# AS
# FURTHER MATHEMATICS
# 7366/1

Paper 1

**Mark scheme**

June 2020

Version: 1.0 Final Mark Scheme

*206A7366/1/MS*

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

# Mark scheme instructions to examiners

## General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

**Key to mark types**

| | |
|---|---|
| M | mark is for method |
| R | mark is for reasoning |
| A | mark is dependent on M marks and is for accuracy |
| B | mark is independent of M marks and is for method and accuracy |
| E | mark is for explanation |
| F | follow through from previous incorrect result |

**Key to mark scheme abbreviations**

| | |
|---|---|
| CAO | correct answer only |
| CSO | correct solution only |
| ft | follow through from previous incorrect result |
| ‘their’ | indicates that credit can be given from previous incorrect result |
| AWFW | anything which falls within |
| AWRT | anything which rounds to |
| ACF | any correct form |
| AG | answer given |
| SC | special case |
| OE | or equivalent |
| NMS | no method shown |
| PI | possibly implied |
| sf | significant figure(s) |
| dp | decimal place(s) |

Examiners should consistently apply the following general marking principles:

**No Method Shown**

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

**Diagrams**

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

**Work erased or crossed out**

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

**Choice**

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

## AS/A-level Maths/Further Maths assessment objectives

| AO | | Description |
|---|---|---|
| **AO1** | AO1.1a | Select routine procedures |
| | AO1.1b | Correctly carry out routine procedures |
| | AO1.2 | Accurately recall facts, terminology and definitions |
| **AO2** | AO2.1 | Construct rigorous mathematical arguments (including proofs) |
| | AO2.2a | Make deductions |
| | AO2.2b | Make inferences |
| | AO2.3 | Assess the validity of mathematical arguments |
| | AO2.4 | Explain their reasoning |
| | AO2.5 | Use mathematical language and notation correctly |
| **AO3** | AO3.1a | Translate problems in mathematical contexts into mathematical processes |
| | AO3.1b | Translate problems in non-mathematical contexts into mathematical processes |
| | AO3.2a | Interpret solutions to problems in their original context |
| | AO3.2b | Where appropriate, evaluate the accuracy and limitations of solutions to problems |
| | AO3.3 | Translate situations in context into mathematical models |
| | AO3.4 | Use mathematical models |
| | AO3.5a | Evaluate the outcomes of modelling in context |
| | AO3.5b | Recognise the limitations of models |
| | AO3.5c | Where appropriate, explain how to refine models |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| 1 | Ticks correct box. | 1.1b | B1 | $2\left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right)$ |
| | **Total** | | **1** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| 2 | Ticks correct box. | 1.2 | B1 | $1 + \mathrm{i}$ and $1$ |
| | **Total** | | **1** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| 3 | Ticks correct box. | 1.1b | B1 | $\{x: x < 1\} \cup \{x: 2 < x < a\}$ |
| | **Total** | | **1** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| 4(a) | Obtains one correct element in terms of $a$.<br>Must be a $2 \times 2$ matrix. | 1.1a | M1 | $\begin{bmatrix} 2 & a & 3 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ -2 & 4a \\ 0 & 5 \end{bmatrix}$ |
| | Obtains the correct product.<br>Accept unsimplified. ISW | 1.1b | A1 | $= \begin{bmatrix} 2-2a+0 & -6+4a^2+15 \\ 0+4+0 & 0-8a+5 \end{bmatrix} = \begin{bmatrix} 2-2a & 4a^2+9 \\ 4 & 5-8a \end{bmatrix}$ |
| 4(b) | Obtains the correct determinant.<br>Accept unsimplified. ISW<br>Follow through their $2 \times 2$ matrix with at least one element in terms of $a$. | 1.1b | B1F | $(2-2a)(5-8a) - 4(4a^2+9)$<br>$= 10 - 16a - 10a + 16a^2 - 16a^2 - 36$<br>$= -26 - 26a$ |
| 4(c) | Selects a method to show that **AB** is singular.<br>e.g.<br>equates their expression for the determinant to zero<br>or<br>substitutes $a = -1$ into their expression for the determinant. | 1.1a | M1 | **AB** is singular when $\det \mathbf{AB} = 0$<br>$-26 - 26a = 0$<br><br>$-26a = 26$<br>$a = -1$ |
| | Completes a fully correct reasoned argument to show that AB is singular, clearly referring to singular ⇔ determinant = 0. | 2.1 | R1 | |
| | **Total** | | **5** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| 5(a) | Completes a rigorous argument to show that<br>$$r^2(r+1)^2-(r-1)^2r^2=4r^3$$<br>Must show at least one intermediate step. | 2.1 | R1 | $r^2(r+1)^2-(r-1)^2r^2=r^2(r^2+2r+1)-r^2(r^2-2r+1)$<br>$=r^4+2r^3+r^2-r^4+2r^3-r^2=4r^3$ |
| 5(b) | Uses the result from (a) with their $p$ to express $\sum r^3$ in terms of $\sum r^2(r+1)^2-(r-1)^2r^2$ with one pair of terms of the sum written correctly. | 1.1a | M1 | $\sum_{r=1}^{n}4r^3=\sum_{r=1}^{n}[r^2(r+1)^2-(r-1)^2r^2]$<br>$=1^2\times2^2-0^2\times1^2$<br>$+2^2\times3^2-1^2\times2^2$<br>$+\dots\dots\dots\dots\dots\dots$<br>$+(n-1)^2n^2-(n-2)^2(n-1)^2$<br>$+n^2(n+1)^2-(n-1)^2n^2$<br>$=n^2(n+1)^2-0^2\times1^2$<br>$\sum_{r=1}^{n}r^3=\frac{1}{4}n^2(n+1)^2$ |
| | Writes down at least three pairs of terms including the first and last pair of terms of the sum. | 1.1b | A1 | |
| | Completes a reasoned argument using the method of differences to show that<br>$$\sum_{r=1}^{n}r^3=\frac{1}{4}n^2(n+1)^2$$ | 2.1 | R1 | |
| | **Total** | | **4** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| 6 | Assesses the validity of Anna's work by identifying her error,<br>e.g. that the angle is not 30°<br>or that $\cos\theta$ has been ignored. | 2.3 | B1 | Anna gave the wrong angle<br>$\sin\theta = \frac{1}{2} \quad \Rightarrow \quad \theta = 30°$ or $\theta = 150°$<br>$\cos\theta = -\frac{\sqrt{3}}{2} \quad \Rightarrow \quad \theta = 150°$ or $\theta = -150°$<br>$\therefore \quad \theta = 150°$ |
| | States the correct angle 150°. | 1.1b | B1 | |
| | **Total** | | **2** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| 7 | Shows that $7^n - 3^n$ is divisible by 4 for $n = 1$. | 1.1b | B1 | $7^1 - 3^1 = 7 - 3 = 4$<br><br>Assume it is true for $n = k$<br><br>$\therefore\ 7^k - 3^k = 4m$<br>where $m$ is an integer<br><br>$7^{k+1} - 3^{k+1} = 7 \times 7^k - 3 \times 3^k$<br>$= 7(4m + 3^k) - 3 \times 3^k$<br>$= 28m + 4 \times 3^k$<br>$= 4(7m + 3^k)$<br><br>$\therefore$ it is also true for $n = k + 1$<br><br>It is true for $n = 1$. If it is true for $n = k$ then it is true for $n = k + 1$. Therefore, by induction, $7^n - 3^n$ is divisible by 4 for all integers, $n \geq 1$. |
| | States the assumption that<br>$7^k - 3^k$ is divisible by 4 and considers $7^{k+1} - 3^{k+1}$ ,<br>by using $7 \times 7^k$ or $3 \times 3^k$. | 2.4 | M1 | |
| | Completes rigorous working to deduce that $7^{k+1} - 3^{k+1}$ is divisible by 4. | 2.2a | R1 | |
| | Concludes a reasoned argument by stating that<br>$7^n - 3^n$ is divisible by 4 for $n = 1$;<br>that if $7^k - 3^k$ is divisible by 4, then $7^{k+1} - 3^{k+1}$ is divisible by 4<br>and hence, by induction, $7^n - 3^n$ is divisible by 4 for $n \geq 1$. | 2.1 | R1 | |
| | **Total** | | **4** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| 8(a) | Writes $\tanh x$ or $\tanh y$ in exponential form. | 1.2 | B1 | Let $y = \tanh^{-1} x$<br>$x = \tanh y$<br>$x = \dfrac{\dfrac{(e^y - e^{-y})}{2}}{\dfrac{(e^y - e^{-y})}{2}}$<br>$= \dfrac{e^{2y} - 1}{e^{2y} + 1}$<br>$x(\mathrm{e}^{2y} + 1) = \mathrm{e}^{2y} - 1$<br>$x\mathrm{e}^{2y} + x = \mathrm{e}^{2y} - 1$<br>$1 + x = \mathrm{e}^{2y} - x\mathrm{e}^{2y}$<br>$1 + x = \mathrm{e}^{2y}(1 - x)$<br>$\dfrac{1 + x}{1 - x} = \mathrm{e}^{2y}$<br>$2y = \ln\left(\dfrac{1 + x}{1 - x}\right)$<br>$y = \dfrac{1}{2}\ln\left(\dfrac{1 + x}{1 - x}\right)$<br>$\tanh^{-1} x = \dfrac{1}{2}\ln\left(\dfrac{1 + x}{1 - x}\right)$ |
| | Selects a method by forming an equation of the form $x = \tanh y$. | 3.1a | M1 | |
| | Forms an equation in $e^{2y}$. | 1.1a | M1 | |
| | Isolates the terms in $e^{2y}$. | 1.1a | M1 | |
| | Completes a rigorous argument to show that<br>$\tanh^{-1} x = \dfrac{1}{2}\ln\left(\dfrac{1 + x}{1 - x}\right)$ | 2.1 | R1 | |

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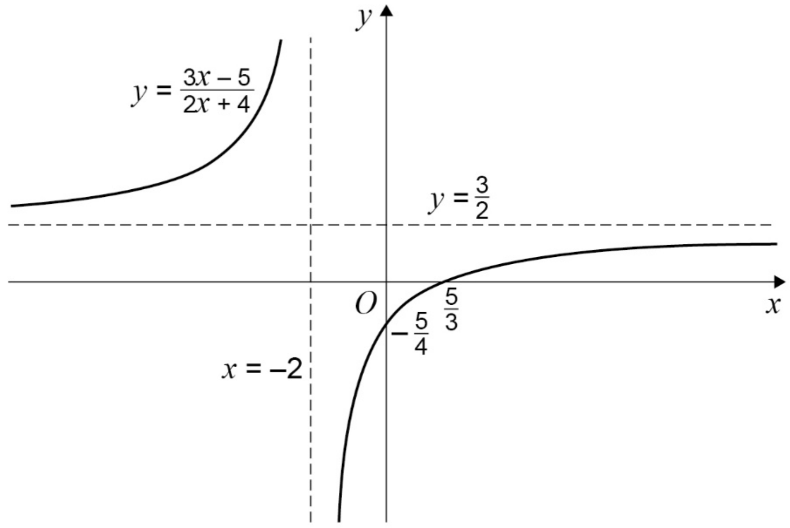
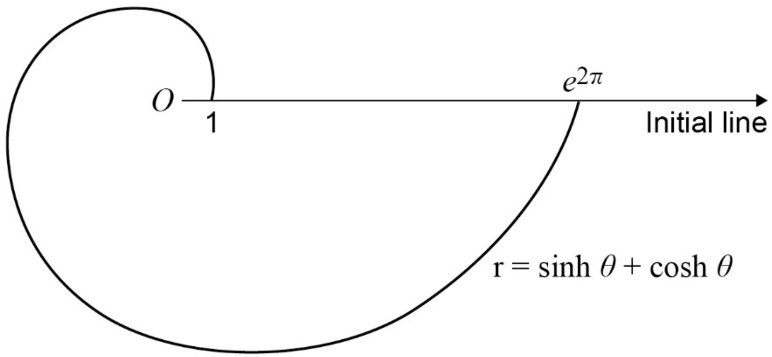
|---|---|---|---|---|
| **8(b)** | Selects a method by assuming, for contradiction, that the graphs do meet and forms the equation $\sinh x = \cosh x$. | 3.1a | M1 | If the graphs meet, then $\sinh x = \cosh x$<br>$\frac{1}{2}(e^x - e^{-x}) = \frac{1}{2}(e^x + e^{-x})$<br>$e^x - e^{-x} = e^x + e^{-x}$<br>$0 = 2e^{-x}$<br>but $e^{-x} > 0$<br>$\therefore$ the graphs of $y = \sinh x$ and $y = \cosh x$ do not intersect |
| | Deduces that $\tanh x = 1$ or that $2e^{-x} = 0$. | 2.2a | A1 | |
| | Completes a rigorous argument to prove the required result that the graphs of $y = \sinh x$ and $y = \cosh x$ do not intersect. | 2.1 | R1 | |
| | **Total** | | **8** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| 9(a)(i) | Obtains the correct value of $\alpha\beta = \frac{3}{2}$. | 1.2 | B1 | $\alpha\beta = \frac{3}{2}$ |
| 9(a)(ii) | Obtains the correct value of $\alpha + \beta = -\frac{p}{2}$. | 1.2 | B1 | $\alpha + \beta = -\frac{p}{2}$ |
| 9(b) | Expresses $(\alpha - \beta)^2$ in the form of $(\alpha + \beta)^2 + m\alpha\beta$. | 1.1a | M1 | $(\alpha - \beta)^2 = \alpha^2 - 2\alpha\beta + \beta^2$<br>$= (\alpha + \beta)^2 - 2\alpha\beta - 2\alpha\beta$<br>$= \left(-\frac{p}{2}\right)^2 - 4 \times \frac{3}{2}$<br>$= \frac{p^2}{4} - 6$ |
| | Obtains the correct value of $(\alpha - \beta)^2 = \frac{p^2}{4} - 6$.<br>(May be unsimplified.)<br>FT their $\alpha\beta$ and $\alpha + \beta$. | 1.1b | A1F | |

| | | | | |
|---|---|---|---|---|
| **9(c)** | Selects a method to find the quadratic equation with roots $\alpha - 1,\ \beta + 1$ by expressing the sum and product of roots in terms of $\alpha$ and $\beta$. | 3.1a | M1 | New sum $= \alpha - 1 + \beta + 1 = \alpha + \beta = -\frac{p}{2}$<br><br>$\alpha - \beta = \sqrt{\frac{p^2}{4} - 6}$<br><br>positive root only as $\alpha > \beta$<br><br>New product $= (\alpha - 1)(\beta + 1) = \alpha\beta + \alpha - \beta - 1$<br><br>$= \frac{3}{2} + \sqrt{\left(\frac{p^2}{4} - 6\right)} - 1$<br><br>$= \frac{1}{2} + \sqrt{\frac{p^2}{4} - 6}$<br><br>$x^2 - x\left(-\frac{p}{2}\right) + \frac{1}{2} + \sqrt{\left(\frac{p^2}{4} - 6\right)} = 0$ |
| | Obtains the sum of roots = their $\alpha + \beta$. | 1.1b | B1F | |
| | Finds an expression for $\alpha - \beta$ in terms of $p$.<br><br>FT their $(\alpha - \beta)^2$. | 1.1b | B1F | |
| | Obtains a correct quadratic equation with roots $\alpha - 1,\ \beta + 1$.<br><br>FT their $(\alpha - \beta)^2$ and their $\alpha + \beta$.<br>Condone a quadratic expression.<br>ISW after a correct equation or expression. | 1.1b | A1F | |
| | **Total** | | **8** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| 10(a) | Selects a method to find the values of $a, b, c$.<br>e.g.<br>by expanding $(x+a)(y+b)=c$<br>or by multiplying the equation by $(2x+4)$and expanding<br>or by dividing the numerator of the equation by its denominator. | 3.1a | M1 | $y(2x+4)=3x-5$<br>$2xy+4y=3x-5$<br>$xy+2y-\frac{3}{2}x+\frac{5}{2}=0$<br>$xy+2y-\frac{3}{2}x-3=-\frac{11}{2}$<br>$(x+2)\left(y-\frac{3}{2}\right)=-\frac{11}{2}$ |
| | Expresses the original equation in a form that allows comparison with $(x+a)(y+b)=c$. | 1.1a | M1 | |
| | Completes a rigorous argument to show that $y=\frac{3x-5}{2x+4}$ can be written as $(x+2)\left(y-\frac{3}{2}\right)=-\frac{11}{2}$. | 2.1 | R1 | |
| 10(b) | Obtains a correct asymptote. | 1.1b | B1 | $x=-2$ |
| | Obtains the other correct asymptote and no incorrect asymptotes. | 1.1b | B1 | $y=\frac{3}{2}$ |

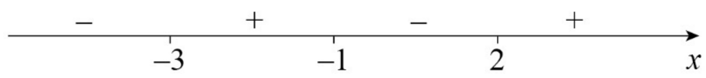
| | | | | |
|---|---|---|---|---|
| **10(c)** | Sketches a curve asymptotic to $x = -2$ or $y = \frac{3}{2}$.<br><br>FT their asymptotes. | 1.1b | B1F |  |
| | Sketches a curve with two branches, asymptotic to their asymptotes. | 1.1a | M1 | |
| | Deduces the shape of the curve and sketches it correctly with a root at $\frac{5}{3}$ and $y$-intercept at $-\frac{5}{4}$.<br><br>FT their asymptotes. | 2.2a | A1F | |
| | **Total** | | **8** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| **11** | Selects an approach to sketch the polar graph of $r = \sinh\theta + \cosh\theta$<br>e.g. by evaluating $r$ for at least 3 values of $\theta$ PI<br>or<br>by finding $\sinh\theta + \cosh\theta = \mathrm{e}^{\theta}$. | 3.1a | M1 | $\sinh\theta + \cosh\theta = \mathrm{e}^{\theta}$ |
| | Draws a spiral. | 1.1a | M1 |  |
| | Completes a fully correct sketch with the spiral beginning and ending on the initial line (not at the pole.) | 1.1b | A1 | |
| | **Total** | | **3** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| 12 | Selects a method to determine the mean value of $h$ by describing the effect of either transformation on the graph of $y = f(x)$.<br>PI by -$m$ or $km \pm 7$. | 3.1a | M1 | mean $= -m$ |
| | Obtains the correct answer $7 - m$. | 1.1b | A1 | mean $= -m + 7$ |
| | **Total** | | **2** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| 13(a) | Obtains the correct direction vector for line $l_1$. | 1.1b | B1 | direction of $l_1$ is $\begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$<br><br>$\begin{bmatrix} 12 \\ a+3 \\ 2b \end{bmatrix} = p\begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$<br><br>$12 = 3p$<br>$p = 4$<br><br>$a + 3 = -2p$ and $2b = -p$<br><br>$a + 3 = -8$ and $2b = -4$<br><br>$a = -11$ and $b = -2$ |
| | Selects a method to find $a$ and $b$ by equating (multiples of) their $l_1$ direction vector and $\begin{bmatrix} 12 \\ a+3 \\ 2b \end{bmatrix}$. | 3.1a | M1 | |
| | Equates all components of their vectors and finds the multiplier '$p$'. | 1.1a | M1 | |
| | Shows correctly that $a = -11$ and obtains $b = -2$. | 2.1 | R1 | |

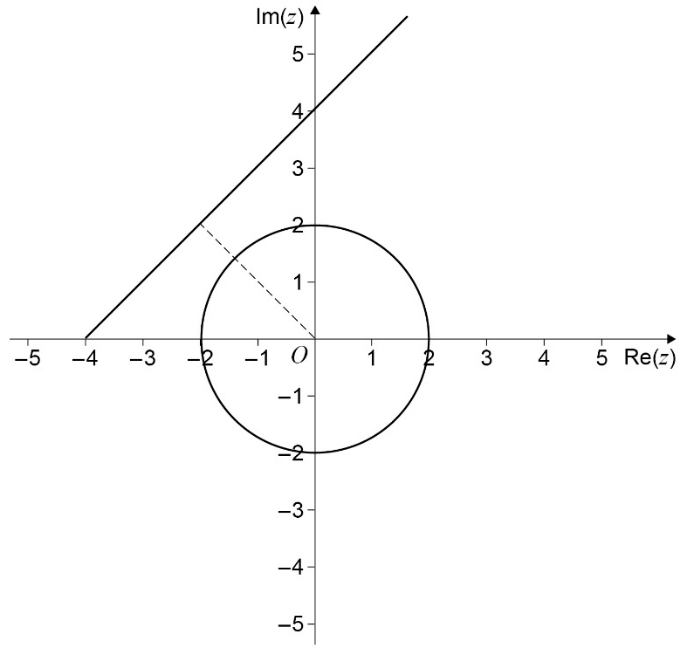
| | | | | |
|---|---|---|---|---|
| **13(b)** | Selects a method to find the intersection of $l_1$and $l_2$ by equating vector equations of the two lines or by substituting components of $l_2$ in the Cartesian equation of $l_1$. | 3.1a | M1 | $\frac{x-2}{3}=\frac{y-\frac{1}{2}}{-2}=\frac{z-0}{-1}$<br>$r=\begin{bmatrix}2\\\frac{1}{2}\\0\end{bmatrix}+\lambda\begin{bmatrix}3\\-2\\-1\end{bmatrix}$<br>$\begin{bmatrix}-7+12\mu\\4+\mu(a+3)\\-2+6\mu\end{bmatrix}=\begin{bmatrix}2+3\lambda\\0.5-2\lambda\\-\lambda\end{bmatrix}$<br>$-7+12\mu=2+3\lambda$<br>$4+\mu a+3\mu=0.5-2\lambda$<br>$-2+6\mu=-\lambda$<br>$-7+12\mu=2+3(2-6\mu)$<br>$\mu=\frac{1}{2}$<br>$\lambda=2-6\times\frac{1}{2}=-1$<br>$4+\frac{1}{2}a+3\times\frac{1}{2}=\frac{1}{2}-2\times-1$<br>$a=-6$ |
| | Forms an equation in $\mu$, or writes at least two simultaneous equations in $\mu$ and another parameter. Allow one arithmetic error. | 1.1a | M1 | |
| | Obtains the correct value of $\mu$. | 1.1b | A1 | |
| | Forms an equation in $a$ by substituting their value of $\mu$ into an appropriate equation, from the intersection of the two lines. | 1.1a | M1 | |
| | Obtains the correct value of $a$. | 1.1b | A1 | |
| | **Total** | | **9** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| **14(a)** | Writes both terms on one side of the inequality, with a common denominator. | 1.1a | M1 | $0 \le \frac{(x+1)^2}{x+1} - \frac{x+7}{x+1}$<br>$0 \le \frac{x^2+2x+1-x-7}{x+1}$<br>$0 \le \frac{x^2+x-6}{x+1}$<br>$\frac{(x+3)(x-2)}{x+1} \ge 0$ |
| | Correctly combines their two fractions into one fraction. | 1.1b | A1F | |
| | Obtains a single fraction in which the numerator is a three-term quadratic. | 1.1a | M1 | |
| | Completes a rigorous argument to show the correct inequality in the required form.<br><br>Accept $0 \le \frac{(x+-2)(x+3)}{x+1}$. | 2.1 | R1 | |
| **14(b)** | Explains that $x = -r$ is a solution of the inequality on the RHS, but not the one of the LHS. | 2.4 | B1 | $x = -r$ is a solution of the inequality on the right, but not the one on the left. |
| **14(c)** | Obtains one correct region, FT their three critical values.<br>Condone $-3 \le x \le -1$. | 1.1a | M1 |  |
| | Obtains both correct regions.<br><br>$-3 \le x < -1\,, \quad x \ge 2$ | 1.1b | A1 | $-3 \le x < -1\,, \quad x \ge 2$ |
| | **Total** | | **7** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| 15(a) | Obtains the correct expression $k = \frac{r}{h}$. | 1.1b | B1 | $\frac{r}{h}$ |
| 15(b) | Uses the formula for volume of revolution $V = \pi \int mx^2 dx$.<br><br>Condone missing $\pi, dx$ and missing or incorrect limits. | 1.1a | M1 | $\text{Volume} = \pi \int_0^h \left(\frac{rx}{h}\right)^2 dx$<br>$= \pi \int_0^h \frac{r^2x^2}{h^2} dx$<br>$= \pi \left[\frac{r^2x^3}{3h^2}\right]_0^h$<br>$= \pi \left(\frac{r^2h^3}{3h^2} - 0\right) = \frac{1}{3}\pi r^2 h$ |
| | Correctly integrates their $(kx)^2$, with an expression for $k$ in terms of $r$ and $h$. | 1.1a | M1 | |
| | Completes a rigorous proof to show that $V = \frac{1}{3}\pi r^2 h$. | 2.1 | R1 | |
| | **Total** | | **4** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| 16(a) | Obtains $\mathbf{I}$.<br>Accept any identity matrix, eg $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$. | 1.2 | B1 | $\mathbf{AA}^{-1} = \mathbf{I}$ |
| 16(b) | Correctly uses pre- or post - multiplication by either $\mathbf{A}^{-1}$, $\mathbf{B}^{-1}$ or $\mathbf{M}^{-1}$ in a way which gives a product equal to $\mathbf{I}$, starting from<br><br>$\mathbf{M} = \mathbf{AB}$<br>or<br>$\mathbf{ABM}^{-1} = \mathbf{I}$<br>or<br>$\mathbf{M}^{-1}\mathbf{AB} = \mathbf{I}$ | 1.1a | M1 | $\mathbf{M} = \mathbf{AB}$<br>$\mathbf{B}^{-1}\mathbf{A}^{-1}\mathbf{M} = \mathbf{B}^{-1}\mathbf{A}^{-1}\mathbf{AB}$<br>$\mathbf{B}^{-1}\mathbf{A}^{-1}\mathbf{M} = \mathbf{B}^{-1}\mathbf{IB}$<br>$\mathbf{B}^{-1}\mathbf{A}^{-1}\mathbf{MM}^{-1} = \mathbf{B}^{-1}\mathbf{BM}^{-1}$<br>$\mathbf{B}^{-1}\mathbf{A}^{-1}\mathbf{I} = \mathbf{IM}^{-1}$<br>$\mathbf{B}^{-1}\mathbf{A}^{-1} = \mathbf{M}^{-1}$ |
| | Correctly simplifies $\mathbf{AA}^{-1} = \mathbf{I}$ OE<br>or $\mathbf{BB}^{-1} = \mathbf{I}$ OE<br>in a correct equation. | 1.1a | M1 | |
| | Completes a rigorous argument to prove that $\mathbf{M}^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$. | 2.1 | R1 | |
| | **Total** | | **4** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| **17** | Selects a method to transform the given equation of $C$ into a standard polar form or Cartesian form, e.g. uses $r^2 = x^2 + y^2$ and $x = r\cos\theta$ and $y = r\sin\theta$;<br>or<br>writes $r$ in the form $R(\cos A\cos B + \sin A\sin B)$. | 3.1a | M1 | $r^2 = a(r\cos\theta + r\sin\theta)$ |
| | Obtains a correct equation in terms of $x$ and $y$ only<br>or<br>obtains<br>$r = a\sqrt{2}\cos\left(\theta - \frac{\pi}{4}\right)$. | 1.1b | A1 | $x^2 + y^2 = a(x + y)$<br>$x^2 - ax + y^2 - ay = 0$<br>$x^2 - ax + \dfrac{a^2}{4} + y^2 - ay + \dfrac{a^2}{4} = \dfrac{a^2}{2}$ |
| | Correctly completes the square of their quadratic expression<br>or<br>states that the circle must pass through $O$, and that the maximum value of $\cos\left(\theta - \frac{\pi}{4}\right)$ is 1. | 1.1a | M1 | $\left(x - \frac{a}{2}\right)^2 + \left(y - \frac{a}{2}\right)^2 = \left(\frac{a}{\sqrt{2}}\right)^2$<br>radius = $\frac{a}{\sqrt{2}}$ |
| | Obtains the correct radius = $\frac{a}{\sqrt{2}}$. | 3.2a | A1 | |
| | **Total** | | **4** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| **18(a)** | Draws a circle with centre $(0, 0)$ and radius 2.<br>Accept a reasonably accurate freehand circle. | 1.1b | B1 |  |
| **18(b)** | Draws a straight line from $(-4, 0)$ at $\frac{\pi}{4}$ to the real axis.<br>Accept a reasonably accurate unruled line. | 1.1b | B1 | |
| **18(c)** | Selects a method to find the required expression by relating it to the shortest distance between the circle and the line.<br>e.g. a perpendicular drawn from the line to the origin (or to the circle).<br>or an indication of the use of the point $(-2, 2)$. | 3.1a | M1 | |
| | Calculates the distance from $(-2, 2)$ to the origin,<br>or the distance from $(-2, 2)$ to $(-\sqrt{2}, \sqrt{2})$. | 1.1a | M1 | $\sqrt{(-2-0)^2+(2-0)^2}$ |
| | Obtains the correct value = $12 - 8\sqrt{2}$<br>ACF, need not be simplified, exact value not required. | 3.2a | A1 | shortest distance $= \sqrt{8} - 2$<br>least possible value $= \left(\sqrt{8} - 2\right)^2$<br>$= 12 - 8\sqrt{2}$ |
| | **Total** | | **5** | |

| | | | | |
|---|---|---|---|---|
| | **Paper total** | | **80** | |