AQA

A-level

FURTHER MATHEMATICS

7367/3D

Paper 3 Discrete

Mark scheme

June 2020

Version: 1.0 Final Mark Scheme

206a7367/3D/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M and is for accuracy
B	mark is independent of M and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
‘their’	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
sf	significant figure(s)
dp	decimal place(s)

Examiners should consistently apply the following general marking principles:

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Q	Marking instructions	AO	Marks	Typical solution
1	Ticks correct box	1.1b	B1	**Supersource:** *V, X* **Supersink:** *U, W*
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
2	Ticks correct box	1.2	B1	Matrix multiplication is associative but not commutative
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
3(a)(i)	Models the situation as that of finding a spanning tree by listing or drawing five labelled arcs (PI). (Condone E for Education etc)	3.3	M1	Education to Refuge Collection: 13 Education to Social Care: 16 Refuge Collection to Transport: 17 Payroll to Social Care: 20 Housing to Social Care: 22 88 m Total cost $= 8 \times 88$ $= £704$
	Uses the model and correctly finds at least four arcs of the minimum spanning tree.	3.4	M1	
	Correctly finds the total minimum length of cable required (PI by £704).	1.1b	A1	
	Translates their total minimum length of cable into the total minimum cost. Must include unit.	3.2a	A1F	
3(a)(ii)	Suggests a plausible reason in the context of the question as to why the total cost of installing the network to the council will be different from the installation cost of the cable from **part (a)(i)**.	2.4	E1	The cost is likely to be higher than £704 as the council may also need to buy telephones for the internal network.
3(b)	Explains correctly that the Education Office is not directly connected to the Transport Office in their minimum spanning tree.	2.4	M1	The Education Office is not directly connected to the Transport Office in the minimum spanning tree, so the total minimum length of cable required does not change. Hence, the cost of installing the cable does not change.
	Deduces correctly that the cost of installing the network does not change.	2.2a	A1	
	Total		**7**	

Q	Marking instructions	AO	Marks	Typical solution
4(a)	Finds the correct Hamiltonian cycle starting at the depot using the nearest neighbour algorithm (Condone missing return to Depot).	3.1a	M1	Depot–*C*–*A*–*F*–*B*–*E*–*D*–Depot (15 + 20 + 21 + 14 + 16 + 34 + 16) = 136
	Correctly obtains the upper bound on L.	1.1b	A1	
4(b)	Correctly determines a minimum spanning tree connecting *A*, *B*, *C*, *D*, *E* and *F* (PI by 95).	1.1b	B1	MST with the depot removed: *A* – *C*: 20 *A* – *F*: 21 *B* – *F*: 14 *B* – *E*: 16 *D* – *F*: 24 = 95 Two arcs with lowest weight connected to the depot are: Depot – *C*: 15 Depot – *D*: 16 = 31 Hence, the lower bound for L is 126
	Correctly identifies the two arcs with the lowest weights that are connected to the depot (PI by 31 or by 15 & 16).	1.1a	M1	
	Correctly finds a lower bound for L by finding the sum of their spanning tree and the two correct lowest weight arcs.	1.1b	A1F	
4(c)	Infers that Joe's minimum total distance is an upper bound which might be possible to improve upon or Infers that Joe's claim may be correct but more information is needed to prove or disprove the claim.	2.2b	B1F	Joe's claim may be correct, but without further study we can only say that the upper bound for L is 134 miles.
	Total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
5(a)	Uses Euler's formula for connected planar graphs or Redraws the graph correctly with no edges crossing (PI).	1.1a	M1	*P* has 8 vertices and 13 edges. Hence, $v - e + f = 2$ $8 - 13 + f = 2$ $f = 7$ Therefore, *P* has 7 faces.
	Determines correctly that the graph *P* has 7 faces.	1.1b	A1	
5(b)	Explains the condition that must be met for a graph to be semi-Eulerian.	2.4	M1	For a graph to be semi-Eulerian it must have exactly two vertices which have an odd degree. As *P* has six odd degree vertices, therefore *P* is not semi-Eulerian and Akwasi's claim is incorrect.
	Uses their explanation to assess the validity of Akwasi's claim with direct reference to graph *P*, concluding that his claim is incorrect.	2.3	A1	
	Total		**4**	

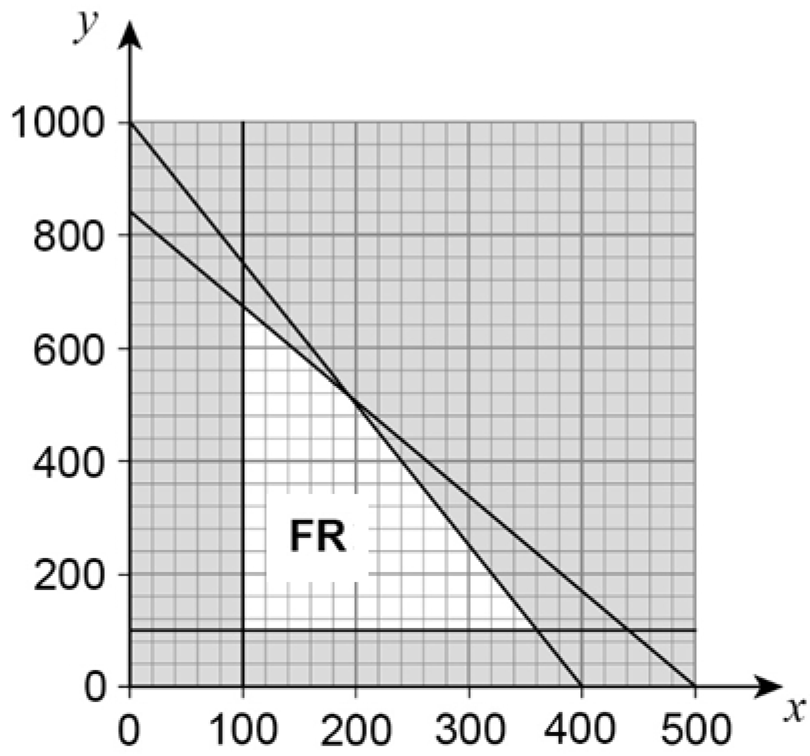
Q	Marking instructions	AO	Marks	Typical solution
6(a)(i)	States the correct order of *G*.	1.1b	B1	6
6(a)(ii)	Multiplies qr with qr and uses the given rule to rewrite either of the qr or Multiplies qr with an unknown element x and equates to e	1.1a	M1	$qr \blacktriangle qr = qr \blacktriangle r^2q$ $= q \blacktriangle r \blacktriangle r^2 \blacktriangle q$ $= q \blacktriangle e \blacktriangle q$ $= q \blacktriangle q$ $= e$ As $qr \blacktriangle qr = e$, the inverse of qr is qr
	Uses the given rules for r^3 or q^2 or Uses the given rules to obtain $xq = r^2$ or $rx = q$	1.1b	A1	
	Constructs a rigorous mathematical proof, including the explicit use of the identity element of the group and its properties, and provides a concluding statement that the inverse of qr is qr (Condone not using ▲).	2.1	R1	

6(b)	Correctly finds at least two missing entries in the Cayley table.	1.1b	B1	
	Correctly finds at least six missing entries in the Cayley table.	1.1b	B1	
	Correctly finds all missing entries in the Cayley table.	1.1b	B1	

▲	e	r	r^2	q	qr	qr^2
e	e	r	r^2	q	qr	qr^2
r	r	r^2	e	qr^2	q	qr
r^2	r^2	e	r	qr	qr^2	q
q	q	qr	qr^2	e	r	r^2
qr	qr	qr^2	q	r^2	e	r
qr^2	qr^2	q	qr	r	r^2	e

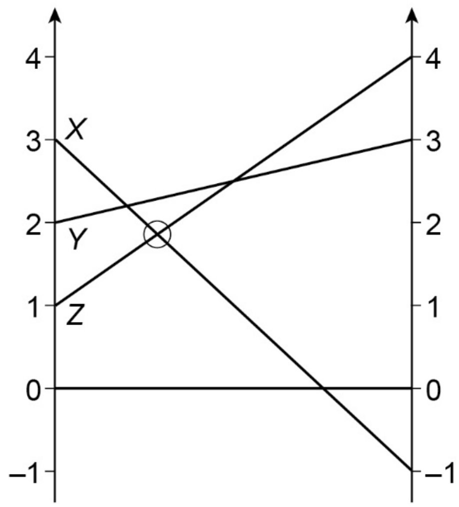
6(c)	States the correct name of a group which G is isomorphic to. (Accept reflections and rotations instead of symmetry) (Accept dihedral group) (Condone missing 'equilateral')	2.5	B1	G is isomorphic to the symmetry group of the equilateral triangle.
	Total		**8**	

Q	Marking instructions	AO	Marks	Typical solution
7(a)	Models the problem as a linear programming problem by introducing two variables, defining at least one of them as 'number of'.	3.3	B1	x = number of brake kits made y = number of clutch kits made $5x + 3y \leq 2500$ $500x + 200y \leq 200\,000$ $x \geq 100$, $y \geq 100$ $P = 2000x + 1000y$ P is maximised at (200, 500) To maximise profit, the engineering company should make 200 brake kits and 500 clutch kits each month.
	Uses their model to write down an inequality for the time taken to make the kits or the cost to the engineering company (Condone equalities).	3.4	M1	
	Uses their model to write down two correct inequalities for the time taken to make the kits and the cost to the engineering company.	1.1b	A1	
	Writes down two inequalities to ensure their variables are each greater than or equal to 100 (PI by graph)	3.1b	M1	
	Correctly plots the lines $x = 100$ and $y = 100$	1.1b	A1	
	Correctly plots the lines $5x + 3y = 2500$ and $5x + 2y = 2000$	1.1b	B1	
	Deduces their feasible region and shows it by labelling or shading on the graph.	2.2a	B1	
	States an objective function or draws an objective line and clearly states the solution to the problem in the context of the question (CAO). (Condone no reference to 'each month')	3.2a	R1	



7(b)	Recognises a limitation of the model by giving a plausible reason in the context of the problem as to why the engineering company may not be able to make the optimal number of each kit.	3.5b	E1	The total time available to work on making the kits may be less than 2500 hours as, for instance, some workers may be ill on some days and not available to work on making the kits.
7(c)	Explains that neither of the conditions requiring at least 100 of each kit be made is active in the solution in **part (a)**.	2.4	M1	The optimal vertex does not occur at (100, …) or (…, 100), so removing this condition does not change the maximum profit the engineering company can make.
	Deduces that removing these conditions has no effect on the maximum profit.	2.2a	A1	
	Total		**11**	

Q	Marking instructions	AO	Marks	Typical solution
8(a)(i)	Finds four or more correct row minima/col maxima.	3.1a	M1	row minima = (–1, 1) col maxima = (3, 3, 4) max(row minima) = 1 min(col maxima) = 3 As max(row minima) = 1 ≠ 3 = min(col maxima), then a stable solution does not exist.
	Correctly finds all row minima/col maxima and states the max(row minima) and min(col maxima).	1.1b	A1	
	Reasons correctly that as the max(row minima) is not equal to the min(col maxima) then a stable solution does not exist.	3.2a	R1	
8(a)(ii)	States the correct interval for the value of the game for Daryl based on their max(row minima) and min(col maxima). (Condone weak inequalities)	3.2a	B1F	$1 < V < 3$
8(b)(i)	Formulates an unsimplified expected gain for Daryl when Clare plays strategy **X**	3.1a	M1	When Clare plays strategy **X**, the expected gain for Daryl is Gain $= -1 \times \sin^2\theta + 3 \times \cos^2\theta$ $= -\sin^2\theta + 3\cos^2\theta$ $= -\sin^2\theta + 3(1 - \sin^2\theta)$ $= 3 - 4\sin^2\theta$
	Uses the identity $\sin^2\theta + \cos^2\theta \equiv 1$ and all necessary steps with no errors or omissions to show $3 - 4\sin^2\theta$	1.1b	A1	

8(b)(ii)	Finds the correct gain for Daryl, $2 + \sin^2\theta$, when Clare plays strategy **Y**.	1.1b	B1	When Clare plays strategy **Y**, the expected gain for Daryl is Gain $= 3 \times \sin^2\theta + 2 \times \cos^2\theta$ $= 2 + \sin^2\theta$
8(b)(iii)	Finds the correct gain for Daryl, $1 + 3\sin^2\theta$, when Clare plays strategy **Z**.	1.1b	B1	When Clare plays strategy **Z**, the expected gain for Daryl is Gain $= 4 \times \sin^2\theta + 1 \times \cos^2\theta$ $= 1 + 3\sin^2\theta$
8(c)	Constructs a graph with at least one vertical axis and plots two expected gains correctly or Forms an equation in $\sin^2\theta$ using using any two of the three expected gains.	3.1a	M1	
	Identifies the correct optimal point of intersection from the graph.	1.1b	A1	
	Forms an equation in $\sin^2\theta$ using using the correct expected gains for **X** and **Z**.	1.1a	M1	$3 - 4\sin^2\theta = 1 + 3\sin^2\theta$ $2 = 7\sin^2\theta$ $\frac{2}{7} = \sin^2\theta$
	Obtains the correct value for θ to 3 significant figures (CSO).	1.1b	A1	$\theta = \sin^{-1}\left(\sqrt{\frac{2}{7}}\right)$ $\theta = 32.3^\circ$
	Total		**12**	