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 Edexcel}

# Mark Scheme (Result) 

October 2020

Pearson Edexcel GCE In A level Further Mathematics
Paper 9FM0/3B

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## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.


## EDEXCEL GCE MATHEMATI CS

## General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:

- M marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- B marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod - benefit of doubt
- ft - follow through
- the symbol $\sqrt{ }$ will be used for correct ft
- cao - correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw - ignore subsequent working
- awrt - answers which round to
- SC: special case
- oe - or equivalent (and appropriate)
- dep - dependent
- indep - independent
- dp decimal places
- sf significant figures
-     * The answer is printed on the paper
- $\quad$ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

| Question | Scheme | Marks | AOs |
| :---: | :---: | :---: | :---: |
| 1(a)(i) | $X \sim \operatorname{Po}(24)$ | B1 | 3.4 |
|  | $\mathrm{P}(X=26)=0.071912 \ldots \quad$ awrt $\underline{\mathbf{0 . 0 7 1 9}}$ | B1 | 1.1b |
|  |  | (2) |  |
| (ii) | $\mathrm{P}(X \geq 21)=1-\mathrm{P}(X \leq 20)[=1-0.24263 \ldots]$ | M1 | 3.4 |
|  | $=0.75736 \ldots$ awrt $\underline{\mathbf{0 . 7 5 7}}$ | A1 | 1.1 b |
|  |  | (2) |  |
| (b) | $\begin{array}{ll} \mathrm{H}_{0}: \lambda=2 & {[\mu=16]} \\ \mathrm{H}_{1}: \lambda<2 & {[\mu<16]} \end{array}$ | B1 | 2.5 |
|  | $\mathrm{P}(Y \leq 10 \mid Y \sim \operatorname{Po}(16))=0.077396 \ldots \quad$ awrt $\mathbf{0 . 0 7 7 4}$ | B1 | 1.1b |
|  | Not significant / Do not reject $\mathrm{H}_{0} / 10$ is not in the CR | M1 | 1.1b |
|  | There is not sufficient evidence to suggest a decrease/change in the rate of customers entering Jeff's supermarket. | A1 | 2.2b |
|  |  | (4) |  |
| (c) | Use of $\mathrm{Po}(8)$ to attempt critical region | M1 | 2.1 |
|  | Critical region is $Y \leq 3 / \mathrm{H}_{0}$ is not rejected when $Y \geq 4$ | A1 | 1.1b |
|  | True distribution is $W \sim \operatorname{Po}(4)$ | B1 | 2.1 |
|  | $\mathrm{P}(W \geq 4 \mid W \sim \mathrm{Po}(4))=1-\mathrm{P}(W \leq 3)[=1-0.43347 \ldots]$ | M1 | 1.1 b |
|  | $=0.56652 \ldots$ awrt $\underline{\mathbf{0 . 5 6 7}}$ | A1 | 1.1b |
|  |  | (5) |  |
| (13 marks) |  |  |  |
| Notes |  |  |  |
| (a)(i) <br> (ii) | B1: For realising the distribution is $\operatorname{Po}(24)$ (May be seen or implied in part (ii)) <br> B1: awrt 0.0719 <br> M1: Writing or using $1-\mathrm{P}(X \leq 20)$ <br> A1: awrt 0.757 |  |  |
| (b) | B1: Both hypotheses correct (must use $\mu$ or $\lambda$ ) <br> B1: awrt 0.0774 Allow awrt 0.08 from a correct probability statement. <br> allow CR: $X \leq 9$ <br> M1: Correct non-contextual conclusion (may be implied by correct contextual conclusion). Allow a f.t. comparison of 'their $p$ ' with 0.05 <br> (Ignore any contradictory contextual comments for this mark) <br> A1: A fully correct solution drawing a correct inference in context with all previous marks in (b) scored. |  |  |
| (c) | M1: Use of $\mathrm{Po}(8)$ to attempt critical region $[\mathrm{P}(Y \leq 3)=0.0423$.. $\mathrm{P}(Y \leq 4)=0.0996$..] <br> A1: Finding critical region for the test $Y \leq 3$ which must come from $\operatorname{Po}(8)$. <br> B1: Identifying the need to use $\operatorname{Po}(4)$ as the true distribution. <br> Allow Po(4) seen or used for this mark. <br> M1: Writing or using $\mathrm{P}\left(W \geq{ }^{\prime} 4\right.$ ') or $1-\mathrm{P}\left(W \leq{ }^{\prime} 3\right.$ ') from $\mathrm{Po}(4)$. Allow f.t. on their identified CR but must be using $\operatorname{Po}(4)$ <br> A1: awrt 0.567 |  |  |


| Question | Scheme | Marks | AOs |
| :---: | :---: | :---: | :---: |
| 2(a) | requires large $n /$ small $p$ so not a good approximation | B1 | 3.5 b |
|  |  | (1) |  |
| (b) | $X$ and $Y$ must be independent | B1 | 2.4 |
|  |  | (1) |  |
| (c) | $\mathrm{P}(X+Y<2.4)$ from $\operatorname{Po}(7) \quad[\mathrm{P}(X+Y \leq 2)]$ | M1 | 3.4 |
|  | $=0.029636 \ldots$ awrt $\underline{0.0296}$ | A1 | 1.1b |
|  |  | (2) |  |
| (4 marks) |  |  |  |
| Notes |  |  |  |
| (a) | B1: Correct reason why the model would not be appropriate and correct conclusion. Condone e.g. ' $p$ is close to 0.5 ' for $p$ is not small. Mean is not equal to variance on its own in B0. |  |  |
| (b) | B1: Correct explanation mentioning independence (oe). Ignore extraneous comments. |  |  |
| (c) | $\begin{aligned} & \text { M1: Using Po(7) with } 2.4 \\ & \text { A1: awrt } 0.0296 \\ & \hline \end{aligned}$ |  |  |


| Question | Scheme | Marks | AOs |
| :---: | :---: | :---: | :---: |
| 3(a) | [ $X \sim$ Geo ( 0.2 ) <br> Suzanne's $4^{\text {th }}$ selection is the $7^{\text {th }}$ selection overall] $\mathrm{P}(X=7)=(0.8)^{6}(0.2) \text { or }(0.64)^{3}(0.2)$ | M1 | 3.3 |
|  | $=0.05242 \ldots \quad$ awrt $\underline{0.0524}$ | A1 | 1.1b |
|  |  | (2) |  |
| (b) | $\mathrm{P}(X \geq 6)\left[=(1-0.2)^{5}\right]$ | M1 | 1.1b |
|  | $=0.32768$ awrt $\underline{0.328}$ | A1 | 1.1b |
|  |  | (2) |  |
| (c) | Mean $=5$ | B1 | 1.1b |
|  | Standard deviation $\left[=\sqrt{\frac{1-0.2}{0.2^{2}}}\right]=\sqrt{20} \quad$ awrt 4.47 | B1 | 1.1b |
|  |  | (2) |  |
| (d) | $\mathrm{P}($ Suzanne wins $)=0.2+(0.8)^{2}(0.2)+(0.8)^{4}(0.2)+\ldots$ | M1 | 3.1b |
|  | Infinite geometric series $=\frac{0.2}{1-0.8^{2}}(\mathrm{oe})$ | M1 | 2.1 |
|  | $=\frac{5}{9}$ | A1 | 1.1b |
|  |  | (3) |  |
| (9 marks) |  |  |  |
| Notes |  |  |  |
| (a) | M1: Selecting geometric distribution with $p=0.2$ and attempting required probability. <br> Allow $(0.8)^{n}(0.2)$ to imply M1 with $n=6$ or $n=3$ <br> A1: awrt 0.0524 Allow exact fraction $\frac{4096}{78125}$ |  |  |
| (b) | M1: $\mathrm{P}(X \geq 6)$ may be implied by $(1-p)^{5}$ or $1-\left(p+p q+p q^{2}+p q^{3}+p q^{4}\right)$ A1: awrt 0.328 Allow exact fraction $\frac{1024}{3125}$ |  |  |
| (c) | B1: Mean = 5 <br> B1: Standard deviation $=\sqrt{20}$ o.e. or awrt 4.47 |  |  |
| (d) | M1: Determining the probability that Suzanne wins with at least three terms seen (may be implied by $2^{\text {nd }} \mathrm{M} 1$ ) <br> M1: Recognising need to sum terms of an infinite geometric series with correct $r$ $=0.8^{2}$ (with numerator less than denominator) <br> A1: $\frac{5}{9}$ (allow awrt 0.556 ) |  |  |



| Qu. | Scheme |  |  |  |  |  | Marks | AOs |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5(a) | $p=\frac{(0)+11+14+6+(0)+5+(0)}{6 \times 40}$ |  |  |  |  |  | M1 | 2.1 |
|  | $p=\underline{\mathbf{0 . 1 5}}$ * |  |  |  |  |  | A1*cso | 1.1b |
|  |  |  |  |  |  |  | (2) |  |
| (b) | $X \sim \mathrm{~B}(6,0.15)$ |  |  |  |  |  | M1 | 3.4 |
|  | $x$ | 2 | 3 | 4 | 5 | 6 |  |  |
|  | $40 \times \mathrm{P}(X=x)$ | 7.04... | 1.65... | 0.219... | 0.015... | 0.00... |  |  |
|  | Require $40 \times \mathrm{P}(X \geq k)>5$ <br> Exp. frequency for $X \geq 2=8.94 \ldots / X \geq 3=1.89 \ldots$ |  |  |  |  |  | M1 | 1.1b |
|  | Combine last 5 cells / only 3 cells in total |  |  |  |  |  | A1 | 2.2a |
|  | 2 is subtracted (as there are 2 restrictions) and the proportion used from data (and 1 equal totals) |  |  |  |  |  | B1 | 2.4 |
|  | $3-2=1$ degree of freedom |  |  |  |  |  | A1 | 1.1b |
|  | $\mathrm{H}_{0}$ : Binomial distribution is a suitable model <br> $\mathrm{H}_{1}$ : Binomial distribution is not a suitable model |  |  |  |  |  | B1 | 3.4 |
|  | Critical value $\chi_{(1,0.10)}^{2}=2.705$ or 2.706 |  |  |  |  |  | B1ft | 1.1b |
|  | Test statistic is not in the critical region, insufficient evidence to reject $\mathrm{H}_{0} \quad(2.689<2.705 / 6)$ <br> Data are consistent with binomial/engineer's/suggested model. |  |  |  |  |  | B1ft | 3.5a |
|  |  |  |  |  |  |  | (8) |  |
| (c) | The total amount/proportion of defective pins remains the same. |  |  |  |  |  | M1 | 2.4 |
|  | The cells for $X \geq 2$ are still combined in the test. |  |  |  |  |  | M1 | 1.1b |
|  | So there is no change to the value of the test statistic. |  |  |  |  |  | A1 | 2.2a |
|  |  |  |  |  |  |  | (3) |  |
|  | (13 marks) |  |  |  |  |  |  |  |
|  | Notes |  |  |  |  |  |  |  |
| (a) | M1: Correct expression for $p$ (may be seen in stages). Allow $\frac{36}{240}$ but not $\frac{6}{40}$ on its own A1*cso: $p=0.15$ stated and no incorrect working seen |  |  |  |  |  |  |  |
| (b) | M1: Attempting to find expected frequencies, at least 2 correct trunc. or rounded 1 dp <br> M1: Recognising need to combine cells (Sight of awrt 8.94 implies M1M1) <br> A1: Combining cells for $X \geq 2$ (to make 3 cells) <br> B1: Justifying why 2 is subtracted with $p$ being calculated from data <br> A1: 1 degree of freedom <br> B1: Correct hypotheses ( 0.15 must not be included) Allow engineer's model. <br> B1ft: Correct critical value ( ft their df) May see $\chi_{(2,0.10)}^{2}=4.605$ or $\chi_{(3,0.10)}^{2}=6.251$ <br> B1ft: Correct inference (ft comparison of their CV with 2.689). <br> Condone $p=0.15$ included here. Do not allow contradictory statements to score here. <br> Hypotheses must be correct way round. |  |  |  |  |  |  |  |
| (c) | M1: Determining the number ( $N=36$ )/proportion $(p=0.15)$ of defective pins has not changed. e.g. $11+12+9+4=36$. But not $7+2+1=6+3+1$ <br> M1: Understanding the cells for $X \geq 2$ are still combined in the test <br> A1: (dep on both M1s) Concluding that there is no change to the value of the test statistic. |  |  |  |  |  |  |  |

\begin{tabular}{|c|c|c|c|}
\hline Question \& Scheme \& Marks \& AOs <br>
\hline \multirow[t]{2}{*}{6(a)} \& $\mathrm{P}(X=3)=\underline{\mathbf{0}}$ \& B1 \& 1.1b <br>
\hline \& \& (1) \& <br>
\hline \multirow[t]{8}{*}{(b)(i)} \& Coefficient of $t^{4}=\frac{1}{64} b^{2}$ \& M1 \& 2.1 <br>
\hline \& $\frac{1}{64} b^{2}=\frac{25}{64}$ \& M1 \& 1.1 b <br>
\hline \& $b=5$ (reject $b=-5$ since $b>0$ ) \& A1 \& 2.3 <br>
\hline \& $$
\begin{aligned}
& \mathrm{G}_{X}(1)=1 \\
& \frac{1}{64}(a+" 5 ")^{2}=1
\end{aligned}
$$ \& M1 \& 2.1 <br>
\hline \& $a=3$ (reject $a=-13$ since $a>0$ ) \& A1 \& 1.1 b <br>
\hline \& $\mathrm{P}(X=2)=$ coefficient of $t^{2}=\frac{1}{64}(2 a b)$ \& M1 \& 3.4 <br>
\hline \& $=\frac{15}{32}$ \& A1 \& 1.1b <br>
\hline \& \& (7) \& <br>
\hline \multirow[t]{4}{*}{(ii)} \& $\mathrm{E}(X)=\mathrm{G}_{X}^{\prime}(1)$ \& M1 \& 2.1 <br>
\hline \& $$
\begin{aligned}
& \mathrm{G}_{X}^{\prime}(t)=\frac{2}{64}\left(" 3 "+" 5 " t^{2}\right) \times " 10 " t \text { or } \\
& \mathrm{G}_{x}^{\prime}(t)=\frac{1}{64}\left(" 60 " t+" 100 " t^{3}\right)
\end{aligned}
$$ \& M1 \& 1.1b <br>
\hline \& $\mathrm{G}_{X}^{\prime}(1)=2.5$ \& A1ft \& 1.1b <br>
\hline \& \& (3) \& <br>
\hline \multirow[t]{3}{*}{(c)} \& $\mathrm{G}_{Y}(t)=t^{2} \mathrm{G}_{X}\left(t^{3}\right)\left[=\frac{t^{2}}{64}\left(a+b\left(t^{3}\right)^{2}\right)^{2}\right]$ \& M1 \& 3.1a <br>
\hline \& $\mathrm{G}_{Y}(t)=\frac{t^{2}}{64}\left(" 3 "+" 5 " t^{6}\right)^{2}$ \& A1ft \& 1.1b <br>
\hline \& \& (2) \& <br>
\hline \multicolumn{4}{|r|}{(13 marks)} <br>
\hline \multicolumn{4}{|c|}{Notes} <br>
\hline (a) \& \multicolumn{3}{|l|}{B1: 0 (Since there is no term in $t^{3}$ )} <br>
\hline (b)(i)

(b)(ii) \& \multicolumn{3}{|l|}{| M1: Realising that $\frac{1}{64} b^{2}$, the coefficient of $t^{4}$, is needed |
| :--- |
| M1: Equating their coefficient of $t^{4}$ to $\frac{25}{64}$ with an attempt to find $b$ |
| A1: $b=5$ only |
| M1: Realising that $\mathrm{G}_{X}(1)=1$ is required |
| A1: $a=3$ only |
| M1: Finding coefficient of $t^{2}$ with their $a>0$ and $b>0$ |
| A1: $\frac{15}{32}$ (condone awrt 0.469) |
| M1: Realising $\mathrm{G}_{X}^{\prime}(1)$ is needed |
| M1: Attempt to differentiate $\mathrm{G}_{X}(t)$ with their values of $a$ and $b$ |
| A1ft: 2.5 (ft (3sf) their values of $a$ and $b, a>0$ and $b>0$ ) $\mathrm{E}(X)=\frac{a b+b^{2}}{16}$ |
| Alternative: |
| M1: Realising $X=0,2$ and 4 only |
| M1: $[0 \times \mathrm{P}(X=0)]+2 \times \mathrm{P}(X=2)+4 \times \mathrm{P}(X=4)$ |} <br>

\hline (c) \& \multicolumn{3}{|l|}{| M1: either $\mathrm{G}_{X}\left(t^{3}\right)$ or $\times t^{2}$ or using $Y=2,8,14$ |
| :--- |
| A1ft: ft their values of $a$ and $b, a>0$ and $b>0$ $\begin{aligned} & \mathrm{G}_{Y}(t)=\frac{t^{2}}{64}\left(" 3 "+" 5 " t^{6}\right)^{2} \text { or } \mathrm{G}_{Y}(t)=\frac{t^{2}}{64}\left(" 9 "+" 30 " t^{6}+" 25 " t^{12}\right) \text { or } \\ & \mathrm{G}_{Y}(t)=\frac{1}{64}\left(" 9 t^{2} "+" 30 " t^{8}+" 25 " t^{14}\right) \end{aligned}$ |} <br>

\hline
\end{tabular}

| Qu. | Scheme | Marks | AOs |
| :---: | :---: | :---: | :---: |
| 7(a) | Realising $S$ has a discrete uniform distribution over $\{1, \ldots 6\}$ | M1 | 3.3 |
|  | $\mathrm{E}(S)=1 \times \frac{1}{6}+2 \times \frac{1}{6}+3 \times \frac{1}{6}+4 \times \frac{1}{6}+5 \times \frac{1}{6}+6 \times \frac{1}{6}$ | M1 | 1.1b |
|  | $\operatorname{Var}(S)=\frac{6^{2}-1}{12}$ or $1^{2} \times \frac{1}{6}+2^{2} \times \frac{1}{6}+3^{2} \times \frac{1}{6}+4^{2} \times \frac{1}{6}+5^{2} \times \frac{1}{6}+6^{2} \times \frac{1}{6}-3.5^{2}$ | M1 | 1.1 b |
|  | $\mathrm{E}(S)=3.5$ and $\operatorname{Var}(S)=\frac{35}{12}$ | A1 | 1.1b |
|  | $\bar{S} \sim \mathrm{~N}(3.5, \ldots$. | M1 | 3.1a |
|  | $\operatorname{Var}(\bar{S})=\frac{\frac{35}{12}}{45}=\frac{7}{108}, \bar{S} \sim \mathrm{~N}(3.5,0.0648 \ldots)$ | A1 | 1.1b |
|  | $\mathrm{P}(\bar{S}<k)=0.05 \rightarrow \frac{k-3.5}{\sqrt{\frac{7}{108}}}=-1.6449$ | M1 | 3.4 |
|  | $k=3.08122 \ldots$ awrt $\mathbf{3 . 0 8}$ | A1 | 1.1b |
|  |  | (8) |  |
| (b) | CLT applies since the sample size is large | B1 | 3.5b |
|  | CLT states that the sample mean/ $\bar{S}$ is (approximately) normally distributed | B1 | 3.5 b |
|  |  | (2) |  |
| (c) | True $\bar{S} \sim \mathrm{~N}\left(4, \frac{3}{45}\right)$ | M1 | 3.3 |
|  | $\mathrm{P}(\bar{S}<3.1)+\mathrm{P}(\bar{S}>3.9) \quad$ or $\quad 1-\mathrm{P}(3.1<\bar{S}<3.9)$ | dM1 | 3.4 |
|  | Power $=$ awrt $\underline{\mathbf{0 . 6 5 1}}$ | A1 | 1.1b |
|  |  | (3) |  |
| (d) | E.g. The increase in sample size would decrease the variance of $\bar{S}$ [leading to an increase in $\mathrm{P}(\bar{S}>3.9)$ and the decrease in $\mathrm{P}(\bar{S}<3.1)$ would be negligible] | B1 | 2.4 |
|  | So the power would increase. | dB1 | 2.2a |
|  |  | (2) |  |
| (15 marks) |  |  |  |
| Notes |  |  |  |
| (a) | M1: Setting up model for $S$ <br> M1: Attempt at expression for $\mathrm{E}(S)$ <br> M1: Attempt at expression for $\operatorname{Var}(S)$ <br> A1: Correct mean and variance for $S$ (may be implied by a correct distribution for $\bar{S}$ ) <br> M1: Use of CLT to find distribution for $\bar{S} \sim \mathrm{~N}\left({ }^{\prime} 3.5^{\prime}, \ldots\right)$ f.t. their 3.5 but variance $\neq \frac{35}{12}$ <br> A1: Correct distribution with correct variance, allow $\sigma^{2}=$ awrt 0.0648 or $\sigma=$ awrt 0.255 <br> M1: Standardising using their model and equating to a $z$-value $1<\|z\|<2$ <br> A1: awrt 3.08 |  |  |
| (b) | B1: Correct explanation about appropriateness of the CLT given large sample size (allow > 30) <br> B1: Requires both sample and mean or $\bar{S}$ |  |  |
| (c) | M1: Writing or using $\bar{S} \sim \mathrm{~N}\left(4, \frac{3}{45}\right)$ allow $\sigma^{2}=$ awrt 0.0667 or $\sigma=$ awrt 0.258 dM1: (dep on $1^{\text {st }} \mathrm{M} 1$ ) correct probability statement for power A1: awrt 0.651 |  |  |
| (d) | B1: Correct reasoning which refers to decrease in variance dB1: (dep on $\left.1^{\text {st }} \mathrm{B} 1\right)$ Correct deduction with no incorrect reasoning |  |  |

