# Mark Scheme (Result)

## October 2020

Pearson Edexcel GCE Advanced Level

in Further Mathematics

Paper 2: Core Pure Mathematics 2 (9FM0/02)

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## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

# EDEXCEL GCE MATHEMATICS

## General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- M marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- B marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- The second mark is dependent on gaining the first mark

**4.** All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 1 | $\dfrac{dy}{dx} = 31\cosh x - 4\cosh 2x$ | B1 | 1.1b |
| | $\dfrac{dy}{dx} = 31\cosh x - 4\left(2\cosh^2 x - 1\right)$ | M1 | 3.1a |
| | $8\cosh^2 x - 31\cosh x - 4 = 0$ | A1 | 1.1b |
| | $(8\cosh x + 1)(\cosh x - 4) = 0 \Rightarrow \cosh = ...$ | M1 | 1.1b |
| | $\cosh x = 4, \left(-\dfrac{1}{8}\right)$ | A1 | 1.1b |
| | $\cosh x = \alpha \Rightarrow x = \ln\left(\alpha + \sqrt{\alpha^2 - 1}\right)$ or $\ln\left(\alpha + \sqrt{\alpha^2 - 1}\right)$<br>or $-\ln\left(\alpha + \sqrt{\alpha^2 - 1}\right)$ or $\ln\left(\alpha - \sqrt{\alpha^2 - 1}\right)$<br>or<br>$\dfrac{e^x + e^{-x}}{2} = 4 \Rightarrow e^{2x} - 8e^x + 7 = 0 \Rightarrow e^x = ... \Rightarrow x = \ln(...)$ | M1 | 1.2 |
| | $\pm\ln\left(4 + \sqrt{15}\right)$ or $\ln\left(4 \pm \sqrt{15}\right)$ | A1 | 2.2a |
| | | **(7)** | |
| | **Alternative**<br>$\dfrac{dy}{dx} = 31\cosh x - 4\cosh 2x$ or $31\left(\dfrac{e^x + e^{-x}}{2}\right) - 4\left(\dfrac{e^{2x} + e^{-2x}}{2}\right)$ | B1 | 1.1b |
| | Using $\cosh x = \left(\dfrac{e^x + e^{-x}}{2}\right)$ and $\sinh x = \left(\dfrac{e^x - e^{-x}}{2}\right)$ as required<br>$\Rightarrow 31\left(\dfrac{e^x + e^{-x}}{2}\right) - 4\left(\dfrac{e^{2x} + e^{-2x}}{2}\right) = 0$<br>leading to $4e^{4x} - 31e^{3x} - 31e^x + 4 = 0$ o.e. | M1<br>A1 | 3.1a<br>1.1b |
| | Solves $4e^{4x} - 31e^{3x} - 31e^x + 4 = 0$<br>$\Rightarrow e^x = ...$ | M1 | 1.1b |
| | $e^x = 4 \pm \sqrt{15}$ or awrt 7.87, 0.13 | A1 | 1.1b |
| | $x = \ln(b)$ where $b$ is a real exact value | M1 | 1.2 |
| | $\ln\left(4 \pm \sqrt{15}\right)$ | A1 | 2.2a |
| | | **(7)** | |
| | | | **(7 marks)** |

**Notes**

B1: Correct differentiation
M1: Identifies a correct approach by using a correct identity to make progress to obtain a quadratic in cosh $x$
A1: Correct 3 term quadratic obtained
M1: Solves their 3TQ
A1: Correct values (may only see 4 here)
M1: Correct process to reach at least one value for $x$ from their cosh $x$

| |
|---|
| A1: Deduces the correct 2 values with no incorrect values or work involving $\cosh x = -\frac{1}{8}$ |
| **Alternative**<br>B1: Correct differentiation<br>M1: Using the exponential form for cosh $x$, and sinh $x$ if required, and forms a quartic equation for $e^x$ with all terms simplified and all on one side<br>A1: Correct quartic equation for $e^x$<br>M1: Solves their quartic equation in $e^x$<br>A1: Correct values to two decimal places or exact values<br>M1: $x = \ln(b)$ where $b$ is a real exact value<br>A1: Deduces the correct 2 values only |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 2(a) | Centre of circle $C$ is $(1, -1)$ | B1 | 1.1b |
| | $r=\sqrt{(5-1)^2+(-4+1)^2}=5$ or $r=\sqrt{(-3-1)^2+(2+1)^2}=5$ or $r=\frac{1}{2}\sqrt{(-3-5)^2+(2+4)^2}=5$ | M1 | 3.1a |
| | $\lvert z-1+\mathrm{i}\rvert=5$ or $\lvert z-(1-\mathrm{i})\rvert=5$ | A1 | 2.5 |
| | | **(3)** | |
| (b) | $(x-1)^2+(y+1)^2=25,\quad (x-2)^2+(y-3)^2=4$ <br> $x^2-2x+1+y^2+2y+1=25$ <br> $x^2-4x+4+y^2-6y+9=4$ <br> $\Rightarrow 2x+8y=32$ | M1 | 3.1a |
| | $(16-4y)^2-4(16-4y)+4+y^2-6y+9=4$ or $x^2-4x+4+\left(\frac{16-x}{4}\right)^2-6\left(\frac{16-x}{4}\right)+9=4$ | M1 | 1.1b |
| | $17y^2-118y+201=0$ or $17x^2-72x+16=0$ | A1 | 1.1b |
| | $17y^2-118y+201=0\Rightarrow(17y-67)(y-3)=0\Rightarrow y=\frac{67}{17},3$ or $17x^2-72x+16=0\Rightarrow(17x-4)(x-4)=0\Rightarrow x=\frac{4}{17},4$ | M1 | 1.1b |
| | $y=\frac{67}{17},3\Rightarrow x=\frac{4}{17},4$ or $x=\frac{4}{17},4\Rightarrow y=\frac{67}{17},3$ | M1 | 2.1 |
| | $4+3\mathrm{i},\ \frac{4}{17}+\frac{67}{17}\mathrm{i}$ | A1 | 2.2a |
| | | **(6)** | |
| | | | **(9 marks)** |

**Notes**

(a)
B1: Correct coordinates of centre
M1: Fully correct strategy for identifying the radius. If the diameter is calculated this must be halved to achieve this mark.
A1: Correct equation using the required notation
(b)
M1: Begins the process of finding $z_1$ and $z_2$ by using the Cartesian equations to obtain the equation of the line of intersection
M1: Substitutes back into the equation of one of the circles to obtain an equation in one variable
A1: Correct 3 term quadratic
M1: Solves their 3TQ
M1: Substitutes to find values of the other variable to complete the process of finding $z_1$ and $z_2$
A1: Correct complex numbers

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 3(a) | $100m^2+60m+13=0\Rightarrow m=-0.3\pm0.2\mathrm{i}$ | M1 | 1.1b |
| | $x=\mathrm{e}^{-0.3t}\left(A\cos0.2t+B\sin0.2t\right)$ | A1 | 1.1b |
| | PI: $x=2$ | B1 | 1.1b |
| | $x=\mathrm{e}^{-0.3t}\left(A\cos0.2t+B\sin0.2t\right)+2$ | A1ft | 2.2a |
| | | **(4)** | |
| (b) | $t=0,\ \ x=0\Rightarrow A=-2$ | M1 | 3.4 |
| | $\frac{\mathrm{d}x}{\mathrm{d}t}=-0.3\mathrm{e}^{-0.3t}\left(-2\cos0.2t+B\sin0.2t\right)+\mathrm{e}^{-0.3t}\left(0.4\sin0.2t+0.2B\cos0.2t\right)$<br>$t=0,\ \frac{\mathrm{d}x}{\mathrm{d}t}=10\Rightarrow B=\ldots$(NB $B=47$) | M1 | 3.4 |
| | $x=\mathrm{e}^{-0.3t}\left(47\sin0.2t-2\cos0.2t\right)+2$ | A1 | 1.1b |
| | $-0.3\mathrm{e}^{-0.3t}\left(47\sin0.2t-2\cos0.2t\right)+\mathrm{e}^{-0.3t}\left(9.4\cos0.2t+0.4\sin0.2t\right)=0$<br>$\Rightarrow t=\ldots$<br>or<br>$x=\sqrt{2213}\mathrm{e}^{-0.3t}\sin(0.2t-0.0425)+2$<br>$\Rightarrow \frac{\mathrm{d}x}{\mathrm{d}t}=-0.3\sqrt{2213}\mathrm{e}^{-0.3t}\sin(0.2t-0.0425)$<br>$+0.2\sqrt{2213}\mathrm{e}^{-0.3t}\cos(0.2t-0.0425)$<br>$\Rightarrow t=\ldots$ | M1 | 3.1b |
| | $\tan0.2t=\frac{100}{137}\Rightarrow0.2t=0.630\ldots$<br>or<br>$\tan(0.2t-0.0425)=\frac{2}{3}\Rightarrow 0.2t=0.630$ | M1 | 2.1 |
| | $t=3.15\ldots$ weeks | A1 | 1.1b |
| | $x=\mathrm{e}^{-0.3\times\text{"}3.15\ldots\text{"}}\left(47\sin\left(0.2\times\text{"}3.15\ldots\text{"}\right)-2\cos\left(0.2\times\text{"}3.15\ldots\text{"}\right)\right)+2$ | M1 | 3.4 |
| | = awrt 12.1 {μg/ml} | A1 | 3.2a |
| | | **(8)** | |
| (c) | $t=10\Rightarrow x=\mathrm{e}^{-3}\left(47\sin(2)-2\cos(2)\right)+2=4.16\ldots$ | M1 | 3.4 |
| | The model suggests that it would be safe to give the second dose | A1ft | 2.2a |
| | | **(2)** | |
| | | **(14 marks)** | |

**Notes**

(a)
M1: Uses the model to form and solve the auxiliary equation
A1: Correct CF, does not need $x=$
B1: Correct PI
A1ft: Deduces the correct GS (follow through their CF + PI). Must have $x=\mathrm{f}(t)$ and PI not 0

(b)
M1: Uses the model and the initial conditions to establish the value of "$A$"
M1: Differentiates their model using the product rule and uses the initial conditions to establish the value of "$B$". Must be using $x=0$ and $\frac{\mathrm{d}x}{\mathrm{d}t}=10$
A1: Correct particular solution. This can be implied by the correct constants found following a correct answer to part (a).

M1: Uses their solution to the model with a correct strategy to obtain the required value of $t$ e.g. differentiates, sets equal to zero and solves for $t$
M1: Uses a correct trigonometric approach that leads to a value for $t$
A1: Correct value for $t$
M1: Uses the model and their value for $t$ to find the maximum concentration.
A1: Correct value
(c)
M1: Uses the model to find the concentration when $t = 10$
A1ft: Makes a suitable comment that is consistent with their calculated value
**Special case:** If the candidate's maximum value is less than 5 then
M1: never reaches 5 as maximum is…. or max is less than 5
A1: yes, it is safe

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 4(a) | $(\cos\theta+\mathrm{i}\sin\theta)^7=\cos^7\theta+\binom{7}{1}\cos^6\theta(\mathrm{i}\sin\theta)+\binom{7}{2}\cos^5\theta(\mathrm{i}\sin\theta)^2+...$ <br> Some simplification may be done at this stage <br> e.g. $c^7+7c^6\mathrm{i}s-21c^5s^2-35c^4\mathrm{i}s^3+35c^3s^4+21c^2\mathrm{i}s^5-7cs^6-\mathrm{i}s^7$ | M1 | 1.1b |
| | $\mathrm{i}\sin7\theta={}^7\mathrm{C}_1c^6\mathrm{i}s+{}^7\mathrm{C}_3c^4\mathrm{i}^3s^3+{}^7\mathrm{C}_5c^2\mathrm{i}^5s^5+\mathrm{i}^7s^7$ <br> or $=7c^6\mathrm{i}s+35c^4\mathrm{i}^3s^3+21c^2\mathrm{i}^5s^5+\mathrm{i}^7s^7$ | M1 | 2.1 |
| | $\sin7\theta=7c^6s-35c^4s^3+21c^2s^5-s^7$ | A1 | 1.1b |
| | $=7(1-s^2)^3s-35(1-s^2)^2s^3+21(1-s^2)s^5-s^7$ <br> $=7(1-3s^2+3s^4-s^6)s-35(1-2s^2+s^4)s^3+21(1-s^2)s^5-s^7$ | M1 | 2.1 |
| | $\{7s-21s^3+21s^5-7s^7-35s^3+70s^5-35s^7+21s^5-21s^7-s^7\}$ <br> leading to <br> $\sin7\theta=7\sin\theta-56\sin^3\theta+112\sin^5\theta-64\sin^7\theta$ * | A1* | 1.1b |
| | | **(5)** | |
| (b) | $1+\sin7\theta=0\Rightarrow\sin7\theta=-1$ | M1 | 3.1a |
| | $7\theta=-450,-90,270,630,...$ <br> or <br> $7\theta=-\frac{5\pi}{2},-\frac{\pi}{2},\frac{3\pi}{2},\frac{7\pi}{2},...$ | A1 | 1.1b |
| | $\theta=-\frac{450}{7},-\frac{90}{7},\frac{270}{7},\frac{630}{7},..\Rightarrow\sin\theta=...$ <br> or <br> $\theta=-\frac{5\pi}{14},-\frac{\pi}{14},\frac{3\pi}{14},\frac{7\pi}{14},...\Rightarrow\sin\theta=...$ | M1 | 2.2a |
| | $x=\sin\theta=-0.901,-0.223,\ 0.623,1$ | A1 <br> A1 | 1.1b <br> 2.3 |
| | | **(5)** | |
| | | **(10 marks)** | |

**Notes**

(a)
M1: Attempts to expand $(\cos\theta+\mathrm{i}\sin\theta)^7$ including a recognisable attempt at binomial coefficients
Some simplification may be done at this stage. (May only see imaginary terms)
M1: Identifies imaginary terms with sin $7\theta$
A1: Correct expression with coefficients evaluated and i's dealt with correctly
M1: Replaces $\cos^2\theta$ with $1-\sin^2\theta$ and applies the expansions of $(1-\sin^2\theta)^2$ and $(1-\sin^2\theta)^3$ to their expression
A1*: Reaches the printed answer with no errors and expansion of brackets seen.
(b)
M1: Makes the connection with part (a) and realises the need to solve $\sin7\theta=-1$
A1: At least one correct value for $7\theta$
M1: Divides by 7 and deduces that $x$ values are found by finding at least one value for $\sin\theta$
A1: Awrt 2 correct values for $x$
A1: Awrt all 4 $x$ values correct and no extras

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 5(a) | $y=\tan^{-1}x \Rightarrow \tan y = x \Rightarrow \dfrac{dx}{dy}=\sec^2 y$<br>$y=\tan^{-1}x \Rightarrow \tan y = x \Rightarrow \dfrac{dy}{dx}\sec^2 y = 1$ | M1 | 3.1a |
| | $\dfrac{dx}{dy}=1+\tan^2 y$ or $\dfrac{dy}{dx}\left(1+\tan^2 y\right)=1$ | M1 | 1.1b |
| | $\dfrac{dy}{dx}=\dfrac{1}{1+\tan^2 y}=\dfrac{1}{1+x^2}$ * | A1* | 2.1 |
| | | **(3)** | |
| (b) | $\dfrac{d\left(\tan^{-1}4x\right)}{dx}=\dfrac{4}{1+16x^2}$ | B1 | 1.1b |
| | $\int x\tan^{-1}4x\,dx = \alpha x^2\tan^{-1}4x - \int \alpha x^2 \times \left(\dfrac{4}{1+16x^2}\right)dx$ | M1 | 2.1 |
| | $\int x\tan^{-1}4x\,dx = \dfrac{x^2}{2}\tan^{-1}4x - \int \dfrac{x^2}{2}\times\dfrac{4}{1+16x^2}dx$ | A1 | 1.1b |
| | $=\ldots-\dfrac{1}{8}\int\dfrac{16x^2+1-1}{1+16x^2}dx = \ldots-\dfrac{1}{8}\int\left(1-\dfrac{1}{1+16x^2}\right)dx$<br>or<br>let $4x=\tan u \Rightarrow \dfrac{1}{8}\int \dfrac{\tan^2 u}{1+\tan^2 u}\times\dfrac{1}{4}\sec^2 u\,du$<br>$\Rightarrow \dfrac{1}{32}\int \tan^2 u\,du = \dfrac{1}{32}\int \sec^2 u - u\,du$ | M1 | 3.1a |
| | $=\dfrac{x^2}{2}\tan^{-1}4x-\dfrac{1}{8}x+\dfrac{1}{32}\tan^{-1}4x+k$ | A1 | 2.1 |
| | | **(5)** | |
| (c) | Mean value $=\left(\dfrac{1}{\frac{\sqrt{3}}{4}-0}\right)\left[\dfrac{x^2}{2}\tan^{-1}4x-\dfrac{1}{8}x+\dfrac{1}{32}\tan^{-1}4x\right]_0^{\frac{\sqrt{3}}{4}}$<br>$=\dfrac{4}{\sqrt{3}}\left(\left(\dfrac{3}{32}\times\dfrac{\pi}{3}-\dfrac{1}{8}\times\dfrac{\sqrt{3}}{4}+\dfrac{1}{32}\times\dfrac{\pi}{3}\right)-0\right)$ | M1 | 2.1 |
| | $=\dfrac{\sqrt{3}}{72}\left(4\pi-3\sqrt{3}\right)$ or $\dfrac{\sqrt{3}}{18}\pi-\dfrac{1}{8}$ oe | A1 | 1.1b |
| | | **(2)** | |
| | | | **(10 marks)** |

**Notes**

(a)
M1: Makes progress in establishing the derivative by taking the tan of both sides and differentiating with respect to $y$ or implicitly with respect to $x$
M1: Use of the correct identity
A1*: Fully correct proof

(b)
B1: Correct derivative
M1: Uses integration by parts in the correct direction
A1: Correct expression
M1: Adopts a correct strategy for the integration by splitting into two fractions or using a substitution of $4x = \tan u$ to get to an integrable form
A1: Correct answer
(c)
M1: Correctly applies the method for the mean value for their integration. The limit of zero can be implied if it comes to 0.
A1: Correct exact answer. Allow exact equivalents e.g. $\frac{4\pi\sqrt{3}-9}{72}$, $\frac{\pi\sqrt{3}}{18}-\frac{1}{8}$

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **6(a)** | $\lvert\mathbf{M}\rvert = k(-1-1)-5(-1-2)+7(1-2)$<br>$\{=8-2k\}$ | M1 | 1.1b |
| | Minors: $\begin{pmatrix}-2 & -3 & -1\\ -12 & -k-14 & k-10\\ -2 & k-7 & k-5\end{pmatrix}$<br>Cofactors: $\begin{pmatrix}-2 & 3 & -1\\ 12 & -k-14 & 10-k\\ -2 & 7-k & k-5\end{pmatrix}$ | M1 | 1.1b |
| | $\mathbf{M}^{-1} = \dfrac{1}{8-2k}\begin{pmatrix}-2 & 12 & -2\\ 3 & -k-14 & 7-k\\ -1 & 10-k & k-5\end{pmatrix}$ | M1<br>A1 | 2.1<br>1.1b |
| | | **(4)** | |
| **(b)** | $\mathbf{M}^{-1} = \dfrac{1}{4}\begin{pmatrix}-2 & 12 & -2\\ 3 & -16 & 5\\ -1 & 8 & -3\end{pmatrix} \Rightarrow \begin{pmatrix}x\\ y\\ z\end{pmatrix} = \mathbf{M}^{-1}\begin{pmatrix}1\\ p\\ 2\end{pmatrix}$<br>Solve the equations simultaneously to achieve values for $x$, $y$ and $z$<br>$y+3z=2p-2$ and $4y+8z=-1 \Rightarrow x=\ldots,\ y=\ldots,\ z=\ldots$ | M1 | 3.1a |
| | $\begin{pmatrix}x\\ y\\ z\end{pmatrix} = \begin{pmatrix}-\frac{1}{2}+3p-1\\ \frac{3}{4}-4p+\frac{5}{2}\\ -\frac{1}{4}+2p-\frac{3}{2}\end{pmatrix}$ | A1ft | 1.1b |
| | $\left(\dfrac{12p-6}{4}, \dfrac{13-16p}{4}, \dfrac{8p-7}{4}\right)$<br>$\left(3p-\dfrac{3}{2}, \dfrac{13}{4}-4p, 2p-\dfrac{7}{4}\right)$ | A1 | 2.2a |
| | | **(3)** | |
| **(c)(i)** | | | |
| | For consistency: E.g. eliminates $z$ to find two equations from $3x+2y=q+2$, $3x+2y=7q-1$, $18x+12y=15$<br>For consistency: E.g. eliminates $x$ to find two equations from $y+3z=1-4q$, $3y+9z=-3$, $y+3z=2q-2$<br>For consistency: E.g. eliminates $y$ to find two equations from $x-2z=2-q$, $-x+2z=1-5q$, $-6x+12z=-9$ | M1 | 3.1a |
| | e.g. $q+2=7q-1 \Rightarrow q=\ldots$<br>e.g. $-3=3(1-4q) \Rightarrow q=\ldots$<br>e.g. $-9=6(1-5q) \Rightarrow q=\ldots$ | M1 | 1.1b |
| | $q=\dfrac{1}{2}$ | A1 | 1.1b |

| | | | |
|---|---|---|---|
| | **Alternative**<br>Equating coefficients leading to two out of three equations and solves to find values for a and b<br>$4a+b=2, 5a+b=1, 7a+b=-1$<br>$\{a=-1, b=6\}$ | M1 | 3.1a |
| | Forms the fourth equation involving $q$ $a+bq=2$ and substitutes in the values of $a$ and $b$ to finds a value for $q$ | M1 | 1.1b |
| | $q=\dfrac{1}{2}$ | A1 | 1.1b |
| | Finds a coordinate of intersection of the planes<br>$4x+5y+7z=1$ and $2x+y-z=2$<br>e.g let $z=0 \Rightarrow 4x+5y=1$ and $2x+y=2 \Rightarrow y=-1, x=1.5$ | M1 | 3.1a |
| | Substitutes the values for $x, y$ and $z$ into $x+y+z=q$ to reach a value for $q$ | M1 | 1.1b |
| | $q=\dfrac{1}{2}$ | A1 | 1.1b |
| **(ii)** | For example:<br>$x=\lambda \Rightarrow 3\lambda+2y=\dfrac{5}{2},\ \lambda-2z=\dfrac{3}{2} \Rightarrow y=\mathrm{f}(\lambda), z=\mathrm{f}(\lambda)$<br>$y=\lambda \Rightarrow 3x+2\lambda=\dfrac{5}{2},\ \lambda+3z=1 \Rightarrow x=\mathrm{f}(\lambda), z=\mathrm{f}(\lambda)$<br>$z=\lambda \Rightarrow 3y+9\lambda=-3,\ -6x+12\lambda=-9 \Rightarrow x=\mathrm{f}(\lambda), y=\mathrm{f}(\lambda)$ | M1 | 3.1a |
| | Let $x=\lambda,\ \lambda=\dfrac{y-\frac{5}{4}}{-\frac{3}{2}}=\dfrac{z+\frac{3}{4}}{\frac{1}{2}}$ or $y=\dfrac{5}{4}-\dfrac{3}{2}\lambda, z=-\dfrac{3}{4}+\dfrac{1}{2}\lambda$<br>Let $y=\lambda,\ \lambda=\dfrac{x-\frac{5}{6}}{-\frac{2}{3}}=\dfrac{z+\frac{1}{3}}{-\frac{1}{3}}$ or $x=\dfrac{5}{5}-\dfrac{2}{3}\lambda, z=-\dfrac{1}{3}-\dfrac{1}{3}\lambda$<br>Let $z=\lambda,\ \lambda=\dfrac{x-\frac{3}{2}}{2}=\dfrac{y+1}{-3}$ or $x=\dfrac{3}{2}+2\lambda, y=-1-3\lambda$ | A1 | 1.1b |
| | $\mathbf{r}=\dfrac{5}{4}\mathbf{j}-\dfrac{3}{4}\mathbf{k}+t(2\mathbf{i}-3\mathbf{j}+\mathbf{k})$ o.e.<br>$\mathbf{r}=\dfrac{5}{6}\mathbf{i}-\dfrac{1}{3}\mathbf{k}+t(2\mathbf{i}-3\mathbf{j}+\mathbf{k})$ o.e.<br>$\mathbf{r}=\dfrac{3}{2}\mathbf{i}-\mathbf{j}+t(2\mathbf{i}-3\mathbf{j}+\mathbf{k})$ o.e. | M1<br>A1 | 1.1b<br>2.5 |
| | **Alternative (ii)**<br>Finds two different coordinates that lie on the line of intersection<br>For example: setting $x=0 \Rightarrow \left(0, \dfrac{5}{4}, -\dfrac{3}{4}\right)$<br>setting $y=0 \Rightarrow \left(\dfrac{5}{6}, 0, -\dfrac{1}{3}\right)$ setting $z=0 \Rightarrow \left(\dfrac{3}{2}, -1, 0\right)$ | M1<br>A1 | 3.1a<br>1.1b |
| | Finds the vector equation of the line passing through their two points | M1 | 1.1b |
| | $\mathbf{r}=\dfrac{5}{4}\mathbf{j}-\dfrac{3}{4}\mathbf{k}+t(2\mathbf{i}-3\mathbf{j}+\mathbf{k})$ o.e. | A1 | 2.5 |

| | | | |
|---|---|---|---|
| | $\mathbf{r}=\frac{5}{6}\mathbf{i}-\frac{1}{3}\mathbf{k}+t\left(2\mathbf{i}-3\mathbf{j}+\mathbf{k}\right)$ o.e.<br>$\mathbf{r}=\frac{3}{2}\mathbf{i}-\mathbf{j}+t\left(2\mathbf{i}-3\mathbf{j}+\mathbf{k}\right)$ o.e. | | |
| | **Alternative (ii): Outside the spec**<br>Finds the cross product of two normal vectors and a coordinate that lies on all three planes | M1 | 3.1a |
| | Correct cross product<br>$\begin{vmatrix}4&5&7\\1&1&1\end{vmatrix}=\begin{vmatrix}4&5&7\\2&1&-1\end{vmatrix}=\begin{pmatrix}2\\3\\-1\end{pmatrix}$ or $\begin{vmatrix}1&1&1\\2&1&-1\end{vmatrix}=\begin{pmatrix}12\\18\\-3\end{pmatrix}$ | A1 | 1.1b |
| | Uses the point and the direction vector to find the equation of the line | M1 | 1.1b |
| | $\mathbf{r}=\frac{5}{4}\mathbf{j}-\frac{3}{4}\mathbf{k}+t\left(2\mathbf{i}-3\mathbf{j}+\mathbf{k}\right)$ o.e. | A1 | 1.1b |
| | | **(7)** | |
| | | | **(14 marks)** |

**Notes**

(a)
M1: Correct method to find the determinant. Condone one sign slip
M1: A correct first step in obtaining the inverse. Could be the matrix of minors or cofactors. Condone sign slips as long as the intention is clear.
M1: Fully correct method to obtain the inverse. Attempts matrix of minors, cofactors, transposes and 1/determinant
A1: Correct matrix
(b)
M1: A complete strategy for solving the given equations e.g. multiplies the given coordinates by their inverse or solves simultaneously to achieve values for $x$, $y$ and $z$
A1ft: Correct calculation on their inverse matrix (unsimplified) or at least on correct value if solving simultaneously
A1: Correct coordinates
(c)(i)
M1: Uses a correct strategy that will lead to establishing a value for $q$. E.g. eliminating one of $x$, $y$ or $z$
M1: Solves a suitable equation to obtain a value for $q$
A1: Correct value
(c)(i) **Alternative 1**
M1: Equating coefficients leading to two out of three equations and solves to find values for a and b
M1: Solves a suitable equation to obtain a value for $q$ using their values for $a$ and $b$
(c)(i) **Alternative 2**
M1: Finds a coordinate of intersection of the planes $4x+5y+7z=1$ and $2x+y-z=2$
M1: Substitutes the values for $x$, $y$ and $z$ into $x+y+z=q$ to reach a value for $q$
A1: Correct value
(ii)
M1: Uses a correct strategy to obtain the Cartesian equation of the line or the general coordinates
A1: Correct Cartesian equation or coordinates in terms of a parameter.

| |
|---|
| M1: Uses their Cartesian equation to correctly extract the position and direction to form a vector equation for the required line<br>A1: Correct equation (o.e.) look out for multiples of the direction vector<br>**Alternative (ii)**<br>M1: Finds two different coordinates that lie on the line of intersection<br>A1: Correct coordinates<br>M1: Uses their coordinates to find the vector equation of the line that passes through them.<br>A1: Correct equation (o.e.), look out for multiples of the direction vector. Must have **r** = …<br>**Alternative (ii) outside spec**<br>M1: Finds the cross product between the normal vectors of two of the planes and a coordinate that lies on all three planes. If a coordinate is found in (i) it must be used in this part to award this mark.<br>A1: Correct cross product<br>M1: Uses the coordinate and the cross product to find the equation of the line<br>A1: Correct equation (o.e.), look out for multiples of the direction vector. Must have **r** = … |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 7(a) | $1=\dfrac{a}{0.5+b},\ 0.5=\dfrac{a}{2.5+b} \Rightarrow a=...,\ b=...$ | M1 | 3.3 |
| | $a=2,\ b=1.5$ | A1 | 1.1b |
| | | **(2)** | |
| (b) | $V_1=\pi\int x^2\mathrm{d}y=\pi\int\left(\dfrac{"2"}{y+"1.5"}\right)^2\mathrm{d}y$ | B1ft | 3.4 |
| | $\pi\int_{0.5}^{2.5}\left(\dfrac{"2"}{y+"1.5"}\right)^2\mathrm{d}y$ | M1 | 1.1a |
| | $=\{4\pi\}\left[-(y+1.5)^{-1}\right]_{0.5}^{2.5}(=\pi)$ | M1 | 1.1b |
| | $x^2+(y-3)^2=0.5$ | B1 | 2.2a |
| | $V_2=\pi\int x^2\mathrm{d}y=\pi\int\left(0.5-(y-3)^2\right)\mathrm{d}y$ or $\pi\int\left(-y^2+6y-8.5\right)\mathrm{d}y$ | M1 | 1.1b |
| | $=\pi\int_{2.5}^{3+\frac{1}{\sqrt{2}}}\left(0.5-(y-3)^2\right)\mathrm{d}y$ or $=\pi\int_{2.5}^{3+\frac{1}{\sqrt{2}}}\left(-y^2+6y-8.5\right)\mathrm{d}y$ | M1 | 3.3 |
| | $=\{\pi\}\left[0.5y-\dfrac{1}{3}(y-3)^3\right]_{2.5}^{3+\frac{1}{\sqrt{2}}}$ or $=\{\pi\}\left[-\dfrac{1}{3}y^3+3y^2-8.5y\right]_{2.5}^{3+\frac{1}{\sqrt{2}}}$ | A1 | 1.1b |
| | $V_1+V_2+\text{cylinder}=\pi+\pi\left(\dfrac{5}{24}+\dfrac{\sqrt{2}}{6}\right)+\dfrac{1}{2}\pi$ | dM1 | 3.4 |
| | $=\pi\left(\dfrac{41}{24}+\dfrac{\sqrt{2}}{6}\right)\approx 6.11\text{cm}^3$ | A1 | 2.2b |
| | | **(9)** | |
| | | **(11 marks)** | |

**Notes**

(a)
M1: Uses the given coordinates correctly in the equation modelling the curve to obtain at least one correct equation and attempts to find the values of $a$ and $b$
A1: Correct values
(b)
B1ft: Uses the model to obtain $\pi\int\left(\dfrac{\text{their } a}{y+\text{their } b}\right)^2\mathrm{d}y$. Note the $p$ can be recovered if appears later.
M1: Chooses limits appropriate to the model i.e. 0.5 and 2.5
M1: Integrates to obtain an expression of the form $k(y+\text{“1.5”})^{-1}$
B1: Deduces the correct equation for the circle
M1: Uses their circle equation and $\pi\int x^2\,\mathrm{d}y$ to attempt the top volume. Note the $p$ can be recovered if appears later.

M1: Identifies limits appropriate to the model i.e. 2.5 and 3 + their radius
A1: Correct integration
dM1: Uses the model to find the volume of the chess piece including the cylindrical base (dependent on all previous method marks)
A1: Correct volume

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