Pearson
Edexcel

Mark Scheme (Results)

Summer 2022

Pearson Edexcel GCE
In A Level Further Mathematics (9FM0)
Paper 3A Further Pure Mathematics 1

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS
General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- □ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.

If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

Question	Scheme	Marks	AOs
1(a)	$b^2 = a^2(1-e_1^2) \Rightarrow 4 = 16(1-e_1^2) \Rightarrow e_1^2 = \ldots$	M1	1.1b
	$e_1^2 = \frac{3}{4}$ or $e_1 = \frac{\sqrt{3}}{2}$	A1	1.1b
	E.g. $b^2 = a^2(e_2^2 - 1) = a^2\left(\frac{1}{e_1^2} - 1\right) = a^2\left(\frac{4}{3} - 1\right)$	dM1	2.1
	$\Rightarrow b^2 = \frac{1}{3}a^2 \Rightarrow a^2 = 3b^2$ * cso	A1*	1.lb
		(4)	
(b)	For the focus of the ellipse $(x =)4 \times {'}\frac{\sqrt{3}}{2}{'}$	M1	1.1b
	For focus of the hyperbola (x =) $a \times {'}\frac{2}{\sqrt{3}}{'} \Rightarrow 2\sqrt{3} = \frac{2a}{\sqrt{3}} \Rightarrow a = \ldots (= 3)$ $\Rightarrow b^2 = \frac{1}{3}a^2 = \ldots$	M1	3.1a
	$\frac{x^2}{9} - \frac{y^2}{3} = 1$ cso	A1	2.2a
		(3)	
		(7 marks)	

Notes:

(a)

M1: Uses "$b^2 = a^2(1-e_1^2)$" with values for a and b to find a value for e_1 or e_1^2. They may just call it e and will likely use a and b before substituted, which is fine. The formula must be correct but allow slips with a and b.

A1: Correct exact value for e_1 or e_1^2. Note: allow M1A1 here if the relevant work is seen in (b).

dM1: Dependent on previous method mark. Uses $e_1 \times e_2 = 1$ with their e_1 or e_1^2 to find an expression between a and b. May find an expression for $e_2{}^{(2)}$ and apply $e_1 \times e_2 = 1$ directly or may first substitute as per scheme. Any full method.

SC: Allow M0A0dM1A0 if $b^2 = a^2(1-e_1)$ and $b^2 = a^2(e_2 - 1)$ are used in an otherwise correct process.

A1*: Achieves $a^2 = 3b^2$ with at least one intermediate unsimplified equation in a and b cso

(b)

M1: Uses/implies x coordinate of focus for the ellipse is $4 \times$ their e_1

M1: For a full process to find values for a and b or their squares. E.g. for focus of hyperbola $x = a \times$ their $e_2 = \frac{a}{e_1}$ sets equal to $4e_1$ and solves for a then attempting to use $a^2 = 3b^2$ to obtain b^2(or b). Other methods are possible.

A1: Deduces the correct equation for the hyperbola.

Question	Scheme	Marks	AOs
2(a)	$(H =)0.3\,sin\left(\frac{30}{60}\right) - 4\,cos\left(\frac{30}{60}\right) + 11.5 = 8.13$ {hours}*	B1*	3.4
		(1)	
(b)	Substitutes $sin\left(\frac{x}{60}\right) = \frac{2t}{1+t^2}$ and $cos\left(\frac{x}{60}\right) = \frac{1-t^2}{1+t^2}$ into H $$(H =)0.3\left(\frac{2t}{1+t^2}\right) - 4\left(\frac{1-t^2}{1+t^2}\right) + 11.5$$	M1	1.1b
	$$(H =)\frac{0.6t - 4 + 4t^2 + 11.5(1+t^2)}{1+t^2} = \frac{15.5t^2 + 0.6t + 7.5}{1+t^2}$$	A1	2.1
		(2)	
(c)	$$H = \frac{15.5t^2 + 0.6t + 7.5}{1+t^2} = 12 \Rightarrow 3.5t^2 + 0.6t - 4.5 = 0$$	M1	3.4
	$\Rightarrow t = \frac{-0.6 \pm \sqrt{0.6^2 - 4(3.5)(-4.5)}}{7}$ $=\ldots(1.051\ldots, -1.222\ldots) \Rightarrow x = 120\,tan^{-1}("1.051..)$ $=\ldots(97.254..)$	dM1	3.1b
	$x =$ awrt 97	A1	1.1b
	8th or 9th April	A1	3.2a
		(4)	
		(7 marks)	

Notes:

(a)

B1*: Uses $x = 30$ to show that $H = 8.13$. Accept $x = 30$ seen substituted followed by 8.13, or x =30 identified followed by 8.133… before rounding to 8.13.

(b)

M1: Uses the correct t-formulae $sin\left(\frac{x}{60}\right) = \frac{2t}{1+t^2}$ and $cos\left(\frac{x}{60}\right) = \frac{1-t^2}{1+t^2}$, attempts to substitute into H.

A1: Fully correct method, expresses as a single fraction with a denominator of $1 + t^2$ to achieve $H = \frac{15.5t^2+0.6t+7.5}{1+t^2}$ (oe with fractions or accept values for *a, b* and c stated).

(c)

M1: Sets H = 12 (or any inequality in between) and rearranges to form a a quadratic equation for t.

dM1: Dependent on the previous method mark. Solves the quadratic by any means (accept one correct answer for their quadratic if no method shown) and uses this to find a value for x.

A1: Correct value for $x =$ awrt 97 or accept 98 following a correct value for t.

A1: Correct day of the year. Accept 8th or 9th April following awrt 97 from a correct method.

Note: Question says hence, so answers by graphical methods or trial and improvement are not acceptable for full credit. They can score a SC M0dM0A0B1 for achieving a correct date.

Question	Scheme	Marks	AOs
3 (a)	$(\lvert OB\rvert =)\sqrt{4^2+(2p)^2+1^2}\left(=\sqrt{17+4p^2}\right)$	B1	1.1b
	$\cos 45 = \dfrac{4}{\sqrt{17+4p^2}} \Rightarrow p = \ldots$	M1	3.1a
	$p = \pm\dfrac{\sqrt{15}}{2}$	A1	1.1b
		(3)	
(b)	$\overrightarrow{OA}\times\overrightarrow{OB} = \begin{pmatrix}2+2p\\-6\\4p-8\end{pmatrix}$	B1	1.1b
	E.g. Sets $\begin{pmatrix}2+2p\\-6\\4p-8\end{pmatrix} = \begin{pmatrix}4\lambda\\-p\lambda\\2\lambda\end{pmatrix}$ and solves to find a value for p	M1	3.1a
	$p = 3$ only	A1	2.2a
		(3)	
(c)	$\frac{1}{2}\lvert\overrightarrow{OA}\times\overrightarrow{OB}\rvert = 3\sqrt{2} \Rightarrow (2+2p)^2+(-6)^2+(4p-8)^2 = \left(6\sqrt{2}\right)^2$	M1	3.1a
	Solves a 3TQ to find a value for p $20p^2-56p+32=0 \Rightarrow p = \ldots$	dM1	1.1b
	$p = 2,\ \dfrac{4}{5}$	A1	1.1b
		(3)	
		(9 marks)	

Notes:

(a)

B1: Correct expression for the magnitude for $\overrightarrow{OB}$ (may be seen in formula)

M1: A complete method to find a value for p. E,g, Sets cos 45 = 4/their magnitude of $\overrightarrow{OB}$ and solves to find a value for p. Note this may arise from attempts using dot products, but the same equation is reached and a full method to find p is required.

A1: $p = \pm\frac{\sqrt{15}}{2}$

(b)

B1: Correct vector product, allow if seen anywhere in the question. Alternatively, if method 2 below is used, the cross product is not necessary, and this mark may be awarded for a correct equation in p from either dot product.

M1: A complete method to find a value for p. E.g.

- Set the vector product equal to a multiple of the parallel vector and solves to find a value for p,

- Attempts dot products of both $\overrightarrow{OA}$ **and** $\overrightarrow{OB}$ with $\begin{pmatrix} 4 \\ -p \\ 2 \end{pmatrix}$, solves and finds an answer from both,
- Finds cross product of $\overrightarrow{OA} \times \overrightarrow{OB}$ with $\begin{pmatrix} 4 \\ -p \\ 2 \end{pmatrix}$ and sets at least one coefficient to zero to find p.

A1cso: A complete argument leading to $p = 3$ only, which must be consistent with their work. Where a method leads to more than one value for p the candidate will need to check which values hold and give the answer $p = 3$ only. No method needs be seen for this but other values must be rejected.

(c)
M1: A complete method to set up a polynomial in p. E.g. sets half magnitude of their vector product $= 3\sqrt{2}$ and reaches a quadratic expression in p. An alternative approach is:

$$\cos \quad \angle AOB = \frac{2 \times 4 + 2 \times 2p - 1 \times 1}{\sqrt{4+4+1}\sqrt{16+4p^2+1}} = \frac{7+4p}{3\sqrt{17+4p^2}}$$

$$\Rightarrow 3\sqrt{2} = \frac{1}{2} OA.OB \sin \angle AOB = \frac{1}{2} 3\sqrt{17+4p^2}\sqrt{1 - \frac{(7+4p)^2}{9(17+4p^2)}}$$

$$\Rightarrow 72 = 9(17+4p^2) - (49 + 56p + 16p^2)$$

dM1: Dependent on the previous method mark. Solve a 3TQ to find a value for p.

A1: $p = 2, \frac{4}{5}$ only. Note: allow this mark if their $\overrightarrow{OA} \times \overrightarrow{OB}$ was correct apart from the **j** component sign.

Question	Scheme	Marks	AOs
4(a)	Identifies $t_0 = 0,\ v_0 = 0,\ \left(\frac{dv}{dt}\right)_0 = 10$ and $h = 0.5$	B1	3.4
	$v_1 = v_0 + h\left(\frac{dv}{dt}\right)_0 \Rightarrow v_1 = 0 + 0.5 \times 10 = \ldots$	M1	1.1b
	$v_1 = 5$	A1	1.1b
	$\left(\frac{dv}{dt}\right)_1 = -0.1(5)^2 + 10 = \ldots\{7.5\}$ $v_2 = v_1 + h\left(\frac{dv}{dt}\right)_1 \Rightarrow v_2 = 5 + 0.5 \times 7.5 = \ldots$	M1	3.4
	$v_2 = 8.75$ so 8.75 ms^{-1}	A1	1.1b
		(5)	
(b)	$\frac{dv}{dt} = -0.1v^2 + A$ where $0 < A < 10$	B1	3.5c
		(1)	
		(6 marks)	

Notes:

(a)

B1: Uses the model to identify the correct initial conditions and requirements for h. May be implied by use in the equation.

M1: Applies the approximation formula with their values for v_0, $\left(\frac{dv}{dt}\right)_0$ and h to find a value for v_1

A1: $v_1 = 5$

M1: Uses their v_1 to find a value for $\left(\frac{dv}{dt}\right)_1$ and applies the approximation formula with their values for v_1, $\left(\frac{dv}{dt}\right)_1$ and h to find a value for v_2

A1: $v_2 = 8.75$ or 8.75 ms^{-1}

(b)

B1: Reduce the value of 10 or explains this is what needs **reducing**, but do not accept 0 or negative values in place of the 10. Note: “change the 10” is B0 if it does not explain how to change it.

Question	Scheme	Marks	AOs
5 (a)	$y+x\frac{dy}{dx}=0 \Rightarrow \frac{dy}{dx}=-\frac{y}{x}=\frac{-\frac{6}{t}}{6t}=-\frac{1}{t^2}$ or $y=\frac{36}{x} \Rightarrow \frac{dy}{dx}=-\frac{36}{x^2}=-\frac{36}{(6t)^2}=-\frac{1}{t^2}$ or $\frac{dy}{dx}=\frac{dy}{dt}\div\frac{dx}{dt}=\frac{-6t^{-2}}{6}=-\frac{1}{t^2}$	M1	1.1b
	$y-\frac{6}{t}=\text{"}-\frac{1}{t^2}\text{"}(x-6t)$	M1	1.1b
	$yt^2+x=12t*$	A1 *	2.1
		(3)	
(b)	$\frac{dy}{dx}=-\frac{y}{x}=\frac{-\frac{3}{t}}{12t}=-\frac{1}{4t^2}$ and $y-\frac{3}{t}=\text{'}-\frac{1}{4t^2}\text{'}(x-12t)$	M1	1.1b
	$y-\frac{3}{t}=-\frac{1}{4t^2}(x-12t)$ o.e such as $4yt^2+x=24t$	A1	1.1b
		(2)	
(c)	E.g. $\left.\begin{matrix}4yt^2+x=24t\\ yt^2+x=12t\end{matrix}\right\\}3yt^2=12t \Rightarrow y=\ldots$ and $x=12t-yt^2=\ldots$	M1	2.1
	$x=8t$ and $y=\frac{4}{t}$	A1	1.1b
	$xy=\ldots$	dM1	1.1b
	$xy=32$ hence **rectangular hyperbola**	A1	2.4
		(4)	
		(9 marks)	

Notes:

(a)

M1: Differentiates implicitly, directly or parametrically to find the gradient at the point *P* in terms of *t*. Allow slips in coefficients, as long as method is clear.

M1: Finds the equation of the tangent at the point *P* using their gradient (not reciprocal etc). If using $y = mx + c$ must proceed to find *c* and substitute back in to equation.

A1*: The correct equation for the tangent at the point *P* from correct working.

(b)

M1: Finds the new gradient (any method as above) and proceeds to find the equation of the tangent at the point *Q*. Alternatively replaces *t* by 2*t* in the answer to (a).

A1: Correct equation - any form, need not be simplified and isw after a correct equation.

(c)

M1: Solves their simultaneous equations to find both the *x* and *y* coordinate for the point *R*.

A1: Correct point of intersection, it does not need to be simplified.

dM1: Dependent on the first method mark. Multiplies *x* by *y* to reach a constant.

A1: Shows that $xy = 32$ and hence **rectangular hyperbola**

Question	Scheme	Marks	AOs
6(a)	Finds any two vectors $\pm\overrightarrow{PQ},\ \pm\overrightarrow{PR}$ or $\pm\overrightarrow{QR}$ $\pm\begin{pmatrix}2\\3\\-9\end{pmatrix}$ or $\pm\begin{pmatrix}1\\2\\-1\end{pmatrix}$ or $\pm\begin{pmatrix}-1\\-1\\8\end{pmatrix}$	M1	1.1b
	A correct equation for the plane $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$ $\mathbf{a} = \begin{pmatrix}1\\-2\\4\end{pmatrix}$ or $\begin{pmatrix}3\\1\\-5\end{pmatrix}$ or $\begin{pmatrix}2\\0\\3\end{pmatrix}$ **b** and **c** are any two vectors from $\pm\begin{pmatrix}2\\3\\-9\end{pmatrix}$ or $\pm\begin{pmatrix}1\\2\\-1\end{pmatrix}$ or $\pm\begin{pmatrix}-1\\-1\\8\end{pmatrix}$	A1	1.1b
		(2)	
(b)	Forms two simultaneous equations by setting $y = 0$ and $z = 0$ e.g. $-2 + 3\lambda + 2\mu = 0$ $4 - 9\lambda - \mu = 0$	M1	3.1a
	Solves their simultaneous equations to find a value for μ and a value for λ e.g. $\left.\begin{matrix}-2 + 3\lambda + 2\mu = 0\\ 4 - 9\lambda - \mu = 0\end{matrix}\right\} \Rightarrow \lambda = 0.4,\ \mu = 0.4$	dM1	1.1b
	Uses their values of μ and λ to find the x coordinate $x = 1 + 2\lambda + \mu = 1 + 2(0.4) + (0.4) = \ldots$	ddM1	1.1b
	$(2.2,\ 0,\ 0)$	A1	1.1b
		(4)	
	Alternative $\begin{vmatrix}2 & 3 & -9\\1 & 2 & -1\end{vmatrix} = (-3+18)\mathbf{i} - (-2+9)\mathbf{j} + (4-3)\mathbf{k}$	M1	3.1a
	$\begin{pmatrix}15\\-7\\1\end{pmatrix}.\begin{pmatrix}1\\-2\\4\end{pmatrix} = 15 + 14 + 4 = 33$ leading to $15x - 7y + z = 33$	dM1	1.1b
	$15x - 7(0) + (0) = 33 \Rightarrow x = \ldots$	ddM1	1.1b
	$(2.2,\ 0,\ 0)$	A1	1.1b
		(4)	
			(6 marks)
Notes:			
Accept alternative vector forms throughout. **(a)**			

M1: Finds any two vectors $\pm\overrightarrow{PQ}$, $\pm\overrightarrow{PR}$or $\pm\overrightarrow{QR}$ by subtracting relevant vectors. Two out of three values correct is sufficient to imply the correct method **A1:** Any correct equation for the plane. Must start with **r** =…
(b) **M1:** Uses their equation for the plane to form two simultaneous equations by setting $y = 0$ and $z = 0$ **dM1:** Dependent on the previous method mark. Solves their simultaneous equations from the y and z coordinates to find a value for $\mu$and a value for λ **ddM1:** Depends on both method marks. Uses their value for $\mu$and their value for λ to find the x coordinate **A1:** Correct coordinates. Accept as a column vector or listed separately ($y = 0$ and $z = 0$ may be implied). Accept equivalent fractions, e.g. $\left(\frac{11}{5}, 0{,}0\right)$ or $\left(\frac{33}{15}, 0{,}0\right)$ **Alternative** **M1:** Finds the cross product of the vectors **b** and **c** for their plane. Allow one slip in expansion. **dM1:** Finds the Cartesian equation of the plane **ddM1:** Depends on both previous method marks. Sets$y = 0$and $z = 0$ to find the x coordinate. **A1:** Correct coordinates. Accept as a column vector or listed separately ($y = 0$ and $z = 0$ may be implied). Accept equivalent fractions, e.g. $\left(\frac{11}{5}, 0{,}0\right)$ or $\left(\frac{33}{15}, 0{,}0\right)$

Question	Scheme	Marks	AOs
7(a)	Considers $x^2 - 8 = -(mx + c) \Rightarrow x^2 + mx - 8 + c = 0$ and sets the discriminant = 0 $\{m^2 - 4(-8 + c) = 0\}$	M1	3.1a
	$m^2 - 4c + 32 = 0$ *	A1*	2.1
		(2)	
(b)	$c = 3m \Rightarrow m^2 - 4[3m] + 32 = 0 \Rightarrow m = \ldots (4,8)$ or $\Rightarrow \left(\frac{c}{3}\right)^2 - 4c + 32 = 0 \Rightarrow c = \ldots (12,24)$	M1	3.1a
	$m = "4" \Rightarrow c = \ldots$ or $c = "12" \Rightarrow m = \ldots$	M1	1.1b
	Deduces that $m = 4$ and $c = 12$ and no other values for *m* and *c*	A1	2.2a
		(3)	
(c)	Solves $x^2 - 8 = 'm'x + 'c'$ and $x^2 - 8 = -('m'x + 'c')$ $x^2 - 8 = 4x + 12$ and $x^2 - 8 = -(4x + 12)$	M1	2.1
	$x = 2 \pm \sqrt{24}$ o.e. and $x = -2$ (follow through $m = 8, c = 24 \Rightarrow x = 4 \pm 4\sqrt{3}, x = -4$)	A1ft	1.1b
	$x \leqslant 2 - 2\sqrt{6}, x \geqslant 2 + 2\sqrt{6}, x = -2$ (oe notation)	A1	2.2a
		(3)	
		(8 marks)	

Notes:

(a) If both case are attempted mark for the correct one.
M1: Considers $x^2 - 8 = -(mx + c)$ collects terms, finds the discriminant and sets = 0. Must see a correct equation (without modulus) initially, though allow if subsequent slips rearranging occur.
A1*: Correct result with no incorrect working seen.

(b)
M1: Substitutes $c = 3m$ into the equation (or their equation as long as it came from an attempt at using the correct equation in (a)) and solves the resulting 3TQ to find a value for *m* or *c*.
M1: Finds the corresponding value of *c* (or *m*) or solved the other 3TQ to get values for *c* (or *m*)
A1: Deduces the correct values for *m* and *c*. If two sets of values are stated this mark is not achieved until the extra set $m = 8$ and $c = 24$are rejected (correct reason for rejection is not needed).

(c)
M1: Correct method to find all the critical values (so solves both equations). Allow if both cases are included and more than three critical values are found. Allow if relevant work was seen in (b).
A1ft: Correct three critical values **only**. May be implied by their final answer. **Follow through on *m* = 8 and *c* = 24 only.** Allow if the −2 was seen in (b).
A1cao: Deduces the correct region. Accept any correct notation. Accept with "and" or "or", but not with $\wedge$

Question	Scheme	Marks	AOs
8(i) (a)	$f(x) = ln\,x \qquad \Rightarrow f(1) = 0$ $f'(x) = \dfrac{1}{x} \qquad \Rightarrow f'(1) = 1$ $f^{'' \frac{1}{x^2} f''}$	M1 A1	1.1b 1.1b
	$(ln\,x) = (0 +)(x-1) - \frac{1}{2}(x-1)^2 + \ldots$	M1 A1	2.5 1.1b
		(4)	
(i) (b)	$\lim\limits_{x \to 1}\left(\dfrac{ln\,x}{x-1}\right) = \lim\limits_{x \to 1}\left(\dfrac{(x-1) - \frac{1}{2}(x-1)^2 + \ldots}{x-1}\right)$ $= \lim\limits_{x \to 1}\left(1 - \dfrac{1}{2}(x-1) + \ldots\right)$	M1	2.1
	$= \lim\limits_{x \to 1}\left(1 - \frac{1}{2}(x-1) + \ldots\right) = 1^*$ cso	A1*	2.2a
		(2)	
(ii)	Writes as an indeterminate form For example $\frac{sin(2x)}{(x+3)\,tan(6x)}$ or $\frac{sin(2x)\,cos(6x)}{(x+3)\,sin(6x)}$	M1	3.1a
	Differentiates numerator and denominator using appropriate rules $\frac{2\,cos(2x)}{tan(6x) + 6(x+3)\,sec^2(6x)}$ or $\frac{2\,cos(2x)\,cos(6x) - 6\,sin(2x)\,sin(6x)}{sin(6x) + 6(x+3)\,cos(6x)}$	M1 A1	1.1b 1.1b
	$\lim\limits_{x \to 0}\left(\frac{1}{(x+3)\,tan(6x)\text{cosec}(2x)}\right) = \frac{2}{18} = \frac{1}{9}$ o.e	A1cso	2.2a
		(4)	
			(10 marks)

Notes:

(i) (a) Notes: ignore extra terms throughout.

M1: Differentiates $f(x) = ln\,x$ twice and finds $f(1)$, $f'(1)$ and $f''(1)$

A1: Correct differentiation and values for $f(1)$, $f'(1)$ and $f''(1)$.

M1: Uses correct mathematical notation to find the Taylor series for $ln\,x$ in powers of $(x-1)$ up to $(x-1)^2$

A1: Correct expansion with simplified coefficients. Do not be concerned with the left hand side.

(i) (b) Question says "hence" so the result of (a) must be used. No marks for l'Hosptial's rule on the original functions (send to review if attempted with their part (a)).

M1: Substitutes their Taylor series for $ln\,x$ in powers of $(x-1)$ up to $(x-1)^2$ into the limit and cancels a factor $(x-1)$ from each term. Allow for the cancelling seeing a relevant strikethrough in all $x-1$ terms.

A1*: $\lim\limits_{x \to 1}\left(\frac{ln\,x}{x-1}\right) = \lim\limits_{x \to 1}\left(1 - \frac{1}{2}(x-1) + \ldots\right) = 1$ cso Must have come from a correct expansion. Must see the $1 - \frac{1}{2}(x-1)$

(ii)

M1: Writes the fraction in an indeterminate form $\frac{f(x)}{g(x)}$ where $\frac{f(0)}{g(0)} = \frac{0}{0}$ or $\frac{f(0)}{g(0)} = \frac{"\infty"}{"\infty"}$

M1: Differentiates numerator and denominator using appropriate rules, ie product rule for a product etc. Allow slips in coefficients but the form should be correct. This mark is available as long as written as $\frac{f(x)}{g(x)}$ even if not an indeterminate form. May be seen written as separate from the fraction, ie f'(x) = … and g'(x) = …

A1: Depend on both Ms. Correct differentiation for derivatives that lead to a limit. Must have a derivative for which g'(0) ≠ 0 and is finite.

A1cso: Deduces the correct limit from fully correct work.

Question	Scheme	Marks	AOs
9(a)	Use of $x = ty$ to give $\frac{dx}{dt} = y + t\frac{dy}{dt}$ or $y = \frac{x}{t} \to \frac{dy}{dt} = -\frac{x}{t^2} + \frac{1}{t}\frac{dx}{dt}$ oe	B1	1.1b
	$\frac{d^2x}{dt^2} = \frac{dy}{dt} + \frac{dy}{dt} + t\frac{d^2y}{dt^2}$ or $\frac{d^2y}{dt^2} = -\frac{1}{t^2}\frac{dx}{dt} + \frac{2x}{t^3} + \frac{1}{t}\frac{d^2x}{dt^2} - \frac{1}{t^2}\frac{dx}{dt}$ oe	M1 A1	2.1 1.1b
	$t^2\left[\text{their } \frac{d^2x}{dt^2}\right] - 2t\left[\text{their } \frac{dx}{dt}\right] + 2[ty] + 16t^2[ty] = 4t^3 \sin 2t$ Or $\left[\text{their } \frac{d^2y}{dt^2}\right] + 16\frac{x}{t} = 4 \sin 2t$	dM1	2.1
	$t^2\left[\frac{dy}{dt} + \frac{dy}{dt} + t\frac{d^2y}{dt^2}\right] - 2t\left[y + t\frac{dy}{dt}\right] + 2[ty] + 16t^2[ty] =$ $4t^3 \sin 2t$ $t^3\frac{d^2y}{dt^2} + 16t^3y = 4t^3 \sin 2t \Rightarrow \frac{d^2y}{dt^2} + 16y = 4 \sin 2t$ * (oe in reverse)	A1	1.1b
		(5)	
(b)	Solves $m^2 + 16 = 0$ to give $m = \ldots$	M1	1.1b
	$(y =) A \cos 4t + B \sin 4t$	A1	1.1b
	Particular integral $(y =) \underline{\lambda \sin 2t} + \mu \cos 2t$	B1	2.2a
	$\frac{dy}{dt} = 2\lambda \cos 2t - 2\mu \sin 2t$ and $\frac{d^2y}{dt^2} = -4\lambda \sin 2t - 4\mu \cos 2t$	M1	1.1b
	Substitutes into the differential equation and finds values for λ and μ $[-4\lambda \sin 2t - 4\mu \cos 2t] + 16[\lambda \sin 2t + \mu \cos 2t] = 4 \sin 2t$ $\Rightarrow \lambda = \ldots,\ \mu = \ldots$	dM1	2.1
	$y = \text{"}A \cos 4t + B \sin 4t\text{"} + \frac{1}{3} \sin 2t$	A1ft	1.1b
	$x = t[\text{their } y]$	M1	3.4
	$x = t\left[A \cos 4t + B \sin 4t + \frac{1}{3} \sin 2t\right]$	A1	2.2a
		(8)	
		(13 marks)	

Notes:

(a)

B1: For a correct suitable first derivative expression linking $\frac{dy}{dt}$ and $\frac{dx}{dt}$.

M1: Uses the product rule to find an equation linking second derivatives from their first derivative expression.

A1: A correct second derivative expression.

dM1: Substitutes the first and second derivatives and replaces x with $ty$to obtain a differential equation in y and t only. Alternatively, may go in reverse and replace y with $\frac{x}{t}$ etc in the second equation to obtain a differential equation in x and t only.

A1*: Simplifies their expression with a correct intermediate stage/working to reach the printed answer. Alternatively, correct working in the other direction to achieve equation (I) from the final equation.
(b) **M1:** Forms the correct auxiliary equation and attempts to solve (any values after the correct AE seen) **A1:** Correct complementary function. Accept for this mark if they give it in terms of *x* - you are looking for the correct form for the CF. **B1:** Deduces a correct form of the particular integral (must include at least $\lambda \sin 2t$ but may be no more than this). SC if by error the CF includes sin 2*t* allow B1 for a PI of form $\lambda t \sin 2t + \mu t \cos 2t$ **M1:** Differentiates the PI twice. **dM1:** Dependent on the previous method mark. Substitutes *y* and $\frac{d^2y}{dt^2}$ into the differential equation leading to values for the constant(s). **A1ft:** Correct general equation for *y* following through their CF, which must be a (non-constant) function of *t*. Must be in terms of *t* and start *y* = … **M1:** Links the solution to the solution of the model equation to find the general solution for the displacement. **A1:** Deduces the correct general solution for the displacement. Must be *x* =…

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