Mark Scheme (Result)

November 2021

Pearson Edexcel GCE Further Mathematics
Advanced Level in
Further Mathematics Paper 4B
9FM0/4B

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- M marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- B marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

Question	Scheme	Marks	AOs
1(a)	$s = 4$	B1	1.1b
	$t = 4.5$	B1	1.1b
		(2)	
(b)	Because there are tied ranks.	B1	2.4
		(1)	
(c)	$H_0: \rho_s = 0 \quad H_1: \rho_s > 0$	B1	2.5
	CV = 0.7143	B1	1.1b
	$r_s = 0.7106$ does not lie in the critical region.	M1	2.1
	There is insufficient evidence to suggest that the higher the rank in the History test, the higher the rank in the Geography test (oe).	A1	2.2b
		(4)	
			(7 marks)

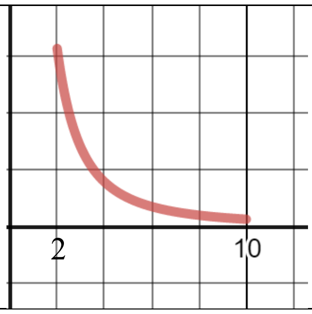
Notes

(a)	**B1:** cao **B1:** cao
(b)	**B1:** Correct explanation
(c)	**B1:** Both hypotheses correct with correct notation (must use ρ_s or ρ) **B1:** Correct critical value 0.7143 or better **M1:** Drawing a correct inference using their CV and 0.7106 **A1:** Drawing a correct inference (condone "marks" instead of ranks) in context using their CV and 0.7106

Question	Scheme	Marks	AOs
2(a)	$\text{H}_0: \mu_{b(lue)} = \mu_{w(hite)} + 5 \quad \text{H}_1: \mu_{b(lue)} > \mu_{w(hite)} + 5$	B1	2.1
	$s.e. = \sqrt{\frac{2.6^2}{90} + \frac{2.4^2}{80}}$	M1	1.1b
	$z = \frac{39.5 - 33.7 - 5}{\sqrt{\frac{2.6^2}{90} + \frac{2.4^2}{80}}} = 2.085773...$ awrt **<u>2.09</u>**	M1 A1	3.1b 1.1b
	CV = 2.3263 [or *p*-value = 0.01849…]	B1	1.1b
	Not significant, insufficient evidence to support Nissim's claim.	A1	2.2b
		(6)	
(b)	Use the *t* - test **or** (Since sample sizes are large,) use s^2 as an approximation to σ^2	B1	2.4
		(1)	
(c)	Since sample size is large, by the Central Limit Theorem, sample means will be (approximately) normally distributed …so no effect as the calculations in part (a) can still be carried out.	B1 dB1	2.4 3.2b
		(2)	
			(9 marks)

Notes

(a)	**B1:** Both hypotheses (oe) correct with correct notation (if using μ_x and μ_y these must be defined). **M1:** Calculation of standard error **M1:** Standardising using normal distribution test statistic for difference of two means with known variance **A1:** awrt 2.09 **B1:** Correct critical value 2.3263 or better **A1:** Drawing a correct inference in context
(b)	**B1:** Correct explanation
(c)	**B1:** Understanding that the assumptions required for the hypothesis test, by CLT sample means follow a normal distribution **dB1:** (dep on previous B1) Correct evaluation

Question	Scheme	Marks	AOs
3(a)	F(5) + (1 – F(8)) $\left[\frac{3}{4}+\left(1-\frac{15}{16}\right)\right]$	M1	2.1
	$=\frac{13}{16}$	A1	1.1b
		(2)	
(b)	$\mathrm{F}(m)=0.5$ $\left[1.25-\frac{2.5}{m}=0.5\right]$	M1	1.1b
	$m=\frac{10}{3}$	A1	1.1b
		(2)	
(c)	$\mathrm{f}(x)[=\frac{\mathrm{d}}{\mathrm{d}x}(\mathrm{F}(x))]=2.5x^{-2}$	M1	2.1
	$\mathrm{E}(X^2)[=\int_2^{10} x^2\mathrm{f}(x)\,\mathrm{d}x]=\int_2^{10} 2.5\,\mathrm{d}x$	M1	1.1b
	$=20$	A1	1.1b
		(3)	
(d)(i)		M1	1.1b
	2 and 10 correctly labelled on horizontal axis	A1	2.1
(ii)	Therefore positive skew.	A1	2.2b
		(3)	
			(10 marks)

Notes

(a)	**M1:** Equivalent correct probability statement, e.g. [1 – (F(8) – F(5))] **A1:** $\frac{13}{16}$ oe
(b)	**M1:** Use of $\mathrm{F}(m)=0.5$ **A1:** $\frac{10}{3}$ oe
(c)	**M1:** Realising that f(x) is required and attempting to differentiate F(x) **M1:** Use of $\int_2^{10} x^2\mathrm{f}(x)\,\mathrm{d}x$ **A1:** 20 cao
(d)(i) and (ii)	**M1:** Correct shape **A1:** Correct labels **A1:** Positive skew provided M1 scored

Question	Scheme	Marks	AOs
4(a)	As **elevation** increases, **temperature** decreases.	B1	3.4
		(1)	
(b)	$S_{xt} = -0.959\sqrt{8\,820\,655 \times 444.7}[= -60\,062.38727]$	M1	2.1
	$b = \dfrac{'-60\,062...'}{8\,820\,655}[= -0.006809...]$	M1	1.1b
	$a = \dfrac{94.62}{20} - 'b'\dfrac{28\,130}{20}[= 14.308...]$	M1	1.1b
	$t = 14.3 - 0.00681x$ *	A1cso*	2.2a
		(4)	
(c)	$[w = \dfrac{x}{1000} \rightarrow]\quad t = 14.3 - 6.81w$	B1	3.3
		(1)	
(d)	$444.7(1-(-0.959)^2)$ or $444.7 - \dfrac{(-60\,062...)^2}{8\,820\,655}$ [= 35.7*]	B1cso*	1.1b
		(1)	
(e)(i)	$(\text{residual})^2 = [1.4 - (14.3 - 0.00681(1100))]^2$ [= 29.2…]	M1	3.4
	[29.2…÷ 35.7 × 100%] awrt 82%	A1	1.1b
		(2)	
(e)(ii)	(As the point representing this data contributes to the majority of the RSS), the point is possibly an outlier and should be investigated.	B1	3.5a
		(1)	
			(10 marks)

Notes

(a)	**B1:** Correct contextual interpretation
(b)	**M1:** Using pmcc to find S_{xt} **M1:** Setting up linear model by attempting to find *b* Note: Allow M2 for $b = r\sqrt{\dfrac{S_{tt}}{S_{xx}}}$ **M1:** Setting up linear model by attempting to find *a* **A1cso*:** Correct model $t = 14.3 - 0.00681x$ with *a* = awrt 14.3 and *b* = awrt –0.00681 dependent upon all previous M marks.
(c)	**B1:** Correct model
(d)	**B1cso*:** Either correct expression
(e)(i) and (ii)	**M1:** Using the model to evaluate the squared residual **A1:** awrt 82% **B1:** Evaluating the result obtained from the model to suggest that this point may be an outlier

Question	Scheme	Marks	AOs
5(a)	$[\mathrm{E}(X)=2\beta]$ $\mathrm{E}(A)=\mathrm{E}\left(\frac{X_1+X_2}{2}\right)=\frac{1}{2}[\mathrm{E}(X)+\mathrm{E}(X)]$ $\mathrm{E}(B)=\mathrm{E}\left(\frac{X_1+2X_2+3X_3}{8}\right)=\frac{1}{8}[\mathrm{E}(X)+2\mathrm{E}(X)+3\mathrm{E}(X)]$ $\mathrm{E}(C)=\mathrm{E}\left(\frac{X_1+2X_2-X_3}{8}\right)=\frac{1}{8}[\mathrm{E}(X)+2\mathrm{E}(X)-\mathrm{E}(X)]$	M1	3.1a
	Bias for $A=\mathrm{E}(A)-\beta=2\beta-\beta=\beta$ Bias for $B=\mathrm{E}(B)-\beta=1.5\beta-\beta=0.5\beta$ Bias for $C=\mathrm{E}(C)-\beta=0.5\beta-\beta=-0.5\beta$	M1 A1 A1 A1	2.1 1.1b 1.1b 1.1b
		(5)	
(b)	$[\mathrm{Var}(X)=\frac{4}{3}\beta^2]$ Better estimator would have the smallest bias and the least variance. *B* and *C* have equal bias, so we select the estimator with the smallest variance $\mathrm{Var}(B)=\mathrm{Var}\left(\frac{X_1+2X_2+3X_3}{8}\right)$ $=\frac{1}{64}[\mathrm{Var}(X)+4\mathrm{Var}(X)+9\mathrm{Var}(X)]$ $\mathrm{Var}(C)=\mathrm{Var}\left(\frac{X_1+2X_2-X_3}{8}\right)$ $=\frac{1}{64}[\mathrm{Var}(X)+4\mathrm{Var}(X)+\mathrm{Var}(X)]$	M1	2.1
	$\mathrm{Var}(B)=\frac{7}{32}\mathrm{Var}(X)[=\frac{7}{24}\beta^2]$ $\mathrm{Var}(C)=\frac{3}{32}\mathrm{Var}(X)[=\frac{1}{8}\beta^2]$	A1 A1	1.1b 1.1b
	(Since both have same bias and) $\mathrm{Var}(C)<\mathrm{Var}(B)$ therefore C is the better estimator.	B1ft	2.2a
		(4)	
(c)	Any unbiased estimator, e.g. $\frac{X_1+X_2+X_3}{6}$	B1	3.5c
		(1)	
			(10 marks)

Notes	
(a)	**M1:** Using independence to calculate the $\mathrm{E}(A)$, $\mathrm{E}(B)$ or $\mathrm{E}(C)$ **M1:** Use of bias = $\mathrm{E}(X)-\beta$ **A1:** Correct bias for *A* **A1:** Correct bias for *B* **A1:** Correct bias for *C* [allow $+0.5\beta$]
(b)	**M1:** Realising that variances need to be compared and attempt at linear combination of variances for *B* or *C* **A1:** Correct Var(*B*) **A1:** Correct Var(*C*) **A1ft:** Correct comparison and deduction that *C* a better estimator than *A* and *B*.
(c)	**B1:** Allow any unbiased estimator, e.g. $\frac{X_1}{2}$

Question	Scheme	Marks	AOs
6(a)	$\mathrm{H}_0 : \sigma_r^{\ 2} = \sigma_{mb}^{\ 2} \quad \mathrm{H}_1 : \sigma_r^{\ 2} \neq \sigma_{mb}^{\ 2}$	B1	2.5
	$s_r^2 = \frac{1}{7}\left(21032 - 8\times\left(\frac{410}{8}\right)^2\right) = 2.7857...$	M1 A1	2.1 1.1b
	$s_{mb}^2 = \frac{1}{5}\left(14426 - 6\times\left(\frac{294}{6}\right)^2\right) = 4$	A1	1.1b
	$\frac{s_{mb}^2}{s_r^2} = 1.4358...$	M1	3.4
	CV $\mathrm{F}_{5,7} = 7.46$	B1	1.1b
	(1.4358… < 7.46 so there is) insufficient evidence to suggest the variances of the wingspans are different.	A1	2.2b
		(7)	
(b)	$\chi^2_{5,\alpha} = \frac{5\times '4'}{1.194}$ or $\chi^2_{5,\alpha} = \frac{5\times '4'}{48.54}$	M1	3.1b
	$\chi^2_{5,\alpha} = 16.75 \to \alpha = 0.005$ or $\chi^2_{5,\alpha} = 0.412 \to \alpha = 0.995$	M1	1.1b
	$k = 99$	A1	1.1b
		(3)	
(c)	$s_p^{\ 2} = \frac{7\times '2.7857...' + 5\times '4'}{8+6-2} [= 3.29...]$	M1	3.1b
	$t_{12} = 2.179$	B1	1.1b
	$(\frac{410}{8} - \frac{294}{6}) \pm '2.179' \times \sqrt{'3.29'}\sqrt{\frac{1}{8}+\frac{1}{6}}$	M1	3.4
	(awrt 0.115, awrt 4.39)	A1 A1	1.1b 1.1b
		(5)	
			(15 marks)

Notes

(a)	**B1:** For both hypotheses in terms of σ **M1:** Correct method for s_r^2 or s_{mb}^2 **A1:** awrt 2.79 (allow $\frac{39}{14}$) **A1:** 4 cao **M1:** Using correct model for test statistic with correct ratio **B1:** Correct CV **A1ft:** Drawing a correct inference in context using their CV and their test statistic (dep on both M marks)
(b)	**M1:** Either correct attempt at $\chi^2_{5,\alpha}$ with $\nu = 5$ **M1:** Using tables to find appropriate probability **A1:** 99 cao
(c)	**M1:** Correct expression for $s_p^{\ 2}$ **B1:** Correct 95% *t*-value **M1:** CI in the correct form (may be implied by either A mark) **A1:** awrt 0.115 **A1:** awrt 4.39

Question	Scheme	Marks	AOs
7(a)	$(A+R) \sim \text{N}(300,\ 12^2+10^2)$	M1 A1	3.3 1.1b
	$(A_1+A_2) \sim \text{N}(320,\ 2\times12^2)$	A1	1.1b
		(3)	
(b)	P(both are apples) $[=\frac{4}{5}\times\frac{3}{4}]=\frac{3}{5}$ P(one apple and one orange) $=\frac{2}{5}$	M1	2.1
	$'\frac{3}{5}'\text{P}(A_1+A_2>310)+'\frac{2}{5}'\text{P}(A+R>310)$	M1	2.1
	=0.5377…. awrt **<u>0.538</u>**	A1	1.1b
		(3)	
(c)	$[W=\sum_1^m A-n\times R]$ $W\sim \text{N}(160m-140n, m\times12^2+n^2\times10^2)$	M1 A1	3.3 1.1b
	$160m - 140n = (1100.08+1499.92)\div2$ [=1300]	M1	2.1
	$2\times1.96\times\sqrt{m\times12^2+n^2\times10^2}=(1499.92-1100.08)$ $[\sqrt{m\times12^2+n^2\times10^2}=102]$	B1 M1	1.1b 1.1b
	$m=\dfrac{1300+140n}{160}\rightarrow\sqrt{(\frac{1300+140n}{160})\times12^2+n^2\times10^2}=102$ $100n^2+126n-9234=0$	dM1	2.1
	$n=9$ ($n=-10.26$ reject)	A1	1.1b
	$m = 16$	A1	1.1b
		(8)	
		(14 marks)	

Notes

(a)	**M1:** Setting up either model for the weights of the two fruit **A1:** Correct distribution for 1 apple 1 orange **A1:** Correct distribution for 2 apples
(b)	**M1:** Finding probability for each possible outcome **M1:** Fully correct method for finding the required probability **A1:** awrt 0.538
(c)	**M1:** Setting up model for W **A1:** correct distribution **M1:** Using given interval to set up equation for mean **B1:** 1.96 **M1:** Using given interval to set up equation for variance **dM1:** Solving simultaneously leading to a 3TQ (dep on previous M mark) **A1:** n = 9 (only) **A1:** m = 16 (only)