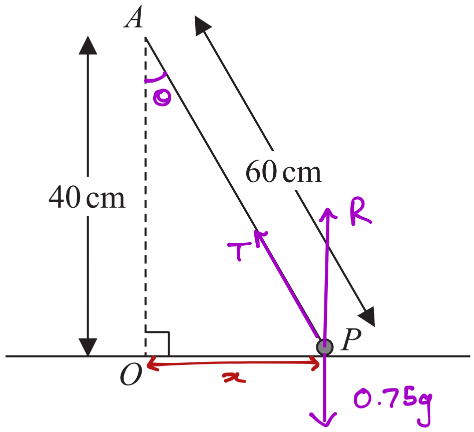


Figure 4

A particle P of mass 0.75 kg is attached to one end of a light inextensible string of length 60 cm. The other end of the string is attached to a fixed point A that is vertically above the point O on a smooth horizontal table, such that $OA = 40$ cm. The particle remains in contact with the table, with the string taut, and moves in a horizontal circle with centre O, as shown in Figure 4.

The particle is moving with a constant angular speed of 3 radians per second.

(a) Find (i) the tension in the string,

(ii) the normal reaction between P and the table.

(7)

The angular speed of P is now gradually increased.

(b) Find the angular speed of P at the instant P loses contact with the table.

(4)

a) Resolving vertically:

$$0.75g = T\cos\theta + R$$
$$\frac{3}{4}g = \frac{2}{3}T + R \longrightarrow ①$$

Resolving Horizontally:

Net Force = Centripetal Force = Horizontal component of T

$$m\omega^2 r = T\sin\theta$$
$$\frac{3}{4} \times 9 \times \frac{\sqrt{20}}{10} = T\frac{\sqrt{20}}{6}$$
$$\Rightarrow T = 6 \times \frac{27}{4} \times \frac{1}{10} = \underline{\underline{4.05 \text{ N}}}$$

$$\therefore R = \frac{3}{4}g - \frac{2}{3} \times 4.05 = \underline{\underline{4.65}} \text{ N}$$

DO NOT WRITE IN THIS AREA

Question 5 continued

b) When the ball lose contact,

$$R = 0$$

$$\Rightarrow \frac{3}{4}g - T\cos\theta = 0$$

$$\therefore \cos\theta = \frac{3g}{4T} \longrightarrow (2)$$

And vertically,

$$m\omega^2 r = T\sin\theta$$

$$\therefore \sin\theta = \frac{m\omega^2 r}{T} \longrightarrow (3)$$

(3) $\div$ (2) : $\tan\theta = \dfrac{m\omega^2 r}{T} \times \dfrac{4T}{3g} = \dfrac{4}{3}\dfrac{m\omega^2 r}{g}$

$$\therefore \omega = \sqrt{\frac{3g\tan\theta}{4mr}}$$

Since $\tan\theta = \frac{\sqrt{20}}{4}$ just before the ball takes off (i.e. when the string is taut as in (a)) :

$$\omega = \sqrt{\frac{3g\sqrt{20}}{16 \cdot \frac{3}{4} \cdot \frac{\sqrt{20}}{10}}}$$

$$= \frac{7}{\sqrt{2}} = 4.949747468\ldots \text{ rad s}^{-1}$$

$$\approx \underline{\underline{4.95 \text{ rad s}^{-1}}}$$

Question 5 continued

Question 5 continued

(Total for Question 5 is 11 marks)

6.

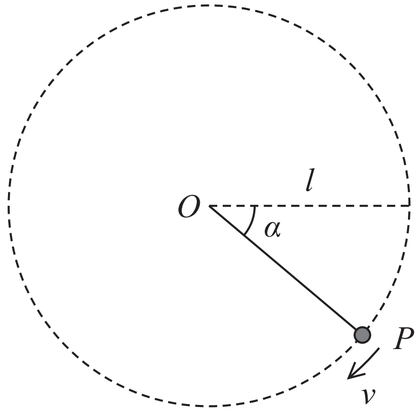


Figure 5

A particle P of mass m is attached to one end of a light inextensible string of length l. The other end of the string is attached to a fixed point O. The particle is held with the string taut and OP horizontal. The particle is then projected vertically downwards with speed u, where $u^2 = \frac{9}{5}gl$. When OP has turned through an angle α and the string is still taut, the speed of P is v, as shown in Figure 5. At this instant the tension in the string is T.

(a) Show that $T = 3mg\sin\alpha + \frac{9}{5}mg$

(6)

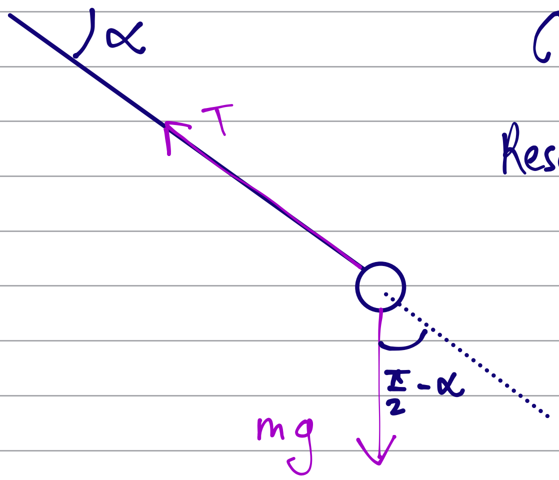
(b) Find, in terms of g and l, the speed of P at the instant when the string goes slack.

(3)

(c) Find, in terms of l, the greatest vertical height reached by P above the level of O.

(4)

(a)



Centripetal Force = $\frac{mv^2}{l}$

Resolving along string:

$T - mg\cos\left(\frac{\pi}{2} - \alpha\right) = \frac{mv^2}{l}$

$T = \frac{mv^2}{l} + mg\sin(\alpha) \rightarrow (1)$

Find v^2 using conservation of energy

KE at P = Initial KE + ΔGPE

Question 6 continued

$$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mgl\sin\alpha$$

$$v^2 = u^2 + 2gl\sin\alpha$$

Plug expression for v^2 into ①

$$\therefore T = \frac{m(u^2 + 2gl\sin\alpha)}{l} + mg\sin(\alpha)$$

$$= \frac{mu^2}{l} + 3mg\sin(\alpha)$$

We also know $u^2 = \frac{9}{5}gl$

$$\Rightarrow T = \frac{9}{5}mg + 3mg\sin\alpha$$

b) When slack, $T = 0$

$$\Rightarrow 3mg\sin\alpha + \frac{9}{5}mg = 0$$

$$\sin\alpha = -\frac{9}{15} = -\frac{3}{5}$$

We found:

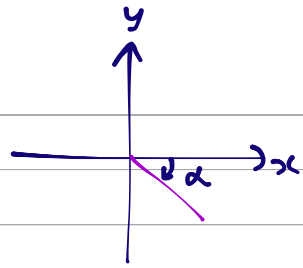
$$v^2 = u^2 + 2gl\sin\alpha = gl\left(\frac{9}{5} + 2\cdot\frac{-9}{15}\right)$$

$$= \frac{3gl}{5}$$

$\therefore v = \sqrt{\frac{3gl}{5}}$ when the string goes slack again

Question 6 continued

c) When $\sin\alpha = \frac{-3}{5}$, $\cos\alpha = \frac{4}{5}$



Initial Vertical component (i.e. when the string goes slack)

$$= \frac{4}{5} \times \sqrt{\frac{3gl}{5}} = \sqrt{\frac{48gl}{125}}$$

Since only its weight now acts, using suvat:

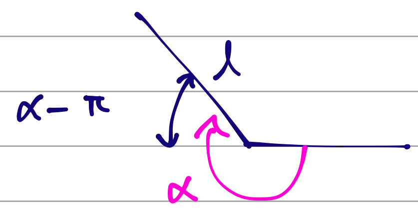
$$0 = \frac{48gl}{125} - 2gh$$

$$h = \frac{24l}{125}$$

Add this h to the height above O we were at when tension just hit O

$$= |\sin\alpha|\, l + \frac{24l}{125} = \frac{3}{5}l + \frac{24}{125}l = \frac{99}{125}l$$

We can take $|\sin\alpha|$ since $\sin\alpha = -\frac{3}{5}$ when α is reflex



$$\sin(\alpha - \pi) = -\sin\alpha$$
$$= |\sin\alpha|$$
$$= \frac{3}{5}$$

Question 6 continued

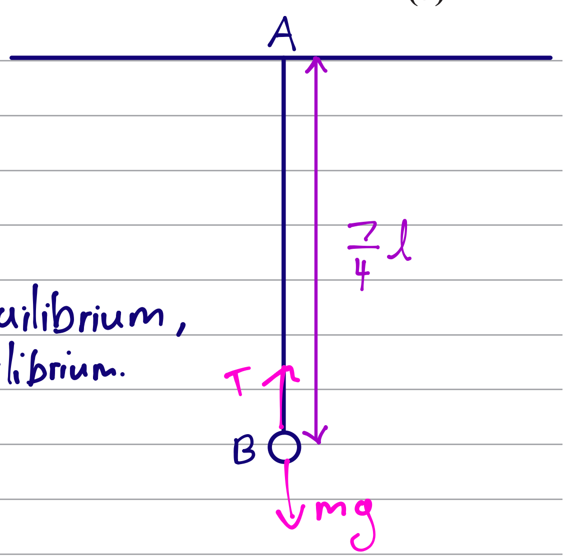
(Total for Question 6 is 13 marks)

7. A light elastic spring has natural length l and modulus of elasticity $4mg$. A particle P of mass m is attached to one end of the spring. The other end of the spring is attached to a fixed point A. The point B is vertically below A with $AB = \frac{7}{4}l$. The particle P is released from rest at B.

(a) Show that P moves with simple harmonic motion with period $\pi\sqrt{\frac{l}{g}}$ **(7)**

(b) Find, in terms of m, l and g, the maximum kinetic energy of P during the motion. **(3)**

(c) Find the time within each complete oscillation for which the length of the spring is less than l. **(5)**



(a)

Tension $= T = \frac{\lambda x}{l} = \frac{4mg(x+e)}{l}$

Where e is is the extension at equilibrium, and x is the displacement from this equilibrium.

$e \rightarrow$ constant
$x \rightarrow$ varies with time

To determine e, at equilibrium:

$$\frac{4mge}{l} = mg$$

$$e = \frac{l}{4}$$

Using Newton's 2^{nd} law:

$$T - mg = -m\frac{d^2x}{dt^2}$$

$$\frac{4mg(x + l/4)}{l} - mg = -m\frac{d^2x}{dt^2}$$

P 6 2 6 8 4 R A 0 2 4 2 8

Question 7 continued

$$-\frac{4gx}{l} = \frac{d^2x}{dt^2}$$

This is of the form $\frac{d^2x}{dt^2} = -\omega^2 x$ [SHM ODE]

$\Rightarrow$ Angular Frequency $= \omega = \sqrt{\frac{4g}{l}} = \frac{2\pi}{T}$

$\therefore$ Period $= T = 2\pi\sqrt{\frac{l}{4g}} = \pi\sqrt{\frac{l}{g}}$

(b) Amplitude $= a =$ Release Position $-$ Equilibrium Position

$$= \frac{7}{4}l - (l + e)$$

$$= \frac{7}{4}l - \left(l + \frac{l}{4}\right)$$

$$= \frac{l}{2}$$

Max speed $= a\omega = \frac{l}{2} \times \sqrt{\frac{4g}{l}} = \sqrt{lg}$

Max KE $= \frac{1}{2}mv_{max}^2 = \frac{1}{2} \times m \times \left(\sqrt{lg}\right)^2 = \frac{mlg}{2}$

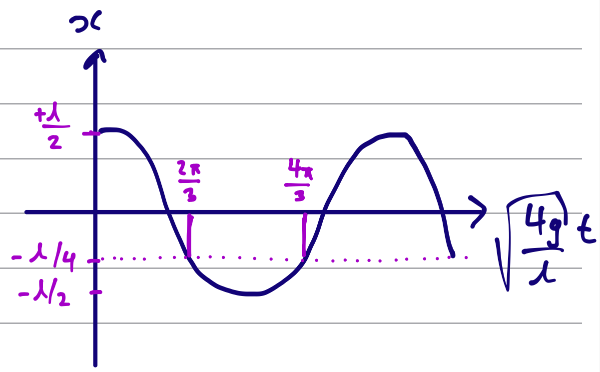
(c)

Equation of Motion: $x = a\cos(\omega t) = \frac{l}{2}\cos\left(\sqrt{\frac{4g}{l}}\,t\right)$

When length of spring $= l$, $x = -e = \frac{-l}{4}$

Question 7 continued

$$\Rightarrow \frac{-l}{4} = \frac{l}{2}\cos\left(\sqrt{\frac{4g}{l}}\,t\right)$$



$$\cos\left(\sqrt{\frac{4g}{l}}\,t\right) = -\frac{1}{2}$$

$$\Rightarrow \sqrt{\frac{4g}{l}}\,t = \frac{2\pi}{3},\ \frac{4\pi}{3}$$

$$t = \frac{\pi}{3}\sqrt{\frac{l}{g}},\ \frac{2\pi}{3}\sqrt{\frac{l}{g}}$$

Length of time $= \frac{2\pi}{3}\sqrt{\frac{l}{g}} - \frac{\pi}{3}\sqrt{\frac{l}{g}} = \frac{\pi}{3}\sqrt{\frac{l}{g}}$

Question 7 continued

Question 7 continued

(Total for Question 7 is 15 marks)

TOTAL FOR PAPER IS 75 MARKS