Mark Scheme (Results)

Summer 2022

Pearson Edexcel GCE

AS Mathematics (8MA0)

Paper 01 Core Mathematics

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 80.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ☐ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response. If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

Question	Scheme	Marks	AOs
1	$x^n \to x^{n+1}$	M1	1.1b
	$\int \left(8x^3 - \frac{3}{2\sqrt{x}} + 5\right) dx = \frac{8x^4}{4} ... + 5x$	A1	1.1b
	$= ... - 2 \times \frac{3}{2} x^{\frac{1}{2}} + ...$	A1	1.1b
	$= 2x^4 - 3x^{\frac{1}{2}} + 5x + c$	A1	1.1b
		(4)	
		(4 marks)	

Notes

M1: For raising any correct power of x by 1 including $5 \to 5x$ (not for $+ c$) Also allow eg $x^3 \to x^{3+1}$

A1: For 2 correct non-fractional power terms (allow unsimplified coefficients) and may appear on separate lines. The indices must be processed. The $+ c$ does not count as a correct term here. Condone the 1 appearing as a power or denominator such as $\frac{5x^1}{1}$ for this mark.

A1: For the correct fractional power term (allow unsimplified) Allow eg $+-2 \times 1.5\sqrt{x^1}$.

Also allow fractions within fractions for this mark such as $\frac{\frac{3}{2}}{\frac{1}{2}} x^{\frac{1}{2}}$

A1: All correct and simplified and on one line including $+ c$. Allow $-3\sqrt{x}$ or $-\sqrt{9x}$ for $-3x^{\frac{1}{2}}$.

Do not accept $+-3x^{\frac{1}{2}}$ for this mark.

Award once a correct expression is seen and isw but if there is any additional/incorrect notation and no correct expression has been seen on its own, withhold the final mark.

Eg. $\int 2x^4 - 3x^{\frac{1}{2}} + 5x + c \, dx$ or $2x^4 - 3x^{\frac{1}{2}} + 5x + c = 0$ with no correct expression seen earlier are both A0.

Question	Scheme	Marks	AOs
2(a)	$f(-3) = 2(-3)^3 + 5(-3)^2 + 2(-3) + 15$ $= -54 + 45 - 6 + 15$	M1	1.1b
	$f(-3) = 0 \Rightarrow (x+3)$ is a factor	A1	2.4
		(2)	
(b)	At least 2 of: $a = 2,\ b = -1,\ c = 5$	M1	1.1b
	All of: $a = 2,\ b = -1,\ c = 5$	A1	1.1b
		(2)	
(c)	$b^2 - 4ac = (-1)^2 - 4(2)(5)$	M1	2.1
	$b^2 - 4ac = -39$ which is < 0 so the quadratic has no real roots so f(x) = 0 has only 1 real root	A1	2.4
		(2)	
(d)	$(x =)\ 2$	B1	2.2a
		(1)	
			(7 marks)

Notes

(a)

M1: Attempts f(−3). Attempted division by $(x+3)$ or $f(3)$ is M0
Look for evidence of embedded values or two correct terms of
$f(-3) = -54 + 45 - 6 + 15 = ...$

A1: Achieves and states f(−3) = 0, and makes a suitable conclusion. Sight of f(x)=0 when $x = -3$ is also acceptable.
It must follow M1. Accept, for example, $f(-3) = 0 \Rightarrow (x+3)$ is a factor

This may be seen in a preamble before finding $f(-3) = 0$ but in these cases there must be a minimal statement ie QED, "proved", tick etc.

(b)

M1: Correct method implied by values for at least 2 correct constants. Allow embedded in their $f(x)$ or within their working if they use algebraic division/other methods which may be seen in part (a) and used in part (b).

A1: All values correct. Allow embedded in their $f(x)$ or seen as the quotient from algebraic division. Isw incorrectly stated values of a b and c following a correct quadratic expression seen.

$$
\begin{array}{r}
2x^2 - x + 5 \\
x+3 \overline{\smash{)}\, 2x^3 + 5x^2 + 2x + 15} \\
2x^3 + 6x^2 \qquad\qquad \\
-x^2 + 2x \qquad \\
-x^2 - 3x \qquad \\
5x + 15 \\
5x + 15 \\
0
\end{array}
$$

scores M1A1

(c)

M1: Either:

- considers the discriminant using their *a*, *b* and *c* (does not need to be evaluated) $\left(b^2-4ac=\right)\ (-1)^2-4(2)(5)$ (the $(-1)^2$ may appear as 1^2 and condone missing brackets for this mark for -1^2). Discriminant $=-39$ is sufficient for M1
- attempts to complete the square so score for $2\left(x\pm\frac{1}{4}\right)^2+...$
- attempts to find the roots of the quadratic using the formula. The values embedded in the formula score this mark.
  $\dfrac{1\pm\sqrt{(-1)^2-4\times2\times5}}{2\times2}$ (the $(-1)^2$ may appear as 1^2 and condone missing brackets for this mark for -1^2)
- Sketches a graph of the quadratic. It must be a U shaped quadratic which does not cross the *x*-axis.

A1: Provides a correct explanation from correct working. They must

- Have a correct calculation
- Explanation that the quadratic has no (real) roots
- Minimal conclusion stating that $f(x)=0$ has only one root

eg $b^2-4ac=-39<0$ so only one root is M1A0 (needs to explain the quadratic has no real roots)

eg $2\left(x-\frac{1}{4}\right)^2+\frac{39}{8}>0$ **so no real roots** (for the quadratic) **so** (f(*x*) has) **only one (**real**) root** is M1A1

The value of the discriminant, completed square form $2\left(x-\frac{1}{4}\right)^2+\frac{39}{8}$ or roots of the quadratic $\left(=\frac{1\pm\sqrt{39}i}{4}\right)$ must be correct.

If they sketch the quadratic graph it must be a U shaped quadratic which crosses the *y*-axis at 5 and has a minimum in the 1st quadrant. They must explain that the graph does not cross the *x*-axis so no real roots for the quadratic so only one root for $f(x)=0$.

(d)

B1: 2 condone (2, 0)

Question	Scheme	Marks	AOs
3(a)	$\overrightarrow{QR}=\overrightarrow{PR}-\overrightarrow{PQ}=13\mathbf{i}-15\mathbf{j}-(3\mathbf{i}+5\mathbf{j})$	M1	1.1a
	$=10\mathbf{i}-20\mathbf{j}$	A1	1.1b
		(2)	
(b)	$\left\lvert\overrightarrow{QR}\right\rvert=\sqrt{"10"^2+"(-20)"^2}$	M1	2.5
	$=10\sqrt{5}$	A1ft	1.1b
		(2)	
(c)	$\overrightarrow{PS}=\overrightarrow{PQ}+\frac{3}{5}\overrightarrow{QR}=3\mathbf{i}+5\mathbf{j}+\frac{3}{5}("10\mathbf{i}-20\mathbf{j}")=...$ or $\overrightarrow{PS}=\overrightarrow{PR}+\frac{2}{5}\overrightarrow{RQ}=13\mathbf{i}-15\mathbf{j}+\frac{2}{5}("-10\mathbf{i}+20\mathbf{j}")=...$	M1	3.1a
	$=9\mathbf{i}-7\mathbf{j}$	A1	1.1b
		(2)	
		(6 marks)	

Notes

(a)

M1: Attempts subtraction either way round. This cannot be awarded for adding the two vectors. If no method shown it may be implied by one correct component. eg $10\mathbf{i}-10\mathbf{j}$ on its own can score M1.

A1: Correct answer. Allow $10\mathbf{i}-20\mathbf{j}$ and $\begin{pmatrix}10\\-20\end{pmatrix}$ but not $\begin{pmatrix}10\mathbf{i}\\-20\mathbf{j}\end{pmatrix}$

(b)

M1: Correct use of Pythagoras. Attempts to "square and add" before square rooting. The embedded values are sufficient. Follow through on their $\overrightarrow{QR}$

A1ft: $10\sqrt{5}$ following (a) of the form $\pm10\mathbf{i}\pm20\mathbf{j}$

(c)

M1: Full attempt at finding a $\overrightarrow{PS}$. They must be attempting $\overrightarrow{PQ}\pm\frac{3}{5}\overrightarrow{QR}$ or $\overrightarrow{PS}=\overrightarrow{PR}\pm\frac{2}{5}\overrightarrow{RQ}$ but condone arithmetical slips after that.

Cannot be scored for just stating eg $\overrightarrow{PQ}\pm\frac{3}{5}\overrightarrow{QR}$

Follow through on their $\overrightarrow{QR}$. Terms do not need to be collected for this mark. If no method shown it may be implied by one correct component following through on their $\overrightarrow{QR}$

A1: Correct vector as shown. Allow $9\mathbf{i}-7\mathbf{j}$ and $\begin{pmatrix}9\\-7\end{pmatrix}$.

Only withhold the mark for $\begin{pmatrix}9\mathbf{i}\\-7\mathbf{j}\end{pmatrix}$ if the mark has not already been withheld in (a) for $\begin{pmatrix}10\mathbf{i}\\-20\mathbf{j}\end{pmatrix}$

Alt (c) (Expressing $\overrightarrow{PS}$ in terms of the given vectors) They must be attempting $\frac{2}{5}\overrightarrow{PQ}+\frac{3}{5}\overrightarrow{PR}$

M1: $(\overrightarrow{PS}=\overrightarrow{PQ}+\frac{3}{5}\overrightarrow{QR}=\overrightarrow{PQ}+\frac{3}{5}(\overrightarrow{PR}-\overrightarrow{PQ}))$

$\Rightarrow \frac{2}{5}\overrightarrow{PQ}+\frac{3}{5}\overrightarrow{PR}=\frac{2}{5}(3\mathbf{i}+5\mathbf{j})+\frac{3}{5}(13\mathbf{i}-15\mathbf{j}) = \ldots$

A1: Correct vector as shown. Allow $9\mathbf{i}-7\mathbf{j}$ and $\begin{pmatrix}9\\-7\end{pmatrix}$.

Only withhold the mark for $\begin{pmatrix}9\mathbf{i}\\-7\mathbf{j}\end{pmatrix}$ if the mark has not already been withheld in (a) for $\begin{pmatrix}10\mathbf{i}\\-20\mathbf{j}\end{pmatrix}$

Question	Scheme	Marks	AOs
4(a)(i)	$(3x+10)^2=(x+2)^2+(7x)^2-2(x+2)(7x)\cos 60°$ oe	M1	3.1a
	Uses cos 60° = ½, expands the brackets and proceeds to a 3 term quadratic equation	dM1	1.1b
	$17x^2-35x-48=0$ *	A1*	2.1
		(3)	
(ii)	$x=3$	B1	3.2a
		(1)	
(b)	$\dfrac{5}{\sin ACB}=\dfrac{19}{\sin 60°}\Rightarrow \sin ACB=...\left(\dfrac{5\sqrt{3}}{38}\right)$ or e.g. $5^2=21^2+19^2-2\times19\times21\cos ACB\Rightarrow \cos ACB=...\left(\dfrac{37}{38}\right)$	M1	1.1b
	$\theta=$ awrt 13.2	A1	1.1b
		(2)	
		(6 marks)	

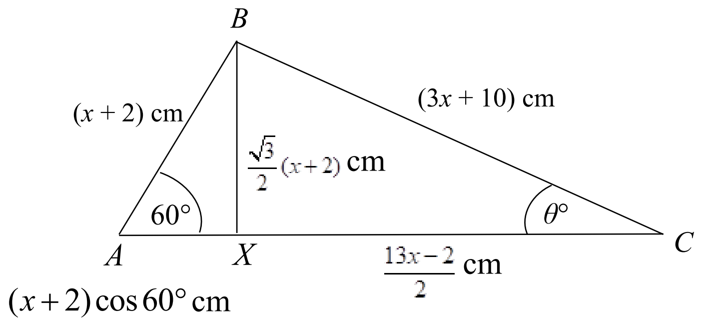
Notes

(a)(i) Mark (a) and (b) together

M1: Recognises the need to apply the cosine rule and attempts to use it with the sides in the correct positions and the formula applied correctly. Condone invisible brackets and slips on $3x+10$ as $3x-10$.
Alternatively, uses trigonometry to find *AX* and then equates two expressions for the length *BX*. You may see variations of this if they use Pythagoras or trigonometry to find *BX* and then apply Pythagoras to the triangle *BXC*. See the diagram below to help you.
The angles and lengths must be in the correct positions. Cos 60 may be $\frac{1}{2}$ from the start

dM1: Uses cos 60° = ½, expands the brackets and proceeds to a 3TQ. You may see the use of cos 60° = ½ in earlier work, but they must proceed to a 3TQ as well to score this mark. It is dependent on the first method mark.

A1*: Obtains the correct quadratic equation with the = 0 with no errors seen in the main body of their solution. Condone the recovery of invisible brackets as long as the intention is clear. You do not need to explicitly see cos 60 to score full marks.



(a)(ii)

B1: Selects the appropriate value i.e. $x = 3$ only. The other root must **either** be rejected if found **or** $x = 3$ must be the only root used in part (b). Can be implied by awrt 13.2 in (b)

(b)

M1: Using their value for x this mark is for either:

- applying the sine rule correctly (or considers 2 right angled triangles) and proceeding to obtain a value for sin ACB or
- applying the cosine rule correctly and proceeding to obtain a value for cos ACB.

Condone slips calculating the lengths AB, BC and AC. At least one of them should be found correctly for their value for x

(Also allow if the sine rule or cosine rule is applied correctly to find a value for sin ABC $\left(=\frac{21\sqrt{3}}{38}\right)$ or cos ACB $\left(=-\frac{11}{38}\right)$)

A1: awrt 13.2 (answers with little working eg just lengths on the diagram can score M1A1)

Question	Scheme	Marks	AOs
5(a)	$p = 10^{0.5}$ (or $\log_{10} p = 0.5$) or $q = 10^{0.03}$ (or $\log_{10} q = 0.03$)	M1	1.1b
	$p =$ awrt 3.162 or $q =$ awrt 1.072	A1	1.1b
	$p = 10^{0.5}$ (or $\log_{10} p = 0.5$) and $q = 10^{0.03}$ (or $\log_{10} q = 0.03$)	dM1	3.1a
	$A = 3.162 \times 1.072^t$	A1	3.3
		(4)	
(b)(i)	The initial mass (in kg) of algae (in the pond).	B1	3.4
(b)(ii)	The ratio of algae from one week to the next.	B1	3.4
		(2)	
(c)(i)	5.5 kg	B1	2.2a
(c)(ii)	$4 = "3.162" \times "1.072"^t$ or $\log_{10} 4 = 0.03t + 0.5$	M1	3.4
	awrt 3.4 (weeks)	A1	1.1b
		(3)	
(d)	• The model predicts unlimited growth. • The weather may affect the rate of growth	B1	3.5b
		(1)	
		(10 marks)	

Notes

(a)

M1: A correct equation in p **or** q. May be implied by a correct value for p or q.
Also score for rearranging the equation to the form $A = 10^{0.5} ... 10^{0.03t}$

A1: For p = awrt 3.162 or q = awrt 1.072. May be embedded within the equation.

dM1: Correct equations in p **and** q. Also score for rearranging the equation to the form $A = 10^{0.5} \times 10^{0.03t}$

A1: Complete equation with p = awrt 3.162 and q = awrt 1.072. **Must be seen in (a)**
If p and q are just stated but the equation is not written with the values embedded then withhold this mark.
Withhold the final mark if the correct values for p and q result from incorrect working such as $A = 10^{0.5} + 10^{0.03t} \Rightarrow A = 3.162 \times 1.072^t$.
If p and q are stated the wrong way round, take the stated equation as their final answer and isw.

(b)
(i)
B1: Must reference mass of algae and relating to initially/at the start/beginning
Examples of acceptable answers:
The mass of algae originally (in the pond)
p is the mass of algae when $t = 0$
Examples of answers we would not accept
p is how much algae there is at the beginning
The relationship between algae and number of weeks

(ii)

B1: Must reference the rate of change/multiplier and the time frame eg per week/every week/each week.

Examples of acceptable answers:

q is the rate at which the mass of algae increases for every week

The amount of algae increases by 7.2% each week (condone amount for mass in ii)

The proportional increase in mass of the algae each week

Examples of answers we would not accept:

q is how much algae will increase when t increases by 1

The amount that grows per unit of time

The rate at which the mass of algae in the small pond increases after t number of weeks

The rate in which the algae mass increases

(c)

B1: cao (including units)

M1: Setting up a correct equation to find t using the given equation or their part (a)
Substitution of $A = 4$ into their equation for A or the given equation is sufficient for this mark.

A1: awrt 3.4 (weeks). Accept any acceptable method (including trial and improvement)
Condone lack of units. isw if they subsequently convert to weeks and days. Allow awrt 3.5 (weeks) following p = awrt 3.16 and q = awrt 1.07.
An answer of only awrt 3.4 is M1A1, but an answer of 4 (weeks) with no working is M0A0

(d)

B1: Any reason why the rate of change, growth or the mass of algae might change or why the model in not realistic.
Be generous with the awarding of this mark as long as the answer has engaged with the context of the problem or the model

Examples of acceptable answers:

Seasonal changes (which would affect the growth rate)

Overcrowding (as it is a small pond)

Algae may stop growing (the model predicts unlimited growth)

Algae may die / be removed / eaten (so the rate of growth may not continue at the same rate)

Examples of answers we would not accept:

There could be other factors that affect the amount of algae (too vague)

The mass of algae might change

Question	Scheme	Marks	AOs
6(a)	3^8 or 6561 as the constant term	B1	1.1b
	$\left(3-\frac{2x}{9}\right)^8 = ...+{}^8C_1(3)^7\left(-\frac{2x}{9}\right)+{}^8C_2(3)^6\left(-\frac{2x}{9}\right)^2+{}^8C_3(3)^5\left(-\frac{2x}{9}\right)^3+...$ $=...+8\times(3)^7\left(-\frac{2x}{9}\right)+28\times(3)^6\left(-\frac{2x}{9}\right)^2+56(3)^5\left(-\frac{2x}{9}\right)^3$	M1 A1	1.1b 1.1b
	$=6561-3888x+1008x^2-\frac{448}{3}x^3+...$	A1	1.1b
		(4)	
(b)	Coefficient of x^2 is $\frac{1}{2}\times"1008"-\frac{1}{2}\times"-\frac{448}{3}"$	M1	3.1a
	$=\frac{1736}{3}$ (or $578\frac{2}{3}$)	A1	1.1b
		(2)	
			(6 marks)

Notes

(a)

B1: Sight of 3^8 or 6561 as the constant term.

M1: An attempt at the binomial expansion. This can be awarded for the correct structure of the 2nd, 3rd or 4th term. The correct binomial coefficient must be associated with the correct power of 3 and the correct power of $(\pm)\frac{2x}{9}$. Condone invisible brackets eg ${}^8C_2(3)^6-\frac{2x}{9}^2$ for this mark.

A1: For a correct simplified or unsimplified **second** or **fourth term** (with binomial coefficients evaluated).

$+8\times(3)^7\left(-\frac{2x}{9}\right)$ or $+56(3)^5\left(-\frac{2x}{9}\right)^3$

A1: $6561-3888x+1008x^2-\frac{448}{3}x^3$ Ignore any extra terms and allow the terms to be listed.

Allow the exact equivalent to $-\frac{448}{3}$ eg $-149.\dot{3}$ but not -149.3.

Condone x^1 and eg $+-3888x$. Do not isw if they multiply all the terms by eg 3

Alt(a)

B1: Sight of $3^8(1+.....)$ or 6561 as the constant term

M1: An attempt at the binomial expansion $\left(1-\frac{2}{27}x\right)^8$. This can be awarded for the correct structure of the 2nd, 3rd or 4th term. The correct binomial coefficient must be associated with the correct power of $(\pm)\frac{2x}{27}$. Condone invisible brackets for this mark.

Score for any of:

$8\times-\frac{2}{27}x,\quad \frac{8\times7}{2}\times\left(-\frac{2}{27}x\right)^2,\quad \frac{8\times7\times6}{6}\times\left(-\frac{2}{27}x\right)^3$ which may be implied by any of

$-\frac{16}{27}x,\ +\frac{112}{729}x^2,\ -\frac{448}{19683}x^3$

A1: For a correct simplified or unsimplified **second** or **fourth** term including being multiplied by 3^8

A1: $6561-3888x+1008x^2-\frac{448}{3}x^3$ Ignore any extra terms and allow the terms to be listed.

Allow the exact equivalent to $-\frac{448}{3}$ eg $-149.\dot{3}$ but not -149.3.

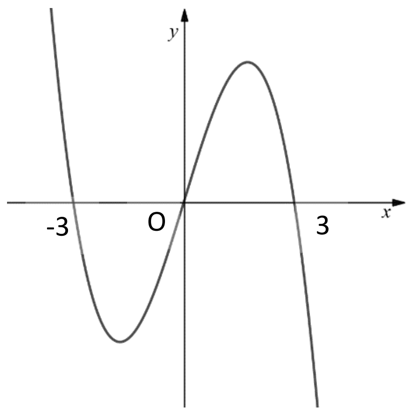
Condone x^1 and eg $+-3888x$

(b)

M1: Adopts a correct strategy for the required coefficient. This requires an attempt to calculate $\pm$ ½ their coefficient of x^2 from part (a) $\pm$ ½ their coefficient of x^3 from part (a).

There must be an attempt to bring these terms together to a single value. ie they cannot just circle the relevant terms in the expansion for this mark. The strategy may be implied by their answer.
Condone any appearance of x^2 or x^3 appearing in their intermediate working.

A1: $\frac{1736}{3}$ or $578\frac{2}{3}$ Do not accept $578.\dot{6}$ or $\frac{1736}{3}x^2$

Question	Scheme	Marks	AOs
7(a)	$9x - x^3 = x\left(9 - x^2\right)$	M1	1.1b
	$9x - x^3 = x\left(3 - x\right)\left(3 + x\right)$ oe	A1	1.1b
		(2)	
(b)	 A cubic with correct orientation	B1	1.1b
	Passes though origin, (3, 0) and (−3, 0)	B1	1.1b
		(2)	
(c)	$y = 9x - x^3 \Rightarrow \dfrac{dy}{dx} = 9 - 3x^2 = 0 \Rightarrow x = \left(\pm\right)\sqrt{3} \Rightarrow y = ...$	M1	3.1a
	$y = \left(\pm\right)6\sqrt{3}$	A1	1.1b
	$\left\{k \in \mathbb{R} : -6\sqrt{3} < k < 6\sqrt{3}\right\}$ oe	A1ft	2.5
		(3)	
			(7 marks)

Notes

(a)

M1: Takes out a factor of x or $-x$. Scored for $\pm x(\pm 9 \pm x^2)$ May be implied by the correct answer or $\pm x\left(\pm x \pm 3\right)\left(\pm x \pm 3\right)$.

Also allow if they attempt to take out a factor of $(\pm x \pm 3)$ so score for $(\pm x \pm 3)(\pm 3x \pm x^2)$

A1: Correct factorisation. $x\left(3 - x\right)\left(3 + x\right)$ on its own scores M1A1.

Allow eg $-x\left(x - 3\right)\left(x + 3\right)$, $x\left(x - 3\right)\left(-x - 3\right)$ or other equivalent expressions

Condone an = 0 appearing on the end and condone eg x written as $(x + 0)$.

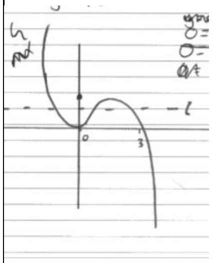
(b)

B1: Correct shape (negative cubic) appearing anywhere on a set of axes. It must have a minimum to the left and maximum to the right. Be tolerant of pen slips. Judge the intent of the shape. (see examples)

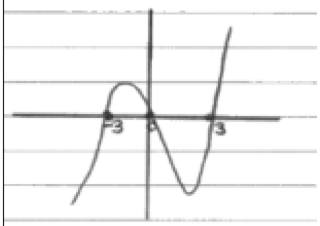
B1: Passes **through** each of the origin, (3, 0) and (−3, 0) and no other points on the x axis. (The graph should not turn on any of these points).
The points may be indicated as just 3 and −3 on the axes. Condone x and y to be the wrong way round eg $(0, -3)$ for (−3, 0) as long as it is on the correct axis but do not allow (−3, 0) to be labelled as (3, 0).

Examples

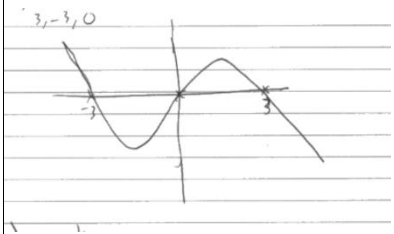
B1B0

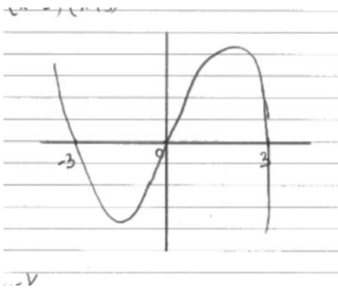


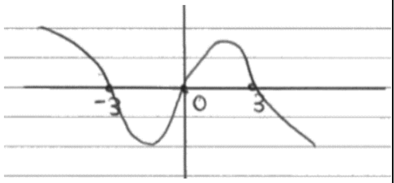
B0B1



B1B1







(c) ***Be aware the value of *y* can be solved directly using a calculator which is not acceptable***

M1: Uses a correct strategy for the y value of either the maximum or minimum. E.g. differentiates to achieve a quadratic, solves $\frac{dy}{dx} = 0$ and uses their x to find y

A1: Either or both of the values $(\pm)6\sqrt{3}$.

Cannot be scored for an answer without any working seen.

A1ft: Correct answer in any acceptable set notation following through their $6\sqrt{3}$.

Condone $\left\{"-6\sqrt{3}" < k < "6\sqrt{3}"\right\}$ or $\left\{"-6\sqrt{3}" < k\right\} \cap \left\{k < "6\sqrt{3}"\right\}$ but not $\left\{"-6\sqrt{3}" < k\right\} \cup \left\{k < "6\sqrt{3}"\right\}$

Note: If there is a contradiction of their solution on different lines of working do not penalise intermediate working and mark what appears to be their final answer.

Must be in terms of k

Question	Scheme	Marks	AOs
8(a)	$(k =)\ 0.8$	B1	1.1b
		(1)	
(b)	$1 = 0.8 + 1.4\mathrm{e}^{-0.5t} \Rightarrow 1.4\mathrm{e}^{-0.5t} = 0.2$	M1	3.1b
	$-0.5t = \ln\left(\frac{0.2}{1.4}\right) \Rightarrow t = ...$	M1	1.1b
	awrt 3.9 minutes	A1	1.1b
		(3)	
(c)	$\left(\frac{\mathrm{d}P}{\mathrm{d}t} =\right)\ -0.7\mathrm{e}^{-0.5t}$ $\left(\frac{\mathrm{d}P}{\mathrm{d}t}\right)_{t=2} = -0.7\mathrm{e}^{-0.5\times 2}$	M1	3.1b
	= awrt 0.258 (kg/cm^2 per minute)	A1	1.1b
		(2)	
	(6 marks)		

Notes

(a)

B1: Completes the equation for the model by obtaining $(k =)$ 0.8 or equivalent.

(b) ***Be aware this could be solved entirely using a calculator which is not acceptable***

M1: For using the model with $P = 1$ and their value for k from (a) and proceeding to $A\mathrm{e}^{\pm 0.5t} = B$. Condone if A or B are negative for this mark.

M1: Uses correct log work to solve an equation of the form $A\mathrm{e}^{\pm 0.5t} = B$ leading to a value for t. They cannot proceed directly to awrt 3.9 without some intermediate working seen.
Eg $t = 2\ln 7$ or $-2\ln\left(\frac{1}{7}\right)$ is acceptable.
Also allow $1.4\mathrm{e}^{-0.5t} = 0.2 \Rightarrow -0.5t = -1.9459... \Rightarrow t = ...$
This cannot be scored from an unsolvable equation (eg when their $k \geqslant 1$ so that $\mathrm{e}^{\pm 0.5t} \leqslant 0$).

A1: Accept awrt 3.9 minutes or t = awrt 3.9 with correct working seen.
eg $1.4\mathrm{e}^{-0.5t} = 0.2 \Rightarrow t = 3.9$ would be M1M0A0

(c) ***Be aware this can be solved entirely using a calculator which is not acceptable***

M1: Links rate of change to gradient and differentiates to obtain an expression of the form $A\mathrm{e}^{-0.5t}$ and substitutes $t = 2$. Do not accept $At\mathrm{e}^{-0.5t}$ as the derivative.
Beware that substituting $t = 2$ and proceeding from e^{-1} to e^{-2} is M0A0

A1: Obtains awrt 0.258 with differentiation seen. (Units not required) Condone awrt -0.258
Awrt ± 0.258 with no working is M0A0. Isw after a correct answer is seen.

(Ignore in (c) any spurious notation on the LHS when differentiating such as $P = ...$ or $\frac{\mathrm{d}y}{\mathrm{d}x} = ...$)

Question	Scheme	Marks	AOs
9(a)(i)	$\log_3\left(\frac{x}{9}\right)=\log_3 x-\log_3 9=p-2$	B1	1.2
(ii)	$\log_3\left(\sqrt{x}\right)=\frac{1}{2}p$	B1	1.1b
		(2)	
(b)	$2\log_3\left(\frac{x}{9}\right)+3\log_3\left(\sqrt{x}\right)=-11\Rightarrow 2p-4+\frac{3}{2}p=-11\Rightarrow p=...$	M1	1.1b
	$p=-2$	A1	1.1b
	$\log_3 x=-2\Rightarrow x=3^{-2}$	M1	1.1b
	$x=\frac{1}{9}$	A1	1.1b
		(4)	
	Alternative for (b) not using (a):		
	$2\log_3\left(\frac{x}{9}\right)+3\log_3\left(\sqrt{x}\right)=-11\Rightarrow \log_3\left(\frac{x}{9}\right)^2+\log_3\left(\sqrt{x}\right)^3=-11$ $\Rightarrow \log_3\frac{x^{\frac{7}{2}}}{81}=-11$	M1	1.1b
	$\Rightarrow \frac{x^{\frac{7}{2}}}{81}=3^{-11}$ or equivalent eg $x^{\frac{7}{2}}=3^{-7}$	A1	1.1b
	$x^{\frac{7}{2}}=81\times3^{-11}\Rightarrow x^{\frac{7}{2}}=3^4\times3^{-11}=3^{-7}\Rightarrow x=\left(3^{-7}\right)^{\frac{2}{7}}=3^{-2}$	M1	1.1b
	$x=\frac{1}{9}$	A1	1.1b
			(6 marks)

Notes

(a)(i)
B1: Recalls the subtraction law of logs and so obtains $p-2$

(a)(ii)
B1: $\frac{1}{2}p$ oe

(b) ***Be aware this should be solved by non-calculator methods***

M1: Uses their results from part (a) to form a linear equation in p and attempts to solve leading to a value for p. Allow slips in their rearrangement when solving. Allow a misread forming the equation equal to 11 instead of −11

A1: Correct value for p

M1: Uses $\log_3 x=p\Rightarrow x=3^p$ following through on what they consider to be their p. It must be a value rather than p

A1: $\left(x=\right)\frac{1}{9}$ cao with correct working seen. Must be this fraction. Do not penalise invisible brackets as long as the intention is clear.

Alternative:

M1: Correct use of log rules to achieve an equation of the form $\log_3 ... = \log_3 ...$ or $\log_3 ... = \text{a number}$ (typically -11). Condone arithmetical slips.

A1: Correct equation with logs removed.

M1: Uses inverse operations to find x. Condone slips but look for proceeding from $x^{\frac{a}{b}} = ... \Rightarrow x = ...^{\frac{b}{a}}$ where they have to deal with a fractional power.

A1: $\left(x=\right)\frac{1}{9}$ cao with correct working seen. Must be this fraction. Do not penalise invisible brackets as long as the intention is clear.

Question	Scheme	Marks	AOs
10(a)	$y=\frac{1}{3}x^2-2\sqrt{x}+3\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2}{3}x-x^{-\frac{1}{2}}$	M1 A1	1.1b 1.1b
	$x=4\Rightarrow y=\frac{13}{3}$	B1	1.1b
	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_{x=4}=\frac{2}{3}\times4-4^{-\frac{1}{2}}\left(=\frac{13}{6}\right)\quad\therefore y-\frac{13}{3}=\frac{13}{6}(x-4)$	M1	2.1
	$13x-6y-26=0$*	A1*	1.1b
		(5)	
(b)	$\int\left(\frac{x^2}{3}-2\sqrt{x}+3\right)\mathrm{d}x=\frac{x^3}{9}-\frac{4}{3}x^{\frac{3}{2}}+3x(+c)$	M1 A1	1.1b 1.1b
	$y=0\Rightarrow x=2$	B1	2.2a
	Area of R is $\left[\frac{x^3}{9}-\frac{4}{3}x^{\frac{3}{2}}+3x\right]_0^4-\frac{1}{2}\times(4-"2")\times"\frac{13}{3}"=\frac{76}{9}-\frac{13}{3}$	M1	3.1a
	$=\frac{37}{9}$	A1	1.1b
		(5)	
		(10 marks)	

Notes

(a) **Calculators: If no algebraic differentiation seen then maximum in a) is M0A0B1M1A0***

M1: $x^n\to x^{n-1}$ seen at least once $...x^2\to...x^1,\ ...x^{\frac{1}{2}}\to...x^{-\frac{1}{2}},\ 3\to0$.

Also accept on sight of eg $...x^{\frac{1}{2}}\to...x^{\frac{1}{2}-1}$

A1: $\frac{2}{3}x-x^{-\frac{1}{2}}$ or any unsimplified equivalent (indices must be processed) accept the use of $0.\dot{6}x$ but not rounded or ambiguous values eg $0.6x$ or eg $0.66....x$

B1: Correct y coordinate of P. May be seen embedded in an attempt of the equation of l

M1: Fully correct strategy for an equation for l. Look for $y-"\frac{13}{3}"="\frac{13}{6}"(x-4)$ where their $\frac{13}{6}$ is from differentiating the equation of the curve and substituting in $x=4$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$ and the y coordinate is from substituting $x=4$ into the given equation.
If they use $y=mx+c$ they must proceed as far as $c=...$ to score this mark.
Do not allow this mark if they use a perpendicular gradient.

A1*: Obtains the printed answer with no errors.

(b) **Calculators: If no algebraic integration seen then maximum in b) is M0A0B1M1A0**

M1: $x^n\to x^{n+1}$ seen at least once. Eg $...x^2\to...x^3,\ ...x^{\frac{1}{2}}\to...x^{\frac{3}{2}},\ 3\to3x^1$. Allow eg $...x^2\to...x^{2+1}$ The $+c$ is not a valid term for this mark.

A1: $\frac{x^3}{9}-\frac{4}{3}x^{\frac{3}{2}}+3x$ or any unsimplified equivalent (indices must be processed) accept the use of exact decimals for $\frac{1}{9}$ ($0.\dot{1}$) and $-\frac{4}{3}$ ($-1.\dot{3}$) but not rounded or ambiguous values.

B1: Deduces the correct value for x for the intersection of l with the x-axis. May be seen indicated on Figure 2.

M1: Fully correct strategy for the area. This needs to include

- a correct attempt at the area of the triangle using their values (**could use integration**)
- a correct attempt at the area under the curve using 0 and 4 in their integrated expression
- the two values subtracted.

Be aware of those who mix up using the y-coordinate of P and the gradient at P which is M0. The values embedded in an expression is sufficient to score this mark.

A1: $\frac{37}{9}$ or exact equivalent eg $4\frac{1}{9}$ or $4.\dot{1}$ but not 4.111…. isw after a correct answer

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Be aware of other strategies to find the area *R*

eg Finding the area under the curve between 0 and 2 and then the difference between the curve and the straight line between 2 and 4:

$$\int_0^2 \frac{x^2}{3}-2\sqrt{x}+3\ \mathrm{d}x+\int_2^4 \frac{x^2}{3}-2\sqrt{x}-\frac{13}{6}x+\frac{22}{3}\ \mathrm{d}x$$

M1 $x^n \to x^{n+1}$ seen at least once on either integral (or on the equation of the line $y=\frac{1}{3}x+3$)

A1 for correct integration of **either** integral $\frac{x^3}{9}-\frac{4}{3}x^{\frac{3}{2}}+3x$ or $\frac{x^3}{9}-\frac{4}{3}x^{\frac{3}{2}}-\frac{13}{12}x^2+\frac{22}{3}x$ (may be unsimplified/uncollected terms but the indices must be processed with/without the $+C$)

B1 Correct value for x can be seen from the top of the first integral (or bottom value of the second integral)

M1 Correct strategy for the area eg.

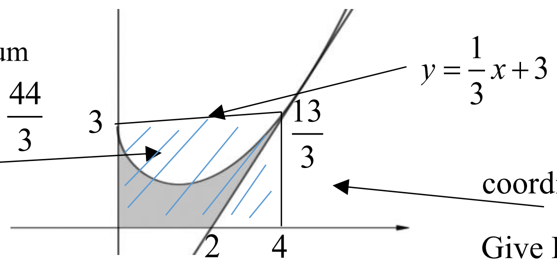
$$\left[\frac{x^3}{9}-\frac{4}{3}x^{\frac{3}{2}}+3x\right]_0^2+\left[\frac{x^3}{9}-\frac{4}{3}x^{\frac{3}{2}}-\frac{13}{12}x^2+\frac{22}{3}x\right]_2^4=\frac{62}{9}-\frac{4}{3}(2)^{\frac{3}{2}}+\frac{76}{9}-\frac{101}{9}+\frac{4}{3}(2)^{\frac{3}{2}}$$

A1: $\frac{37}{9}$ or exact equivalent eg $4\frac{1}{9}$ or $4.\dot{1}$ but not 4.1 or 4.111….

You could also see use of the area of a trapezium and/or the use of the line $y=\frac{1}{3}x+3$ to find other areas which could be combined or used as part of the strategy to find *R*. Ignore areas which are not used. The marks should still be able to be applied as per the scheme

Area of large trapezium

$=\frac{1}{2}\times\left(3+\frac{13}{3}\right)\times 4=\frac{44}{3}$



Area of triangle and the y coordinate of point P are both $\frac{13}{3}$

Give BOD if in doubt of intention

Area of trapezium – (Area between $y=\frac{1}{3}x+3$ and curve C + area of triangle)

$$=\frac{44}{3}-\frac{56}{9}-\frac{13}{3}=\frac{37}{9}$$

Question	Scheme	Marks	AOs
11(a)	$(x\pm5)^2+(y\pm4)^2$	M1	1.1b
	(i) Centre is (5, 4)	A1	1.1b
	(ii) Radius is 3	A1	1.1b
		(3)	
(b)	$2y+x+6=0 \Rightarrow y=-\frac{1}{2}x+\ldots \Rightarrow -\frac{1}{2} \to 2$	B1	2.2a
	$m_N=2 \Rightarrow y-4=2(x-5)$ $y-4=2(x-5),\ 2y+x+6=0 \Rightarrow x=\ldots,\ y=\ldots$	M1	3.1a
	Intersection is at $\left(\frac{6}{5},\ -\frac{18}{5}\right)$ oe	A1	1.1b
	Distance from centre to intersection is $\sqrt{\left(5-\frac{6}{5}\right)^2+\left(4+\frac{18}{5}\right)^2}$ So distance required is $\sqrt{\left(\text{"}5\text{"}-\text{"}\frac{6}{5}\text{"}\right)^2+\left(\text{"}4\text{"}+\text{"}\frac{18}{5}\text{"}\right)^2}-\text{"}3\text{"}$	dM1	3.1a
	$=\frac{19\sqrt{5}}{5}-3$ (or awrt 5.50)	A1	1.1b
		(5)	
			(8 marks)

Notes

(a)

M1: Attempts to complete the square for both x and y terms $(x\pm5)^2 \ldots\ (y\pm4)^2$ which may be implied by a centre of (±5, ±4)

A1: Centre (5, 4)

A1: Radius 3

(b)

B1: Deduces the gradient of the perpendicular to l is 2. May be seen in the equation for the perpendicular line to l

M1: A fully correct strategy for finding the intersection. This requires use of their gradient of the perpendicular which cannot be the gradient of l
Look for $y-\text{"}4\text{"}=\text{"}2\text{"}(x-\text{"}5\text{"})$ where $(5,4)$ is their centre being solved simultaneously with the equation of l
Do not be concerned with the mechanics of their rearrangement when solving simultaneously.
Many are finding the y-intercept of l $(0,-3)$ which is M0

A1: $\left(\frac{6}{5}, -\frac{18}{5}\right)$ or equivalent eg $(1.2, -3.6)$

They do not have to be written as coordinates and may be seen within their working rather than explicitly stated. They may also be written on the diagram.

dM1: Fully correct strategy for finding the required distance e.g. correct use of Pythagoras to find the distance between their centre and their intersection and then completes the problem by subtracting their radius. Condone a sign slip subtracting their $-\frac{18}{5}$.

It is dependent on the previous method mark.

Alternatively, they solve simultaneously their $y = 2x - 6$ with the equation of the circle and then find the distance between this intersection point and the point of intersection between *l* and the normal. They must choose the smaller positive root of the solution to their quadratic.

Eg

$(x-5)^2 + (2x-6-4)^2 = 9 \Rightarrow 5x^2 - 50x + 125 = 9$

$x = \frac{25-3\sqrt{5}}{5}, \ y = \frac{20-6\sqrt{5}}{5}$

Distance between two points:

$$\sqrt{\left(\text{"}\frac{25-3\sqrt{5}}{5}\text{"} - \text{"}\frac{6}{5}\text{"}\right)^2 + \left(\text{"}\frac{20-6\sqrt{5}}{5}\text{"} + \text{"}\frac{18}{5}\text{"}\right)^2}$$

A1: Correct value e.g. $\sqrt{\frac{361}{5}} - 3$ or $\frac{19\sqrt{5}-15}{5}$). Also allow awrt 5.50

Isw after a correct answer is seen.

Alt (b) Be aware they may use vector methods:

B1M1: Attempts to find the perpendicular distance between their $(5,4)$ and $x + 2y + 6 = 0$ by substituting the values into the formula to find the distance between a point (x, y) and a line $ax + by + c = 0$

$$\Rightarrow \frac{|ax+by+c|}{\sqrt{a^2+b^2}} = \frac{|\text{"}5\text{"}\times\text{"}1\text{"}+\text{"}4\text{"}\times\text{"}2\text{"}+\text{"}6\text{"}|}{\sqrt{\text{"}1\text{"}^2+\text{"}2\text{"}^2}}$$

A1: $\frac{|5\times1+4\times2+6|}{\sqrt{1^2+2^2}} \left(= \frac{19}{\sqrt{5}}\right)$

dM1: Distance $= \text{"}\frac{19\sqrt{5}}{5}\text{"} - 3$

A1: $\frac{19\sqrt{5}-15}{5}$

Question	Scheme	Marks	AOs
12(a)	$V=\pi r^2 h=355 \Rightarrow h=\dfrac{355}{\pi r^2}$ $\left(\text{or } rh=\dfrac{355}{\pi r} \text{ or } \pi rh=\dfrac{355}{r}\right)$	B1	1.1b
	$C=0.04\left(\pi r^2+2\pi rh\right)+0.09\left(\pi r^2\right)$	M1	3.4
	$C=0.13\pi r^2+0.08\pi rh=0.13\pi r^2+0.08\pi r\left(\dfrac{355}{\pi r^2}\right)$	dM1	2.1
	$C=0.13\pi r^2+\dfrac{28.4}{r}$ *	A1*	1.1b
		(4)	
(b)	$\dfrac{\mathrm{d}C}{\mathrm{d}r}=0.26\pi r-\dfrac{28.4}{r^2}$	M1 A1	3.4 1.1b
	$\dfrac{\mathrm{d}C}{\mathrm{d}r}=0 \Rightarrow r^3=\dfrac{28.4}{0.26\pi} \Rightarrow r=...$	M1	1.1b
	$r=\sqrt[3]{\dfrac{1420}{13\pi}}=3.26...$	A1	1.1b
		(4)	
(c)	$\left(\dfrac{\mathrm{d}^2C}{\mathrm{d}r^2}=\right)\ 0.26\pi+\dfrac{56.8}{r^3}=0.26\pi+\dfrac{56.8}{"3.26"^3}$	M1	1.1b
	$\left(\dfrac{\mathrm{d}^2C}{\mathrm{d}r^2}=\right)\ (2.45..)>0$ Hence minimum (cost)	A1	2.4
		(2)	
(d)	$C=0.13\pi("3.26")^2+\dfrac{28.4}{"3.26"}$	M1	3.4
	$(C=)13$	A1	1.1b
		(2)	
		(12 marks)	

Notes

(a)

B1: Correct expression for h or rh or πrh in terms of r. This may be implied by their later substitution.

M1: Scored for the sum of the three terms of the form $0.04...r^2$, $0.09...r^2$ and $0.04\times...rh$

The $0.04\times...rh$ may be implied by eg $0.04\times...r\times\dfrac{355}{\pi r^2}$ if h has already been replaced

dM1: Substitutes h or rh or πrh into their equation for C which must be of an allowable form (see above) to obtain an equation connecting C and r.
It is dependent on a correct expression for h or rh or πrh in terms of r

A1*: Achieves given answer with no errors. Allow Cost instead of C but they cannot just have an expression.
As a minimum you must see
- the separate equation for volume
- the two costs for the top and bottom separate before combining
- a substitution before seeing the $\frac{28.4}{r}$ term

Eg $355 = \pi r^2 h$ and $C = 0.04\pi r^2 + 0.09\pi r^2 + 0.04 \times 2\pi rh = 0.13\pi r^2 + 0.08\pi \times \left(\frac{355}{\pi r}\right)$

(b)

M1: Differentiates to obtain at least $r^{-1} \rightarrow r^{-2}$

A1: Correct derivative.

M1: Sets $\frac{\mathrm{d}C}{\mathrm{d}r} = 0$ and solves for r. There must have been some attempt at differentiation of the equation for C ($...r^2 \rightarrow ...r$ or $...r^{-1} \rightarrow ...r^{-2}$) Do not be concerned with the mechanics of their rearrangement and do not withhold this mark if their solution for r is negative

A1: Correct value for r. Allow exact value or awrt 3.26

(c)

M1: Finds $\frac{\mathrm{d}^2C}{\mathrm{d}r^2}$ at their (positive) r or considers the sign of $\frac{\mathrm{d}^2C}{\mathrm{d}r^2}$.

This mark can be scored as long as their second derivative is of the form $A + \frac{B}{r^3}$ where A and B are non zero

A1: Requires
- A correct $\frac{\mathrm{d}^2C}{\mathrm{d}r^2}$
- Either
  - deduces $\frac{\mathrm{d}^2C}{\mathrm{d}r^2} > 0$ for $r > 0$ (without evaluating). There must be some minimal explanation as to why it is positive.
  - substitute their positive r into $\frac{\mathrm{d}^2C}{\mathrm{d}r^2}$ without evaluating and deduces $\frac{\mathrm{d}^2C}{\mathrm{d}r^2} > 0$ for $r > 0$
  - evaluate $\frac{\mathrm{d}^2C}{\mathrm{d}r^2}$ (which must be awrt 2.5) and deduces $\frac{\mathrm{d}^2C}{\mathrm{d}r^2} > 0$ for $r > 0$

(d)

M1: Uses the model and their positive r found in (b) to find the minimum cost. Their r embedded in the expression is sufficient. May be seen in (b) but must be used in (d).

A1: $(C =)\ 13$ ignore units

Question	Scheme		Marks	AOs
13(a)	$\frac{1}{\cos\theta}+\tan\theta=\frac{1+\sin\theta}{\cos\theta}$ or $\frac{(1+\sin\theta)\cos\theta}{\cos^2\theta}$		M1	1.1b
	$=\frac{1+\sin\theta}{\cos\theta}\times\frac{1-\sin\theta}{1-\sin\theta}=\frac{1-\sin^2\theta}{\cos\theta(1-\sin\theta)}=\frac{\cos^2\theta}{\cos\theta(1-\sin\theta)}$ or $\frac{(1+\sin\theta)\cos\theta}{\cos^2\theta}=\frac{(1+\sin\theta)\cos\theta}{1-\sin^2\theta}=\frac{(1+\sin\theta)\cos\theta}{(1+\sin\theta)(1-\sin\theta)}$		dM1	2.1
	$=\frac{\cos\theta}{1-\sin\theta}$ *		A1*	1.1b
			(3)	
(b)	$\frac{1}{\cos 2x}+\tan 2x=3\cos 2x$ $\Rightarrow 1+\sin 2x=3\cos^2 2x=3(1-\sin^2 2x)$	$\frac{\cos 2x}{1-\sin 2x}=3\cos 2x$ $\Rightarrow \cos 2x=3\cos 2x(1-\sin 2x)$	M1	2.1
	$\Rightarrow 3\sin^2 2x+\sin 2x-2=0$	$\Rightarrow \cos 2x(2-3\sin 2x)=0$	A1	1.1b
	$\sin 2x=\frac{2}{3},\ (-1)\Rightarrow 2x=...\Rightarrow x=...$		M1	1.1b
	$x=20.9°,\ 69.1°$		A1 A1	1.1b 1.1b
			(5)	
				(8 marks)

Notes

(a) If starting with the LHS: Condone if another variable for θ is used except for the final mark

M1: Combines terms with a common denominator. The numerator must be correct for their common denominator.

dM1: Either:

- $\frac{1+\sin\theta}{\cos\theta}$: Multiplies numerator and denominator by $1-\sin\theta$, uses the difference of two squares and applies $\cos^2\theta=1-\sin^2\theta$
- $\frac{(1+\sin\theta)\cos\theta}{\cos^2\theta}$: Uses $\cos^2\theta=1-\sin^2\theta$ on the denominator, applies the difference of two squares

It is dependent on the previous method mark.

A1*: Fully correct proof with correct notation and no errors in the main body of their work. Withhold this mark for writing eg sin instead of $\sin\theta$ anywhere in the solution and for eg $\sin\theta^2$ instead of $\sin^2\theta$

Alt(a) If starting with the RHS: Condone if another variable is used for θ except for the final mark

M1: Multiplies by $\frac{1+\sin\theta}{1+\sin\theta}$ leading to $\frac{\cos\theta(1+\sin\theta)}{1-\sin^2\theta}$ or

Multiplies by $\frac{\cos\theta}{\cos\theta}$ leading to $\frac{\cos^2\theta}{\cos\theta(1-\sin\theta)}$

dM1: Applies $\cos^2\theta = 1-\sin^2\theta$ and cancels the $\cos\theta$ factor from the numerator and denominator leading to $\frac{1+\sin\theta}{\cos\theta}$ or

Applies $\cos^2\theta = 1-\sin^2\theta$ and uses the difference of two squares leading to $\frac{(1+\sin\theta)(1-\sin\theta)}{\cos\theta(1-\sin\theta)}$

It is dependent on the previous method mark.

A1*: Fully correct proof with correct notation and no errors in the main body of their work. If they work from both the LHS and the RHS and meet in the middle with both sides the same then they need to conclude at the end by stating the original equation.

(b) ***Be aware that this can be done entirely on their calculator which is not acceptable***

M1: Either multiplies through by cos $2x$ and applies $\cos^2 2x = 1-\sin^2 2x$ to obtain an equation in sin $2x$ only or alternatively sets $\frac{\cos 2x}{1-\sin 2x} = 3\cos 2x$ and multiplies by $1-\sin 2x$

A1: Correct equation or equivalent. The = 0 may be implied by their later work (Condone notational slips in their working)

M1: Solves for sin $2x$, uses arcsin to obtain at least one value for $2x$ and divides by 2 to obtain at least one value for x. The roots of the quadratic can be found using a calculator. They cannot just write down values for x from their quadratic in sin$2x$

A1: For 1 of the required angles. Accept awrt 21 or awrt 69. Also accept awrt 0.36 rad or awrt 1.21 rad

A1: For both angles (awrt 20.9 and awrt 69.1) and no others inside the range. If they find $x = 45$ it must be rejected. (Condone notational slips in their working)

Question	Scheme	Marks	AOs
14(i)	The statement is **not true** because e.g. when $x = -4$, $x^2 = 16$ (which is > 9 but $x < 3$)	B1	2.3
		(1)	
(ii)	$n^3 + 3n^2 + 2n = n\left(n^2 + 3n + 2\right) = n\left(n+1\right)\left(n+2\right)$	M1	2.1
	$n(n + 1)(n + 2)$ is the product of 3 consecutive integers	A1	2.2a
	As $n(n + 1)(n + 2)$ is a multiple of 2 **and** a multiple of 3 it must be a multiple of 6 and so $n^3 + 3n^2 + 2n$ is divisible by 6 for all integers n	A1	2.4
		(3)	
			(4 marks)

Notes

(i)

B1: Identifies the error in the statement by giving

- a counter example and a reason eg $x = -4$ with $x^2 = 16$ eg $x = -4$ with $(-4)^2 > 9$
- concludes **not true**

There should be no errors seen including the use of brackets. The conclusion could be a preamble. Do not accept "sometimes true" or equivalent.

Alternatively, explains why the statement is **not true**

Eg. It is not true as when $x < -3$ then $x^2 > 9$ so x does not have to be greater than 3.

Eg. $x^2 > 9 \Rightarrow x < -3$ or $x > 3$ so not true

(ii)

M1: Takes out a factor of n and attempts to factorise the resulting quadratic.

A1: Deduces that the expression is the product of 3 consecutive integers

A1: Explains that as the expression is a multiple of 3 **and** 2, it must be a multiple of 6 and so is divisible by 6

If you see any method which appears to be credit worthy but is not covered by the scheme then send to review