Mark Scheme (Result)

November 2021

Pearson Edexcel GCE Further Mathematics
Advanced Level in Further Mathematics
Decision 2
Paper 9FM0/4D

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- M marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- B marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

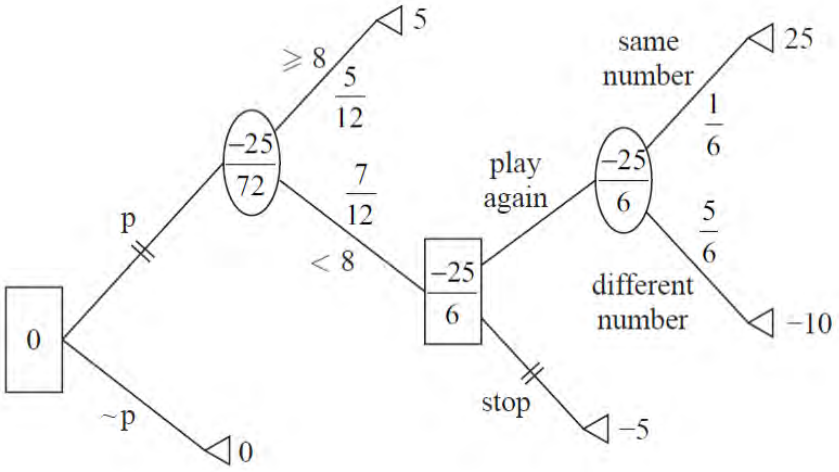
Question	Scheme	Marks	AOs
1	Let x_{ij} be 0 or 1 $\begin{cases}1 & \text{if worker } (i) \text{ does task } (j)\\ 0 & \text{otherwise}\end{cases}$	B1	3.3
	where $i \in \{\text{A},\text{B},\text{C},\text{D}\}$ and $j \in \{1,2,3,4\}$	B1	2.5
	minimise $C = 53x_{\text{A1}} + '100'x_{\text{A2}} + 62x_{\text{A3}} + 48x_{\text{B1}} + 57x_{\text{B2}} + 59x_{\text{B3}}$ $+55x_{\text{C1}} + 63x_{\text{C2}} + 58x_{\text{C3}} + 69x_{\text{D1}} + 49x_{\text{D2}} + '100'x_{\text{D3}}$	M1 A1	3.3 1.1b
	Subject to $\sum x_{\text{A}j} = 1, \sum x_{\text{B}j} = 1, \sum x_{\text{C}j} = 1, \sum x_{\text{D}j} = 1$ $\sum x_{i1} = 1, \sum x_{i2} = 1, \sum x_{i3} = 1, \sum x_{i4} = 1$	M1 A1	3.3 1.1b
		(6)	
			(6 marks)

Notes:

B1: Defining x_{ij} correctly

B1: Correct definition of the values that *i* and *j* can take
M1: Attempt at 12 term expression, coefficients 'correct', 2 'large' values included, condone 2 slips.
A1: cao including **'**minimise'
M1: At least four correct equations, each in three or four variables, unit coefficients, equal to 1
A1: cao (all eight equations)

No dummy column can score a maximum of **B1B0M1A1M0A0**
No 'large' values in A2 and D3 can score a maximum **B1B1M0A0M1A1**
No 'large' values or dummy column can score a maximum of **B1B0M0A0M0A0**

Question	Scheme	Marks	AOs
2		M1 B1 A1 M1 M1 A1	3.3 1.1b 1.1b 3.4 3.4 1.1b
	EMV is £0 and Alka should not play the game	B1	3.2a
		(7)	
			(7 marks)

Notes:

M1: Tree diagram with at least five end pay-offs, two decision nodes and two chance nodes
B1: Correct probabilities for rolling an 8 or more **and** obtaining the same number on both dice
A1: Correct structure for the tree diagram with each arc labelled correctly (including probabilities)
M1: At least three end-pay offs consistent with their stated probabilities; all five attempted
M1: Chance nodes attempted with their probabilities
A1: cao for chance and decision nodes including double line through inferior option
B1: Correct EMV and analysis in context

Question	Scheme	Marks	AOs
3(a)		B1	1.1b
		(1)	
(b)		M1 A1	2.1 1.1b
		(2)	
(c)		M1 A1 M1 A1	1.1b 1.1b 1.1b 2.2a
		(4)	
(d)	(£)2258	B1	1.1b
		(1)	

	P	Q	R	S
A	25	3		
B		38	5	
C			29	
D			6	30

	P	Q	R	S
A		$3-\theta$		θ
B		$38+\theta$	$5-\theta$	
C				
D			$6+\theta$	$30-\theta$

	P	Q	R	S
A	25			3
B		41	2	
C			29	
D			9	27

		18	12	11	13
		P	Q	R	S
0	A	X	7	6	X
3	B	−5	X	X	3
11	C	−8	−6	X	−1
8	D	−10	0	X	X

	P	Q	R	S
A	$25-\theta$			$3+\theta$
B				
C				
D	θ			$27-\theta$

	P	Q	R	S
A				28
B		41	2	
C			29	
D	25		9	2

Entering cell is DP and exiting cell is AP

(e)		M1 A1	2.1 1.1b
	A negative II so solution is not optimal	A1	2.4
		(3)	
			(11 marks)

		8	12	11	13
		P	Q	R	S
0	A	10	7	6	X
3	B	5	X	X	3
11	C	2	−6	X	–1
8	D	X	0	X	X

Notes:

(a)
B1: cao

(b)
M1: A valid route shown, only one empty square used, θ's balance
A1: cao

(c)
M1: Finding all 8 shadow costs and the 9 improvement indices for the correct 9 entries
A1: Shadow costs and II cao
M1: A valid route shown, their most negative II chosen, only one empty square used, θ's balance
A1: cao – including the deduction (and stating) of all entering and exiting cells

(d)
B1: cao

(e)
M1: Finding all 8 shadow costs **and** all 9 negative improvement indices **or** sufficient number of shadow costs for at least 1 negative II found – this mark is dependent on the previous **M** mark in **(b)** which will therefore indicate a correct mathematical argument leading from the initial solution to the confirmation or not of the optimality of the current solution
A1: cao negative II from correct working
A1: cso including the correct reasoning that the solution is not optimal because there is a negative II

Question	Scheme	Marks	AOs
4(a)	$x_{n+2} = 2y_{n+1} + 3 \Rightarrow x_{n+2} = 2\left(-4x_n + 3x_{n+1}\right) + 3$ Leading to $x_{n+2} - 6x_{n+1} + 8x_n = 3$ *	 B1	 2.2a
		(1)	
(b)	aux equation $m^2 - 6m + 8 = 0 \Rightarrow m = 2, m = 4$	B1	2.1
	$x_n = A(2)^n + B(4)^n$	B1	1.1b
	particular solution try $x_n = \lambda$ $\therefore \lambda - 6\lambda + 8\lambda = 3 \Rightarrow \lambda[=1]$	M1	1.1b
	$x_n = A(2)^n + B(4)^n + 1$	A1	2.2a
	$x_1 = 1 \Rightarrow 2A + 4B = 0$	M1	1.1b
	$y_1 = a \Rightarrow x_2 = 2a + 3$	B1	3.1a
	$4A + 16B + 1 = 2a + 3$	M1	1.1b
	$A = -\frac{(a+1)}{2}, B = \frac{(a+1)}{4} \Rightarrow x_n = (a+1)(4)^{n-1} - (a+1)(2)^{n-1} + 1$ (oe)	A1	2.2a
		(8)	
(c)	As $x_7 = 28225 \Rightarrow (a+1)(4)^6 - (a+1)(2)^6 + 1 = 28225$ leading to $a = ...$	M1	3.4
	$a = 6$	A1	2.2a
		(2)	
			(11 marks)

Notes:

(a) B1: Correct reasoning to derive given result – sufficient working must be shown as recurrence equation given in question

(b) B1: cao for auxiliary equation and corresponding solutions (this mark can be implied by the correct complementary function)

B1: cao for the complementary function

M1: Substitute $x_n = \lambda$ into their second order recurrence relation and solve for λ

A1: Correct general solution

M1: Forms one equation in A and B using $x_1 = 1$

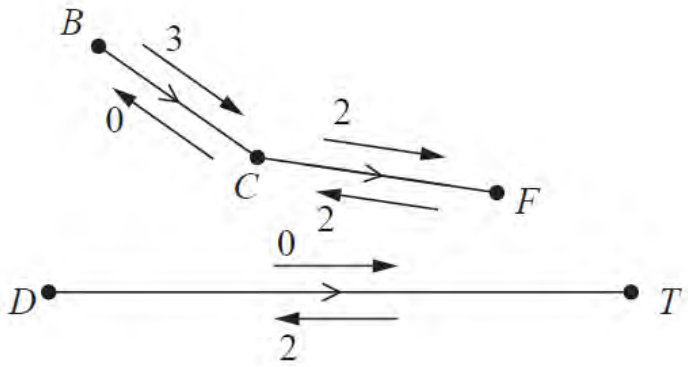
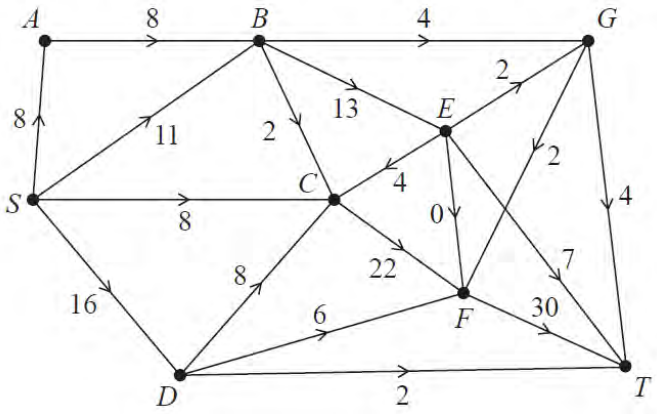
B1: Uses original recurrence relation for x_{n+1} to derive the expression $2a + 3$

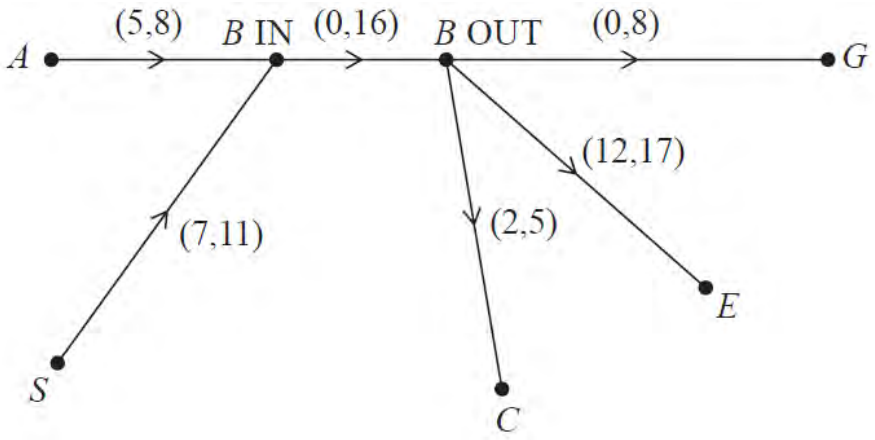
M1: Setting up a second equation in A and B

A1: cao (oe e.g., $x_n = 0.25(a+1)(4)^n - 0.5(a+1)(2)^n + 1)$

(c) M1: Using $x_7 = 28225$ to form a linear equation in a and attempt to solve for a

A1: cao for a

Question	Scheme	Marks	AOs
5 (a)(i) **(ii)**	$C_1 = 8+11+8+10+6+2 = 45$ $C_2 = 8+17-4-0-1+30+2 = 52$	B1 B1	1.1b 1.1b
		(2)	
(b)	Deduces the maximum possible flow is ≤ 45 litres per second	B1ft	2.2a
		(1)	
(c)	Initial flow = 36	B1	1.1b
		(1)	
(d)		M1 A1	1.1b 1.1b
		(2)	
(e)	e.g., SABGT – 2, SCFT – 2, SDCET – 2, SABET – 1 e.g., SABGT – 3, SCFT – 2, SDCET – 2	M1 A1 A1	1.1b 1.1b 1.1b
		(3)	
(f)	e.g. 	B1	2.2a
		(1)	

(g)	Use of max-flow min-cut theorem Identification of cut through AB, SB, BC, EC, CF, DF and DT Value of flow = 43 Therefore it follows that flow is maximal	M1 A1 A1	2.1 3.1a 2.2a
		(3)	
(h)	(i)  	B1 B1	3.3 3.3
	(ii) maximum flow = 40	B1ft	2.2a
		(3)	
			(16 marks)

Notes:

(a)(i) B1: cao

(ii) B1: cao

(b) B1ft: Deduced from their least value given in **(a)** - must include 'less than or equal to' (oe)

(c) B1: cao

(d) M1: Two numbers on each arc and at least two arcs or four numbers correct (so correct numbers with the correct arrows)

A1: cao do give bod since they might well cross these numbers out

(e) M1: One flow augmenting route found from S to T

A1: Two correct routes + flow values

A1: cso – increasing the flow by 7

(f) B1: cao

(g) M1: Construct argument based on max-flow min-cut theorem (e.g., attempt to find a cut through saturated arcs)

A1: Use appropriate process of finding a minimum cut: cut + value correct

A1: Correct deduction that the flow is maximal

(h) B1: Flows into B go to B_{IN} and flows out of B go from B_{OUT}

B1: Arc of capacity 16 from B_{IN} to B_{OUT}

B1ft: value of their maximum flow – 3

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>6(a)</td><td>Minimax</td><td>B1</td><td>1.2</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td>(b)</td><td>
<table>
<tr><th>Stage</th><th>State</th><th>Action</th><th>Destination</th><th>Value</th></tr>
<tr><td rowspan="3">1</td><td>H</td><td>HT</td><td>T</td><td>47*</td></tr>
<tr><td>I</td><td>IT</td><td>T</td><td>48*</td></tr>
<tr><td>J</td><td>JT</td><td>T</td><td>49*</td></tr>
<tr><td rowspan="6">2</td><td rowspan="2">D</td><td>DH</td><td>H</td><td>Max(49, 47) = 49*</td></tr>
<tr><td>DI</td><td>I</td><td>Max(50, 48) = 50</td></tr>
<tr><td rowspan="2">E</td><td>EH</td><td>H</td><td>Max (51, 47) = 51</td></tr>
<tr><td>EJ</td><td>J</td><td>Max (46, 49) = 49*</td></tr>
<tr><td rowspan="3">F</td><td>FH</td><td>H</td><td>Max (51, 47) = 51</td></tr>
<tr><td>FI</td><td>I</td><td>Max (52, 48) = 52</td></tr>
<tr><td>FJ</td><td>J</td><td>Max (50, 49) = 50*</td></tr>
<tr><td rowspan="2">G</td><td>GI</td><td>I</td><td>Max (53, 48) = 53</td></tr>
<tr><td>GJ</td><td>J</td><td>Max (51, 49) = 51*</td></tr>
<tr><td rowspan="8">3</td><td rowspan="3">A</td><td>AD</td><td>D</td><td>Max(53, 49) = 53</td></tr>
<tr><td>AE</td><td>E</td><td>Max(52, 49) = 52*</td></tr>
<tr><td>AF</td><td>F</td><td>Max (53, 50) = 53</td></tr>
<tr><td rowspan="3">B</td><td>BE</td><td>E</td><td>Max(51, 49) = 51</td></tr>
<tr><td>BF</td><td>F</td><td>Max(50, 50) = 50*</td></tr>
<tr><td>BG</td><td>G</td><td>Max(46, 51) = 51</td></tr>
<tr><td rowspan="2">C</td><td>CE</td><td>E</td><td>Max(50, 49) = 50*</td></tr>
<tr><td>CG</td><td>G</td><td>Max(47, 51) = 51</td></tr>
<tr><td rowspan="3">4</td><td rowspan="3">S</td><td>SA</td><td>A</td><td>Max(52, 52) = 52</td></tr>
<tr><td>SB</td><td>B</td><td>Max(48, 50) = 50*</td></tr>
<tr><td>SC</td><td>C</td><td>Max(50, 50) = 50*</td></tr>
</table>
</td><td>B1

M1
A1
A1

M1
A1ft
A1

M1
A1ft</td><td>3.1a

3.1a
1.1b
1.1b

1.1b
1.1b
1.1b

1.1b
1.1b</td></tr>
<tr><td></td><td></td><td>(9)</td><td></td></tr>
<tr><td>(c)</td><td>Route 1: S – B – F – J – T
Route 2: S – C – E – J – T</td><td>B1ft
B1</td><td>2.2a
2.2a</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td colspan="4">(12 marks)</td></tr>
</table>

Notes:

(a) B1: cao

Throughout (b):
• Condone lack of destination column and/or reversed stage numbers throughout
• Only penalise incorrect result in value – ie ignore working values
• Penalise absence of state or action column with first two A marks earned only
• Penalise empty/errors in stage column with first A mark earned only
• If maximin, minimum, maximum, etc. then only the B mark and M marks can be awarded in (b) (so 4 out of 9 maximum) and one B mark can be awarded in (c) following through a correct route from their table

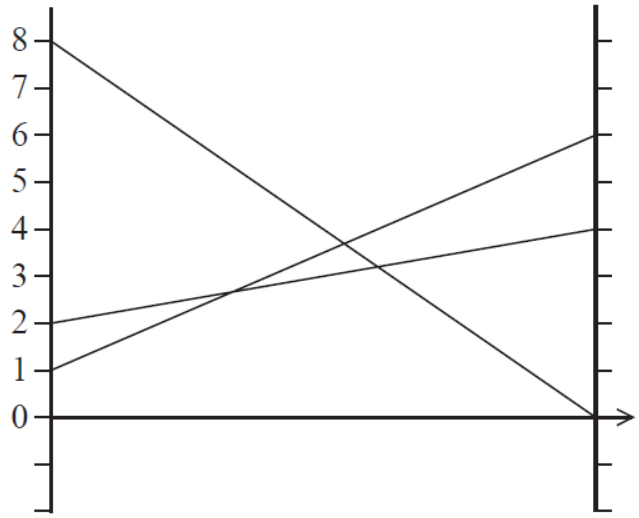
M marks in (b) - must bring earlier optimal results into calculations at least once

(b)
B1: Stage 1 correct
M1: Stage 2 completed with 4 states and at least 9 rows. Bod if something in each cell
A1: Any two states in Stage 2 correct
A1: cao all 4 states correct in Stage 2 (no extra rows)
M1: Stage 3 completed with 3 states and at least 8 rows. Bod if something in each cell
A1ft: cao any 2 states correct in Stage 3 on the follow through
A1: cao all 3 states in Stage 3 (no extra rows)
M1: Stage 4 completed with 1 state and at least 3 rows. Bod if something in each cell
A1ft: cao for Stage 4 following through their * values (no extra rows)

(c)
B1ft: One correct route (dependent on first B mark and all M marks in **(b)**) following through their least values (oe) at each stage in **(b)**
B1: Both routes correct (dependent on first B mark and all M marks in **(b)**)

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>7(a)</td><td>
<table>
<tr><td></td><td>Option X</td><td>Option Y</td><td>Option Z</td></tr>
<tr><td>Option Q</td><td>3</td><td>–3</td><td>1</td></tr>
<tr><td>Option R</td><td>–2</td><td>5</td><td>–1</td></tr>
</table>
</td><td>M1
A1</td><td>3.1a
2.2a</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td>(b)</td><td>If B plays option X, A's gains are $6p_1 + p_2 = 6p_1 + (1-p_1) = 5p_1 + 1$
If B plays option Y, A's gains are $8p_2 = 8(1-p_1) = -8p_1 + 8$
If B plays option Z, A's gains are
$4p_1 + 2p_2 = 4p_1 + 2(1-p_1) = 2p_1 + 2$</td><td>M1

A1</td><td>3.1a

1.1b</td></tr>
<tr><td></td><td> 
</td><td>M1
A1</td><td>1.1b
1.1b</td></tr>
<tr><td></td><td>$2+2p_1 = 8-8p_1 \Rightarrow p_1 = 0.6$</td><td>A1</td><td>1.1b</td></tr>
<tr><td></td><td>Alexis should play option Q with probability 0.6 and option R with probability 0.4</td><td>A1ft</td><td>3.2a</td></tr>
<tr><td></td><td></td><td>(6)</td><td></td></tr>
<tr><td>(c)</td><td>Value of the game = $2+2\left(\frac{3}{5}\right)-3 = \frac{1}{5}$</td><td>B1</td><td>2.2a</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td>(d)</td><td>$4q_3 = \frac{16}{5}, 8q_2 + 2q_3 = \frac{16}{5}$ or $-3q_2 + q_3 = \frac{1}{5}, 5q_2 - q_3 = \frac{1}{5}$</td><td>M1
A1</td><td>3.1a
1.1b</td></tr>
<tr><td></td><td>$q_2 = \frac{1}{5}, q_3 = \frac{4}{5} \Rightarrow$ Becky should play option X never, option Y with probability 0.2 and option Z with probability 0.8</td><td>A1</td><td>3.2a</td></tr>
<tr><td></td><td></td><td>(3)</td><td></td></tr>
<tr><td colspan="4">(12 marks)</td></tr>
</table>

Notes:

(a)

M1: Either one correct row or column

A1: cao

SC M1 A0 for $\begin{pmatrix} 6 & 0 & 4 \\ 1 & 8 & 2 \end{pmatrix}$

(b)

M1: Setting up three expressions in terms of p_1 (either in terms of the original or modified game) or p_2

A1: All three expressions correct – or equivalent e.g., $5p_1 - 2, -8p_1 + 5, 2p_1 - 1$

M1: Axes correct, at least one line correctly drawn for their expressions

A1: Correct graph

A1: Using a correct graph to obtain the correct probability expressions leading to the correct value of p_1 or p_2

A1ft: Interpret their values in the context of the question – must refer to play and the associated probabilities

(c)

B1: cao

(d)

M1: Setting up two equations in q_2 and q_3 with their value of either the original or modified game

A1: Correct two equations

A1: Interpret their values in the context of the question – must refer to play and the associated probabilities

For part **(d)** candidates might set up three equations in three unknowns

e.g.,

$$3q_1 - 3q_2 + q_3 = \frac{1}{5}$$

$$-2q_1 + 5q_2 - q_3 = \frac{1}{5}$$

$$q_1 + q_2 + q_3 = 1$$