

Please write clearly in block capitals.

Centre number

--	--	--	--	--

Candidate number

--	--	--	--

Surname

Forename(s)

Candidate signature

I declare this is my own work.

AS

FURTHER MATHEMATICS

Paper 1

Monday 11 May 2020

Afternoon

Time allowed: 1 hour 30 minutes

Materials

- You must have the AQA formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	
18	
TOTAL	



Answer **all** questions in the spaces provided.

1 Express the complex number $1 - i\sqrt{3}$ in modulus-argument form.

Tick (✓) **one** box.

[1 mark]

$$2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) \quad \square$$

$$2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) \quad \square$$

$$2\left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right) \quad \square$$

$$2\left(\cos\left(-\frac{2\pi}{3}\right) + i\sin\left(-\frac{2\pi}{3}\right)\right) \quad \square$$

2 Given that $1 - i$ is a root of the equation $z^3 - 3z^2 + 4z - 2 = 0$, find the other two roots.

Tick (✓) **one** box.

[1 mark]

$$-1 + i \text{ and } -1 \quad \square$$

$$1 + i \text{ and } 1 \quad \square$$

$$-1 + i \text{ and } 1 \quad \square$$

$$1 + i \text{ and } -1 \quad \square$$



3 Given $(x - 1)(x - 2)(x - a) < 0$ and $a > 2$

Find the set of possible values of x .

Tick (✓) **one** box.

[1 mark]

$$\{x : x < 1\} \cup \{x : 2 < x < a\}$$

$$\{x : 1 < x < 2\} \cup \{x : x > a\}$$

$$\{x : x < -a\} \cup \{x : -2 < x < -1\}$$

$$\{x : -a < x < -2\} \cup \{x : x > -1\}$$

Turn over for the next question

Turn over ►



4 The matrices **A** and **B** are such that

$$\mathbf{A} = \begin{bmatrix} 2 & a & 3 \\ 0 & -2 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 1 & -3 \\ -2 & 4a \\ 0 & 5 \end{bmatrix}$$

4 (a) Find the product **AB** in terms of a .

[2 marks]

4 (b) Find the determinant of **AB** in terms of a .

[1 mark]



4 (c) Show that **AB** is singular when $a = -1$

[2 marks]

Turn over for the next question

Turn over ►



5 (a) Show that

$$r^2(r+1)^2 - (r-1)^2r^2 = pr^3$$

where p is an integer to be found.

[1 mark]



5 (b) Hence use the method of differences to show that

$$\sum_{r=1}^n r^3 = \frac{1}{4}n^2(n+1)^2$$

[3 marks]

Turn over ▶



6

Anna has been asked to describe the transformation given by the matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$$

She writes her answer as follows:

The transformation is a rotation about the x -axis through an angle of θ , where

$$\sin \theta = \frac{1}{2} \quad \text{and} \quad -\sin \theta = -\frac{1}{2}$$

$$\theta = 30^\circ$$

Identify and correct the error in Anna's work.

[2 marks]



8 (b) Prove that the graphs of

$$y = \sinh x \quad \text{and} \quad y = \cosh x$$

do **not** intersect.

[3 marks]

Turn over ►



9 The quadratic equation $2x^2 + px + 3 = 0$ has two roots, α and β , where $\alpha > \beta$.

9 (a) (i) Write down the value of $\alpha\beta$.

[1 mark]

9 (a) (ii) Express $\alpha + \beta$ in terms of p .

[1 mark]

9 (b) Hence find $(\alpha - \beta)^2$ in terms of p .

[2 marks]



9 (c) Hence find, in terms of p , a quadratic equation with roots $\alpha - 1$ and $\beta + 1$

[4 marks]

Turn over ►



10 (a) Show that the equation

$$y = \frac{3x - 5}{2x + 4}$$

can be written in the form

$$(x + a)(y + b) = c$$

where a , b and c are integers to be found.

[3 marks]

10 (b) Write down the equations of the asymptotes of the graph of

$$y = \frac{3x - 5}{2x + 4}$$

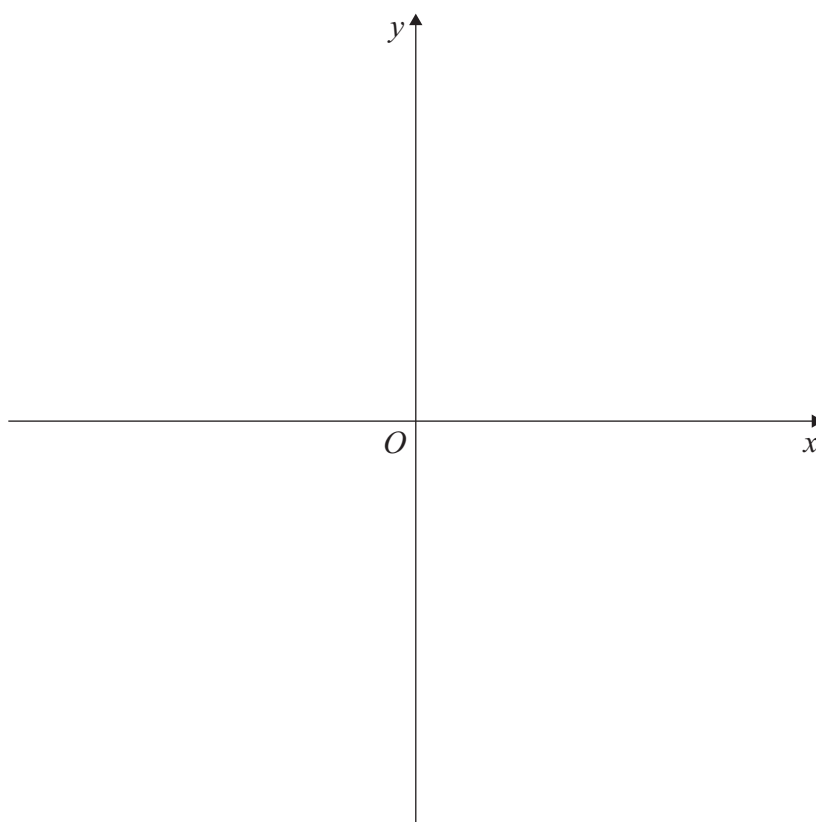
[2 marks]



10 (c) Sketch, on the axes provided, the graph of

$$y = \frac{3x - 5}{2x + 4}$$

[3 marks]



Turn over ►



11 Sketch the polar graph of

$$r = \sinh \theta + \cosh \theta$$

for $0 \leq \theta \leq 2\pi$

[3 marks]

O —————→ Initial line



12

The mean value of the function f over the interval $1 \leq x \leq 5$ is m .

The graph of $y = g(x)$ is a reflection in the x -axis of $y = f(x)$.

The graph of $y = h(x)$ is a translation of $y = g(x)$ by $\begin{bmatrix} 3 \\ 7 \end{bmatrix}$

Determine, in terms of m , the mean value of the function h over the interval $4 \leq x \leq 8$

[2 marks]

Turn over for the next question

Turn over ►

13 Line l_1 has equation

$$\frac{x - 2}{3} = \frac{1 - 2y}{4} = -z$$

and line l_2 has equation

$$\mathbf{r} = \begin{bmatrix} -7 \\ 4 \\ -2 \end{bmatrix} + \mu \begin{bmatrix} 12 \\ a + 3 \\ 2b \end{bmatrix}$$

13 (a) In the case when l_1 and l_2 are parallel, show that $a = -11$ and find the value of b .

[4 marks]



14 (a) Given

$$\frac{x+7}{x+1} \leq x+1$$

show that

$$\frac{(x+a)(x+b)}{x+c} \geq 0$$

where a , b , and c are integers to be found.**[4 marks]**

14 (b) Briefly explain why this statement is incorrect.

$$\frac{(x+p)(x+q)}{x+r} \geq 0 \Leftrightarrow (x+p)(x+q)(x+r) \geq 0$$

[1 mark]



*Do not write
outside the
box*

14 (c) Solve

$$\frac{x + 7}{x + 1} \leq x + 1$$

[2 marks]

Turn over for the next question

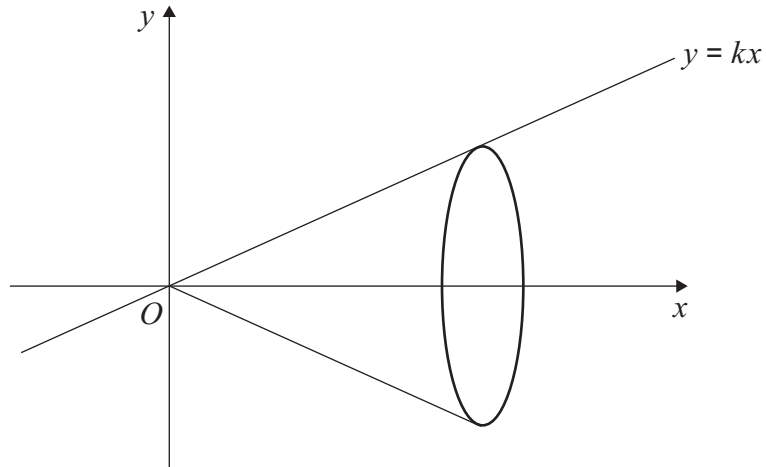
Turn over ►



15 A segment of the line $y = kx$ is rotated about the x -axis to generate a cone with vertex O .

The distance of O from the centre of the base of the cone is h .

The radius of the base of the cone is r .



15 (a) Find k in terms of r and h .

[1 mark]



Do not write
outside the
box

16 **A** and **B** are non-singular square matrices.

16 (a) Write down the product \mathbf{AA}^{-1} as a single matrix.

[1 mark]

16 (b) **M** is a matrix such that $\mathbf{M} = \mathbf{AB}$.

Prove that $\mathbf{M}^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$

[3 marks]



17 The polar equation of the circle C is

$$r = a(\cos \theta + \sin \theta)$$

Find, in terms of a , the radius of C .

Fully justify your answer.

[4 marks]

Turn over ▶

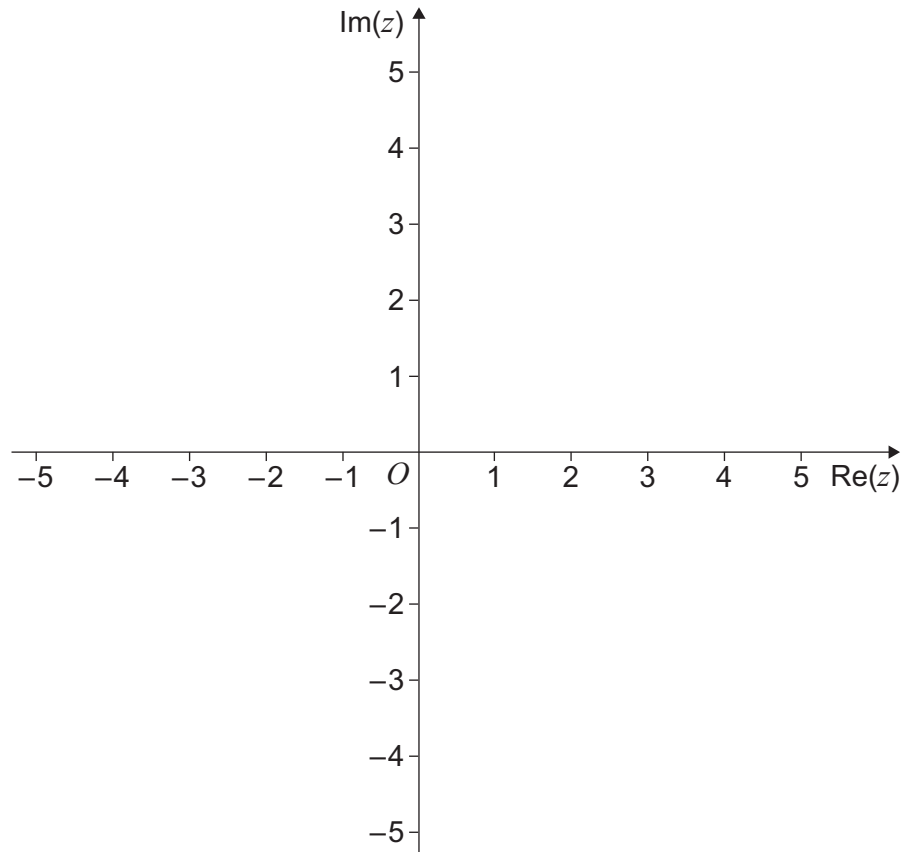


18 The locus of points L_1 satisfies the equation $|z| = 2$

The locus of points L_2 satisfies the equation $\arg(z + 4) = \frac{\pi}{4}$

18 (a) Sketch L_1 on the Argand diagram below.

[1 mark]



18 (b) Sketch L_2 on the Argand diagram above.

[1 mark]



18 (c) The complex number $a + ib$, where a and b are real, lies on L_1

The complex number $c + id$, where c and d are real, lies on L_2

Calculate the least possible value of the expression

$$(c - a)^2 + (d - b)^2$$

[3 marks]

END OF QUESTIONS



There are no questions printed on this page

*Do not write
outside the
box*

**DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED**



Do not write
outside the
box

Question number	<p style="text-align: center;">Additional page, if required. Write the question numbers in the left-hand margin.</p>
	<p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p>
	<p>Copyright information</p> <p>For confidentiality purposes, all acknowledgements of third-party copyright material are published in a separate booklet. This booklet is published after each live examination series and is available for free download from www.aqa.org.uk.</p> <p>Permission to reproduce all copyright material has been applied for. In some cases, efforts to contact copyright-holders may have been unsuccessful and AQA will be happy to rectify any omissions of acknowledgements. If you have any queries please contact the Copyright Team.</p> <p>Copyright © 2020 AQA and its licensors. All rights reserved.</p>

