A-level

FURTHER MATHEMATICS 7367/3M

Paper 3 Mechanics

Mark scheme

June 2022

Version: 1.0 Final Mark Scheme

2 2 6 A 7 3 6 7 / 3 M / M S

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
'their'	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
sf	significant figure(s)
dp	decimal place(s)

Examiners should consistently apply the following general marking principles:

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Q	Marking instructions	AO	Marks	Typical solution
1	Circles correct answer.	1.1b	B1	17.5 N s
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
2	Circles correct answer.	1.1b	B1	4320 W
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
3	Circles correct answer.	1.1b	B1	8 cm
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
4(a)	States correct dimensions of force.	1.2	B1	MLT^{-2}
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
4(b)	Obtains a correct expression for the dimensions of G.	1.1b	B1	$[G] = MLT^{-2} \times L^2M^{-2}$ $= M^{-1}L^3T^{-2}$ $LT^{-1} = M^{-a}L^{3a}T^{-2a} \times M^b \times L^c$ $1 = 3a + c$ $0 = b - a$ $-1 = -2a$ $a = \frac{1}{2}$ $b = \frac{1}{2}$ $c = -\frac{1}{2}$
	Forms a dimensional analysis equation using their dimensions for G and at least two of the other three dimensions correct.	1.1a	M1	
	Deduces at least one of the values of a, b or c correctly.	2.2a	M1	
	Obtains the correct values for a, b and c.	1.1b	A1	
	Total		**4**	

	Question total		**5**	

Q	Marking instructions	AO	Marks	Typical solution
5(a)	Uses integration to find work done to compress the buffer. Condone missing or incorrect limits.	3.4	M1	$\text{WD} = \int_0^{0.2} \left(Ax + 9000x^2\right)\mathrm{d}x$ $= \left[\frac{Ax^2}{2} + \frac{9000x^3}{3}\right]_0^{0.2}$ $= \frac{A}{50} + 24$
	Obtains a correct simplified expression for the work done in terms of A.	1.1b	A1	
	Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
5(b)	Uses their expression for work done from **(a)** and the correct KE to form an equation in terms of A.	3.3	M1	$\frac{1}{2} \times 10000 \times 0.3^2 = \frac{A}{50} + 24$ $450 = \frac{A}{50} + 24$ $A = 21300$
	Obtains the correct value for A from their expression in part (a), provided their expression is only in terms of A.	1.1b	A1F	
	Total		**2**	

	Question total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
6(a)	Differentiates to obtain the velocity with at least one component correct.	1.1a	M1	$\mathbf{v} = 6\cos 3t\mathbf{i} - 6\sin 3t\mathbf{j}$ $\mathbf{v}.\mathbf{r} = 12\sin 3t\cos 3t - 12\cos 3t\sin 3t$ $= 0$ As the scalar product is zero the vectors are perpendicular.
	Obtains correct velocity.	1.1b	A1	
	Calculates the scalar product of their velocity and the position vector. Or Finds expressions for the gradient of the line segment represented by each vector.	3.1a	M1	
	Shows that the scalar product is zero and explains that the two vectors are perpendicular. Or Shows that the product of the gradients of the line segment represented by the vectors is -1 and concludes that the vectors are perpendicular.	2.1	R1	
	Total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
6(b)	Obtains the correct acceleration.	1.1b	B1	$\mathbf{a} = -18\sin 3t\mathbf{i} - 18\cos 3t\mathbf{j}$ $\mathbf{F} = -90\sin 3t\mathbf{i} - 90\cos 3t\mathbf{j}$ $F = \sqrt{(-90\sin 3t)^2 + (-90\cos 3t)^2}$ $= \sqrt{90^2(\sin^2 3t + \cos^2 3t)}$ $= 90 \text{ N}$ So, the magnitude of the resultant force is constant.
	Finds the magnitude of their acceleration or applies Newton's second Law.	1.1a	M1	
	Applies Newton's second law and simplifies their result to show that the magnitude is constant. Must show use of the trigonometric identity. Condone missing units. Conclusion does not have to be stated.	2.1	R1	
	Total		**3**	

	Question total		**7**	

Q	Marking instructions	AO	Marks	Typical solution
7(a)	Recognises that the magnitudes of the perpendicular components of the momentum will be the same for each ball. Condone missing reference to magnitudes.	3.3	M1	The components of the momentum perpendicular to the sides of the table will be equal. As the balls have equal masses the components of the velocity perpendicular to the sides of the table will be equal. Hence as both balls are the same distance from the sides of the table, they will both hit the sides of the table at the same time.
	Uses the equal masses to argue that the perpendicular components of the velocity will be equal and that because both balls are the same distance from the sides of the table the balls hit the side of the table at the same time.	2.4	R1	
	Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
7(b)	Forms conservation of momentum equations for both components, with or without vector notation.	3.3	M1	$m\begin{bmatrix}2\\0\end{bmatrix}=m\begin{bmatrix}0.8\cos 30°\\0.8\sin 30°\end{bmatrix}+m\begin{bmatrix}v\cos\theta\\-v\sin\theta\end{bmatrix}$ $v\cos\theta=2-0.8\cos 30°$ $=2-\dfrac{2\sqrt{3}}{5}$ $v\sin\theta=0.8\sin 30°=0.4$ $\tan\theta=\dfrac{0.4}{2-\dfrac{2\sqrt{3}}{5}}=0.3060...$ $\theta=17.014...=17.0$ to 1 dp
	Obtains a correct pair of equations.	3.4	A1	
	Forms equation for $\tan\theta$ for their pair of equations.	1.1a	M1	
	Obtains correct value for θ Must see $\theta=17.01$ or better. Accept 17 AG	2.1	A1	
	Total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
7(c)	Uses their momentum equation or Pythagoras' theorem to find v	1.1a	M1	$v = \dfrac{0.8\sin 30°}{\sin 17.0°}$ $v = 1.37$
	Obtains the correct value for v AWRT 1.4	1.1b	A1	
	Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
7(d)	Uses their values and the given information to create an argument which contains two comparable quantities. Eg Calculates components of velocity parallel to the sides of the table. Argument based on the comparative size of angles. Argument based on speed and time. Not necessary to see numeric values in their argument.	3.1b	M1	$0.8\cos 30° = 0.693$ $1.37\cos 17.0° = 1.31$ Parallel component for the red ball is greater, so the red ball has the greater speed and travels the greater distance as both balls take the same time to reach the edges of the table.
	Completes a reasoned argument to conclude that the red ball travels the greater distance.	2.2a	R1	
	Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
7(e)	Suggests including air resistance or taking account of the rotational motion of the balls. Must be a refinement and not a criticism.	3.5c	B1	Include the rotational motion of the balls in the model.
	Total		**1**	

	Question total		**11**	

Q	Marking instructions	AO	Marks	Typical solution
8(a)	Obtains correct value of or expression for the normal reaction force.	3.3	B1	$R + 240\sin 30° = 60 \times 9.8$ $R = 468$ $240\cos 30° \times 5 - 0.4 \times 468 \times 5 = \frac{1}{2} \times 60v^2$ $600\sqrt{3} - 936 = 30v^2$ $103.23 = 30v^2$ $v = 1.8549...$ $= 1.9 \text{ m s}^{-1}$
	Forms a three-term energy equation with at least two correct terms. Condone consistent wrong use of trigonometry within their solution.	3.3	M1	
	Obtains a correct energy equation. Follow through use of $R = 60g$	1.1b	A1F	
	Uses an energy method to obtain the maximum speed AWRT 1.9 Condone missing units.	1.1b	A1	
	Total		**4**	

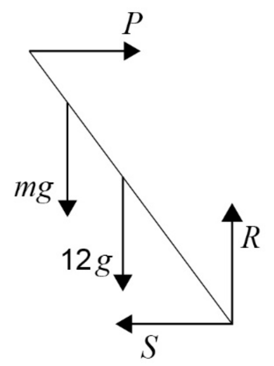
Q	Marking instructions	AO	Marks	Typical solution
8(b)	Forms an energy equation to find the additional distance using their speed / energy from part (a). Must use normal reaction as 588.	3.4	M1	$103.23 = 0.4 \times 588 \times d$ $103.23 = 235.2 \times d$ $d = 0.44$ Crate travels 5.4 metres
	Obtains a correct total distance. Final answer must be given to 2 sf and with the correct units.	3.2a	A1	
	Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
8(c)	States that air resistance has been ignored.	3.5b	E1	As air resistance has been ignored it is unlikely that the crate will travel as far as the 5.4 metres predicted. Hence the student's claim is quite reasonable.
	Makes a reasoned comment on the student's claim based on their answer to part **(b)** and the impact of air resistance.	2.2b	E1	
	Total		**2**	

	Question total		**8**	

Q	Marking instructions	AO	Marks	Typical solution
9(a)	Forms an equation to find the distance of the centre of mass from *AF*. Condone one error in their distances.	3.3	M1	$\bar{x} = \dfrac{9\times10+1\times18}{10}$ $=10.8$
	Obtains correct distance.	1.1b	A1	
	Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
9(b)(i)	Obtains correct vertical reaction force.	1.1b	B1	$R=(12+m)g$
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
9(b)(ii)	Takes moments about any point on the rod, with at least two correct terms.	3.4	M1	Rod:  $P=S$ Take moments about B $P\sin60°=12g\times0.5\cos60°+mg\times0.75\cos60°$ $S=\dfrac{g(6+0.75m)}{\tan60°}$ $=\dfrac{g(6+0.75m)}{\sqrt{3}}$
	Obtains a correct moment equation.	1.1b	A1	
	Completes a reasoned argument to obtain the correct expression for the horizontal reaction force at *B*. AG	2.1	R1	
	Total		**3**	

Q	Marking instructions	AO	Marks	Typical solution
9(b)(iii)	Forms equation / inequality with at least three terms to find m when on the point of toppling by taking moments about E with the moment of the horizontal force correct.	3.3	M1	Composite Body on point of toppling: $10g\times 9.2+(12+m)g\times 4=20\times\frac{g(6+0.75m)}{\sqrt{3}}$ $140\sqrt{3}+4m\sqrt{3}=120+15m$ $m=\frac{140\sqrt{3}-120}{15-4\sqrt{3}}=15.2$
	Obtains correct equation / inequality.	1.1b	A1	Composite Body on point of sliding:
	Obtains correct m for the point of toppling. AWRT 15	1.1b	A1	$\frac{g(6+0.75m)}{\sqrt{3}}=0.3\times(22+m)g$ $6+0.75m=6.6\sqrt{3}+0.3\sqrt{3}m$ $m=\frac{6-6.6\sqrt{3}}{0.3\sqrt{3}-0.75}=23.6$
	Forms equation / inequality to find m when on the point of sliding using $F=\mu R$ Condone omission of the mass of the block.	3.3	M1	So the maximum value of m is 15.2
	Obtains correct m for the point of sliding. AWRT 24	1.1b	A1	
	Deduces that the value of m is AWRT 15 from correct working.	2.2a	R1	
	Total		**6**	

	Question total		**12**	

	Paper total		**50**	