Mark Scheme (Results)

Summer 2022

Pearson Edexcel GCE
In A Level Further Mathematics (9FM0)
Paper 4A Further Pure Mathematics 2

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS
General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol √ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⎾ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.

If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

Question	Scheme	Marks	AOs
1(a)	$\{e =\}\begin{pmatrix}1 & 2 & 3 & 4\\1 & 2 & 3 & 4\end{pmatrix}$	B1	1.1b
		(1)	
(b)	$\begin{pmatrix}1 & 2 & 3 & 4\\4 & 3 & 1 & 2\end{pmatrix}$	B1	1.1b
		(1)	
(c)	Demonstrates that, for example: $[a \circ b] \circ c = \left[\begin{pmatrix}1 & 2 & 3 & 4\\3 & 4 & 2 & 1\end{pmatrix} \circ \begin{pmatrix}1 & 2 & 3 & 4\\2 & 4 & 3 & 1\end{pmatrix}\right] \circ \begin{pmatrix}1 & 2 & 3 & 4\\4 & 1 & 2 & 3\end{pmatrix}$ $= \begin{pmatrix}1 & 2 & 3 & 4\\4 & 1 & 2 & 3\end{pmatrix} \circ \begin{pmatrix}1 & 2 & 3 & 4\\4 & 1 & 2 & 3\end{pmatrix} = \begin{pmatrix}1 & 2 & 3 & 4\\3 & 4 & 1 & 2\end{pmatrix}$ $a \circ [b \circ c] = \begin{pmatrix}1 & 2 & 3 & 4\\3 & 4 & 2 & 1\end{pmatrix} \circ \left[\begin{pmatrix}1 & 2 & 3 & 4\\2 & 4 & 3 & 1\end{pmatrix} \circ \begin{pmatrix}1 & 2 & 3 & 4\\4 & 1 & 2 & 3\end{pmatrix}\right]$ $= \begin{pmatrix}1 & 2 & 3 & 4\\3 & 4 & 2 & 1\end{pmatrix} \circ \begin{pmatrix}1 & 2 & 3 & 4\\1 & 2 & 4 & 3\end{pmatrix} = \begin{pmatrix}1 & 2 & 3 & 4\\3 & 4 & 1 & 2\end{pmatrix}$	M1	2.1
	So $[a \circ b] \circ c = a \circ [b \circ c]$ or associative	A1	2.4
		(2)	
(d)	The order of the group is 24 or 4!	B1	1.1b
	4 is a factor of 24 or **4/24** therefore **it is possible** for a subgroup to have order 4.	B1ft	2.4
		(2)	
		(6 marks)	

Notes:

(a)
B1: See scheme

(b)
B1: See scheme

(c)
M1: Shows two calculations in an attempt to show associative, e,g, $[a \circ b] \circ c$and $a \circ [b \circ c]$. There must be an intermediate line of working with evidence of using the permutations. Condone the wrong order for this mark.
A1: Correct calculations leading to $[a \circ b] \circ c = a \circ [b \circ c]$ or states associative

Note Incorrect order scores M1 A0

$$[a \circ b] \circ c = \left[\begin{pmatrix}1 & 2 & 3 & 4\\3 & 4 & 2 & 1\end{pmatrix} \circ \begin{pmatrix}1 & 2 & 3 & 4\\2 & 4 & 3 & 1\end{pmatrix}\right] \circ \begin{pmatrix}1 & 2 & 3 & 4\\4 & 1 & 2 & 3\end{pmatrix}$$
$$= \begin{pmatrix}1 & 2 & 3 & 4\\3 & 1 & 4 & 2\end{pmatrix} \circ \begin{pmatrix}1 & 2 & 3 & 4\\4 & 1 & 2 & 3\end{pmatrix} = \begin{pmatrix}1 & 2 & 3 & 4\\2 & 4 & 3 & 1\end{pmatrix}$$

$= \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 2 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 1 \end{pmatrix}$
(d) **B1:** Order is 24 or 4! **B1ft:** Follow through on their order of the group, draws the correct conclusion

Question	Scheme	Marks	AOs
2(a)	$\begin{vmatrix}1-\lambda & 0 & a\\ -3 & b-\lambda & 1\\ 0 & 1 & a-\lambda\end{vmatrix}$ $= (1-\lambda)[(b-\lambda)(a-\lambda)-1]+a(-3)(=0)$	M1	1.1b
	$\lambda^3-(a+b+1)\lambda^2+(a+b+ab-1)\lambda+(3a+1-ab)(=0)$ o.e. $-\lambda^3+(a+b+1)\lambda^2-(a+b+ab-1)\lambda+(ab-3a-1)(=0)$ o.e.	A1	1.1b
	$\lambda^2 \Rightarrow\ a+b+1=7$ and $\lambda \Rightarrow\ a+b+ab-1=13$ Solves simultaneously e.g. $a+b=6, ab=8$ For example: leading to $a^2-6a+8=0 \Rightarrow a=\dots$	M1	3.1a
	$a=2, b=4$	A1	1.1b
	$c=-1$	A1	2.2a
		(5)	
(b)	$\mathbf{M}^3-7\mathbf{M}^2+13\mathbf{M}+$ 'their c' $\mathbf{I}=0$	B1ft	1.1b
	$I=M^3-7M^2+13M \Rightarrow M^{-1}=M^2-7M+13I$ $\Rightarrow M^{-1}=\begin{pmatrix}1&0&2\\-3&4&1\\0&1&2\end{pmatrix}^2-7\begin{pmatrix}1&0&2\\-3&4&1\\0&1&2\end{pmatrix}+\begin{pmatrix}13&0&0\\0&13&0\\0&0&13\end{pmatrix}=\dots$ $=\begin{pmatrix}1&2&6\\-15&17&0\\-3&6&5\end{pmatrix}-7\begin{pmatrix}1&0&2\\-3&4&1\\0&1&2\end{pmatrix}+\begin{pmatrix}13&0&0\\0&13&0\\0&0&13\end{pmatrix}=\dots$	M1	3.1a
	$M^{-1}=\begin{pmatrix}7&2&-8\\6&2&-7\\-3&-1&4\end{pmatrix}$	A1	1.1b
		(3)	
		(8 marks)	

Notes:

(a)

M1: Correct method to find the characteristic equation for **M**, condone missing = 0, and one slip as long as the intention is clear

A1: Multiplies out to achieve a correct characteristic equation, condone missing = 0

M1: A complete method to find the values of the constants a or b. Equates their coefficients for $\lambda^2$and λ and solves simultaneously to find values for a or b.

A1: Deduces the correct values for a and b. $(a<b)$ following correct simultaneous equations

A1: Deduces the correct value for c.

(b)

B1ft: Uses Cayley-Hamilton theorem to produce equation replacing λ with **M** and constant term with constant multiple of the identity matrix **I**. Follow through on their value for c. This mark may be implied by the M mark.

M1: A complete method to find M^{-1}using the Cayley-Hamilton theorem. The minimum is for writing an expression for M^{-1} from their characteristic equation, for example $M^{-1} = M^2 - 7M + 13I$ and then stating an answer for M^{-1}, they may have used their calculator, there is no need to check.
A1: Correct M^{-1}

Question	Scheme	Marks	AOs
3 (a)	Implies moves from A • Has to hop to different lily pad to A • Will not hop on the same pad • They can't stay on lily pad A • Goes to B or C but not A • Can't be where it started • Can't stay on A	B1	2.4
		(1)	
(b)	$p_1 = \frac{2}{3}\left(-\frac{1}{2}\right)^1 + \left(\frac{1}{3}\right) = 0$ Minimum needed $p_1 = \frac{2}{3}\left(-\frac{1}{2}\right) + \left(\frac{1}{3}\right) = 0$	B1	2.1
	Assume result is true for $n = k$ Or $p_k = \frac{2}{3}\left(-\frac{1}{2}\right)^k + \left(\frac{1}{3}\right)$	M1	2.4
	Finds an expression for $p_{k+1} = \frac{1}{2}\left(1 - \left[\frac{2}{3}\left(-\frac{1}{2}\right)^k + \left(\frac{1}{3}\right)\right]\right)$	M1	1.1b
	$p_{k+1} = \frac{1}{3} + \frac{2}{3}\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right)^k$ or $\frac{1}{2} - \frac{1}{6} + \frac{2}{3}\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right)^k$ $p_{k+1} = \frac{1}{3} - \frac{1}{3} \times -2\left(-\frac{1}{2}\right)^{k+1}$ or $\frac{1}{2} - \frac{1}{6} - \frac{1}{3} \times -2\left(-\frac{1}{2}\right)^{k+1}$	M1	2.1
	$p_{k+1} = \frac{2}{3}\left(-\frac{1}{2}\right)^{k+1} + \left(\frac{1}{3}\right)$	A1	1.1b
	<u>If true for *n* = *k* then it is true for *n* = *k* + 1</u> and as it is <u>true for *n* = 1</u>, the statement is <u>true for all *n*</u>. (Allow 'for all values')	A1	2.2a
		(6)	
(c)	As $n \to \infty, \left(-\frac{1}{2}\right)^n \to 0 \therefore p_n \to \frac{1}{3}$	B1	3.4
		(1)	
		(8 marks)	

Notes:

(a)

B1: Explains has to move from A

(b)

B1: Shows the statement is true for *n* = 1. Needs to show that $p_1 = 0$and conclusion true for *n* = 1, this statement can be recovered in their conclusion if says e.g. true for *n* = 1

M1: Makes a statement that assumes the result is true for *n* = *k*. Assume (true for) $n = k$ is sufficient. This mark may be recovered in their conclusion if they say e.g. if true for $n = k$ then …etc

M1: Finds an expression forp_{k+1} using the recurrence relation and substitutes in for p_k

M1: Shows a correct intermediate stage $p_{k+1} = \frac{1}{3} + \frac{2}{3}\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right)^k$

Condone $\frac{1}{3} - \frac{2}{3}\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)^k$

A1: $p_{k+1} = \frac{2}{3}\left(-\frac{1}{2}\right)^{k+1} + \left(\frac{1}{3}\right)$ cso

A1: Correct complete conclusion. This mark is dependent on previous four marks, the B mark is not required. It is gained by conveying the ideas of **all** underlined points either at the end of their solution **or** as a narrative in their solution.

Note Using $p_{k+1} = \frac{1}{2} - \frac{1}{2}p_k = \frac{1}{2} - \frac{1}{2}\left[\frac{2}{3}\left(-\frac{1}{2}\right)^k + \left(\frac{1}{3}\right)\right] = \frac{1}{2} + \frac{2}{3}\left(-\frac{1}{2}\right)^{k+1} - \left(\frac{1}{6}\right)$ scores 2nd M1 3rd M1

(c)

B1: see scheme

Question	Scheme		Marks	AOs
4(a)	$124 = 17 \times 7 + 5$ $17 = 3 \times 5 + 2$ $5 = 2 \times 2 + 1$ $\{2 = 2 \times 1\}$		M1	2.1
	Since the **gcd is 1**, 124 and 17 are **relatively prime (coprime)**		A1	2.4
			(2)	
(b)	$1 = 5 - 2 \times 2$ $1 = 5 - 2(17 - 3 \times 5) \Rightarrow 1 = 7 \times 5 - 2 \times 17$ $1 = 7(124 - 17 \times 7) - 2 \times 17$ $\Rightarrow 1 = 7 \times 124 - 51 \times 17$		M1 A1	2.1 1.1b
	$\Rightarrow 10 = 70 \times 124 - 510 \times 17$ $x = 70, y = -510$		A1	2.2a
	Alternative For example $124 = 17 \times 7 + 5$ $5 = 124 - 7 \times 17$	$1 = 5 - 2 \times 2$ $1 = 5 - 2(17 - 3 \times 5) \Rightarrow 4 = 2 \times 17 - 6 \times 5$ $4 = 2 \times 17 - 6(124 - 17 \times 7)$ $\Rightarrow 4 = 44 \times 17 - 6 \times 124$	M1 A1	2.1 1.1b
	$\Rightarrow 10 = 2 \times 124 - 14 \times 17$ $x = 2, y = -14$	$\Rightarrow 10 = 110 \times 17 - 15 \times 124$ $x = -15, y = 110$	A1	2.2a
			(3)	
(c)	A complete method using modulo arithmetic to achieve $x \equiv \dots \bmod 17$ For example $7 \times 124x \equiv 7 \times 6 \bmod 17 \Rightarrow x \equiv \dots \bmod 17$		M1	2.1
	$x \equiv 8 \bmod 17$ o.e.		A1	1.1b
			(2)	
			(7 marks)	

Notes:

(a)

M1: Complete attempt at the Euclidean algorithm to find the gcd (hcf) of 124 and 17, condone a numerical slip.

A1: A fully correct application of the Euclidean algorithm and draws the conclusion that since the **gcd/hcf = 1** therefore 124 and 17 are **relatively prime (coprime).**

(b)

M1: Attempts to find the Bezout's identity, condone sign slips

A1: Correct Bezout's identity.

A1: Deduces a set of correct values of x and y.

Alternatively **M1:** Uses the solution to (a) to find an identity of the form $a \times 124 + b \times 17 = c$ **A1:** Correct identity **A1:** Deduces a set of correct values of x and y.
(c) **M1:** A complete method to reach using modulo arithmetic to achieve $x \equiv \ldots \mod 17$ For example Multiplies through by their multiplicative inverse of 124 and reaches $x \equiv \ldots \mod 17$. Bezout's identity used in part (b) follow through on their multiple of 124 **A1**: $x \equiv 8 \mod 17$o.e.

Question	Scheme	Marks	AOs
5(a)	$(x)^2+(y+a)^2=9[(x-a)^2+y^2]$ or $\sqrt{(x)^2+(y+a)^2}=3\sqrt{(x-a)^2+y^2}$	M1	2.1
	$8x^2-18ax+8y^2-2ay+8a^2\{=0\}$ o.e.	A1	1.1b
	$x^2-\frac{9}{4}ax+y^2-\frac{1}{4}ay+a^2=0$ $\Rightarrow\left(x-\frac{9}{8}a\right)^2-\left(\frac{9}{8}a\right)^2+\left(y-\frac{1}{8}a\right)^2-\left(\frac{1}{8}a\right)^2+a^2=0$ $\Rightarrow r^2=\left(\frac{9}{8}a\right)^2+\left(\frac{1}{8}a\right)^2-a^2=\left(\frac{3}{2}\sqrt{2}\right)^2\Rightarrow a=\ldots$	M1	1.1b
	$a=-4$ cso	A1	2.2a
		(4)	
(b)	$\left(x-\frac{9}{8}(-4)\right)^2+\left(y-\frac{1}{8}(-4)\right)^2=\ldots$ $(x-\alpha)^2+(y-\beta)^2=\ldots$ implies centre$(\alpha,\ \beta)$	M1	1.1b
	centre $\left(-\frac{9}{2},\ -\frac{1}{2}\right)$	A1	2.2a
		(2)	
			(6 marks)

Notes:

(a)

M1: Obtains an equation in terms of *x* and *y* using the given information. Condone $(x)^2+(y+a)^2=3[(x-a)^2+y^2]$ for this mark

A1: Expands and simplifies the algebra, collects terms and obtains a correct simplified equation. Condone missing = 0

M1: Completes the square for their equation using $x^2+Ax=\left(x+\frac{A}{2}\right)^2+\ldots$ Sets their radius squared $=\left(\frac{3}{2}\sqrt{2}\right)^2=\frac{18}{4}=\frac{9}{2}$ or radius $=\left(\frac{3}{2}\sqrt{2}\right)$ and finds a value for *a*.

Note the correct values are $r^2=\frac{9}{32}a^2, r=\frac{3a\sqrt{2}}{8}$

A1: Deduces that $a=-4$cso

(b)

M1: Substitutes their value for *a* into their equation and finds their centre.

A1: Deduces the correct centre.

Question	Scheme	Marks	AOs
6 (a)	$\lambda^2 - 2\lambda + 1 = 0 \Rightarrow \lambda = \ldots\{\lambda = 1\}$	M1	1.1b
	$u_n = A + Bn$	A1	2.2a
	$u_n = \mu(2)^n \Rightarrow \mu(2)^n = 2\mu(2)^{n-1} - \mu(2)^{n-2} + (2)^n$ $\Rightarrow 4\mu(2)^{n-2} = 4\mu(2)^{n-2} - \mu(2)^{n-2} + 4(2)^{n-2}$ $\Rightarrow \mu = \ldots\{4\}$	M1	1.1b
	$u_n = A + Bn + 4(2)^n$ or $u_n = A + Bn + (2)^{n+2}$	A1	1.1b
		(4)	
(b)	Uses the information to find the values of the constants For example $u_0 = 2u_1 \Rightarrow A + 4 = 2(A + B + 4(2)) \Rightarrow \ldots\{A + 2B = -12\}$ $u_4 = 3u_2 \Rightarrow A + 4B + '4'(2)^4 = 3(A + 2B + 4(2)^2) \Rightarrow \ldots\{2A + 2B = 16\}$ Solves simultaneous equations to find values for *A* and *B*. **Alternatively** $u_2 = 2u_1 - u_0 + 2^2 = u_0 - u_0 + 4 = 4$ leading to $u_4 = 3u_2 = 3 \times 4 = 12$ $u_3 = 2u_2 - u_1 + 2^3 = 2 \times 4 - \left(\frac{A+4}{2}\right) + 8 = 16 - \left(\frac{A+4}{2}\right)$ $u_4 = 2u_3 - u_2 + 2^4 = 2\left(16 - \left(\frac{A+4}{2}\right)\right) - 4 + 16 = 12$ Leading $A = \ldots$ $u_2 = '28' + 2B + 4(2^2) = 4 \Rightarrow B = \ldots$	M1	3.1a
	$u_n = 28 - 20n + 4(2)^n$ or $u_n = 28 - 20n + (2)^{n+2}$	A1	1.1b
		(2)	
		(6 marks)	

Notes:

(a)

M1: Forms and solves the auxiliary equation.

A1: Correct complementary function.

M1: Correct form for the particular solution, substitutes into the recurrence relation to find the PS.

A1: Correct general solution

(b)

Note: They must have two constants to score any marks in this part

M1: A complete method to find the constants using the information given. Form two equations and solves simultaneously to find values for *A* and *B*.

A1: Correct solution.

Question	Scheme	Marks	AOs
7(i) (a)	120	B1	1.1b
		(1)	
(i) (b)	300	B1	1.1b
		(1)	
(ii)(a)	As divisible by 11 implies $a - b + c = 11p$ where p is an integer Due to the restrictions on a, b and c $\Rightarrow a - b + c = 11$ or 0	M1	2.4
	For example $a + b + c$ is even $\Rightarrow (a + b + c) - (2b)$ is even $\Rightarrow a - b + c$ is even or Sum of digits is even implies that $a + b + c = 2q$ $(a - b + c = (a + b + c) - 2b = 2q - 2b = 2n)$ **Alternative approach 1** $a + b + c = 2q$where $q \in \mathbb{Z}$ either $a + b + c = 2q$ and $a - b + c = 0 \Rightarrow 2b = 2q \Rightarrow b = q$ valid $a + b + c = 2q$ and $a - b + c = 11 \Rightarrow 2b = 2q - 11 \Rightarrow b = q -$ 5.5 not valid Or $a + b + c = 2q$ and $a - b + c = 0 \Rightarrow 2a + 2c = 2q$ valid $a + b + c = 2q$ and $a - b + c = 11 \Rightarrow 2a + 2c = 2q + 11$ not valid as $2(a + c)$ even or This approach may be in words **Alternative approach 2** If sum $a - b + c$ is odd/11 then either one or all three numbers odd. This would mean sum $a + b + c$ would be odd/not even **Alternative approach 3** If $a - b + c = 11$ this implies $a + b + c \equiv 1 \ mod\ 2$ (contradiction) **<u>Conclusion</u>** therefore $a - b + c = 0$* cso	A1*	2.1
		(2)	
(ii)(b)	$N = 100a + 10b + c \Rightarrow a + b + c \equiv 8 \text{ mod } 9$ o.e or $N + 1 \equiv 0 \text{ mod } 9$ ($N + 1$ is a multiple of 9) o.e.	B1	1.1b

	$\Rightarrow a+b+c+1 \equiv 0 \text{ mod } 9$ ($a+b+c+1$ is a multiple of 9) o.e		
	Solves $a-b+c=\ 0$ and $a+b+c \equiv 8 \text{ mod } 9$ either $(a+b+c \equiv 8$ or $26)$ to find a value for b (= 4) and uses this value to form an equation $a+c=\ldots(4)$ Or $2a+2c \equiv 8 \text{ mod } 9 \Rightarrow a+c \equiv 4 \text{ mod } 9$ leading to $a+c=\ldots(4)$ or $2a+2c \equiv 8 \text{ mod } 9 \Rightarrow a+c \equiv 4 \text{ mod } 9 \Rightarrow b \equiv 4 \text{ mod } 9$ leading to $b=\ldots(4)$ Or $a-b+c=0$, $b=a+c$ so $2b+1$ is a multiple of 9, $2b+1=9$ or 18 leading to $b=\ldots$ (4)	M1	2.1
	$N=143,\ 242,\ 341{,}440$	M1 A1	1.1b 2.2a
		(4)	
	Alternative $(N)=11n \equiv 8 \text{ mod } 9$	B1	1.1b
	Finds multiplicative inverse of 11 using $5\times 9-4\times 11=1$ $-4\times 11n \equiv -4\times 8 \text{ mod } 9 \Rightarrow n \equiv -32 \text{ mod } 9 \Rightarrow n \equiv 4 \text{ mod } 9$ So $N=11(9n+4)$	M1	2.1
	$N=143,\ 242,\ 341{,}440$	M1	1.1b
		A1	2.2a
		(4)	
			(8 marks)

Notes:

(i)(a)
B1: See scheme

(i)(b)
B1: See scheme

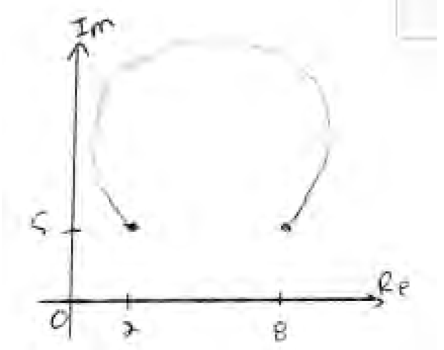
(ii) (a)
M1: Uses the divisibility rule for 11 and the restrictions on the values a, b and c leading to $a-b+c=0$ or 11 only
A1*: Uses the information that the sum of the digits is even and that $a-b+c=0$ or 11 from the divisibility rule for 11 to **show** that $a-b+c=0$ cso

(ii) (b)
B1: States $a+b+c \equiv 8 \text{ mod } 9$ or $a+b+c+1 \equiv 0 \text{ mod } 9$

M1: Uses their equations to find a value for $a + c =$ ... or a value for b
M1: Uses their values to find at least two correct values for N
A1: Deduces all four correct values of N and no extra values

Alternative
B1: States $(N) = 11n \equiv 8 \text{ mod } 9$
M1: Finds the multiplicative inverse of 11 and uses this to find an expression for N
M1: Uses their expression for N to find at least two values for N
A1: Deduces all four correct values of N and no extra values

Trial and error
B1: For identifying $a + c = b$
M1: For one correct value, this implies B1
M1: For two correct values
A1: Deduces all four correct values of N and no extra values

Question	Scheme	Marks	AOs
8 (a)	 The major arc of a circle drawn anywhere	B1	1.1b
	The end points of their arc (2, 5) and (8, 5) and the arc drawn above the coordinates	B1	1.1b
		(2)	
(b)	The centre lies on the **perpendicular bisector/midpoint/equidistant** of **2 and 8**	B1	2.4
		(1)	
(c)	A complete method to find the radius of the circle $$sin\left(\frac{\pi}{3}\right)=\frac{3}{r}\Rightarrow r=\ldots$$ Or $$6^2=r^2+r^2-2\times r\times r\times cos\left(\frac{2\pi}{3}\right)\Rightarrow r=\ldots$$ Or $$h=\frac{3}{tan\left(\frac{\pi}{3}\right)}=\sqrt{3}\Rightarrow r=\sqrt{\left(\sqrt{3}\right)^2+3^2}=\ldots$$ or $$tan[(arg(x+yi-(8+5i))-arg(x+yi-(2+5i))]=tan\left(\frac{\pi}{3}\right)$$ $$\frac{tan[arg(x+yi-(8+5i))]-tan[arg(x+yi-(2+5i))]}{1+tan[arg(x+yi-(8+5i))]\,tan[arg(x+yi-(2+5i))]}=\sqrt{3}$$ $$\frac{\frac{y-5}{x-8}-\frac{y-8}{x-2}}{1+\frac{y-5}{x-8}\times\frac{y-8}{x-2}}=\sqrt{3}$$ Leading to an equation of a circle by completing the square $(x-a)^2+(y-b)^2=r^2$ leading to $r=\ldots$	M1	3.1a
	$$r=\frac{6}{\sqrt{3}}\text{ or }2\sqrt{3}\text{ o.e.}$$	A1	1.1b
		(2)	
(d)	$$y=5+h$$ where $h=\frac{3}{tan\left(\frac{\pi}{3}\right)}$ or $h='2\sqrt{3}'\,cos\left(\frac{\pi}{3}\right)$ or $h=\sqrt{\left('2\sqrt{3}'\right)^2-3^2}$	M1	3.1a

	$y = 5 + \sqrt{3}$	A1	2.2a
		(2)	
			(7 Marks)

Notes:

(a)
B1: Major arc drawn anywhere
B1: Correct end points for their arc drawn above the end points, condone written as complex numbers

(b)
B1: States perpendicular bisector or midpoint of 2 and 8. Condone "in between 2 and 8" if they write $\frac{2+8}{2} = 5$
Note: $\frac{2+8}{2} = 5$on its own is B0
In between 2 and 8 on its own is B0

(c)
M1: A complete method to find the radius of the circle
A1: Correct radius

(d)
M1: Any correct complete strategy. If they attempt to find the height (even if incorrect method) in part (c) then y = 5 + their height
A1: Correct answer

Question	Scheme	Marks	AOs
9 (a)	$I_n = \int \sin^n 2x \, dx = \int \sin^{n-1} 2x \sin 2x \, dx$ Leading to $I_n = \left[\lambda \sin^{n-1} 2x \cos 2x\right] - \mu \int \sin^{n-2} 2x \cos^2 2x dx$	M1	2.1
	$I_n = \left[-\frac{1}{2}\sin^{n-1} 2x \cos 2x\right] + \int (n-1)\sin^{n-2} 2x \cos^2 2x dx$	A1	1.1b
	$I_n = 0 + (n-1)\int \sin^{n-2} 2x\left(1-\sin^2 2x\right)dx$ $= (n-1)\int \sin^{n-2} 2x \, dx - (n-1)\int \sin^n 2x \, dx$	dM1	1.1b
	$nI_n = (n-1)I_{n-2} \Rightarrow I_n = \frac{n-1}{n} I_{n-2}$ *	A1*	2.1
		(4)	
	Alternative: On epen MAMA make sure marks are recorded in the correct place $I_n = \int \sin^n 2x \, dx = \int \sin^{n-2} 2x \sin^2 2x \, dx$ $= \int \sin^{n-2} 2x \,(1- \cos^2 2x) \, dx$ $= \int \sin^{n-2} 2x \, dx - \int \sin^{n-2} 2x \cos^2 2x \, dx$ Leading to an attempt at integration	M1	1.1b
	$I_n = I_{n-2} - \int (\sin^{n-2} 2x \cos 2x)(\cos 2x) \, dx$ $I_n = I_{n-2} - \left[\lambda \sin^{n-1} 2x \cos 2x\right] + \mu \int \sin 2x \sin^{n-1} 2x dx$	dM1	2.1
	$I_n = I_{n-2} - \left[\frac{1}{2(n-1)}\sin^{n-1} 2x \cos 2x\right] - \frac{1}{n-1}\int \sin^n 2x dx$	A1	1.1b
	$I_n = I_{n-2} - \frac{1}{n-1} I_n \Rightarrow (n-1)I_n = (n-1)I_{n-2} - I_n \Rightarrow I_n = \frac{n-1}{n} I_{n-2}$*	A1*	2.1
		(4)	
(b)	$\int 64 \sin^5 x \cos^5 x \, dx = \int A \sin^5 2x \, dx$ **Note** $A = 2$	M1	2.1
	$I_5 = \frac{4}{5} I_3$, $I_3 = \frac{2}{3} I_1$and $I_1 = \int_0^{\frac{\pi}{2}} \sin 2x \;\; dx = [\alpha \cos 2x]_0^{\frac{\pi}{2}} = \ldots$	M1	1.1b
	$= 2 \times \left(\frac{4}{5}\right) \times \left(\frac{2}{3}\right) \times 1 = \frac{16}{15}$	A1	1.1b
		(3)	
			(7 marks)

Notes:
(a) **M1:** Writes $\sin^n 2x$ as $\sin^{n-1} 2x \; \sin 2x$ and integrates using by parts to the form $I_n = \left[\lambda \sin^{n-1} 2x \cos 2x\right] - \mu \int \sin^{n-2} 2x \cos^2 2x \,\mathrm{d}x$ **A1:** Correct integration, may be unsimplified **dM1:** Substitutes the limits of 0 and $\frac{\pi}{2}$ into 'uv', this may be implied by 0. Replaces $\cos^2 2x = 1 - \sin^2 2x$ and multiplies out into separate integrals. **A1*:** Achieves the printed answer following a correct intermediate line and no errors. Cso **Alternative** **M1:** Writes $\sin^n 2x$ as $\sin^{n-2} 2x \; \sin^2 2x = \sin^{n-2} 2x \; (1 - \cos^2 2x)$, writes as $= \int \sin^{n-2} 2x \;\mathrm{d}x - \int \sin^{n-2} 2x \cos^2 2x \,\mathrm{d}x$ and attempts to integrate **dM1:** Integrates using by parts to the form $I_n = I_{n-2} - \left[\lambda \sin^{n-1} 2x \cos 2x\right] + \mu \int \sin 2x \sin^{n-1} 2x \mathrm{d}x$ **A1:** Correct integration, may be unsimplified **A1*:** Achieves the printed answer following a correct intermediate line and no errors. cso
(b) **M1:** Uses the identity $\sin 2x = 2 \sin x \cos x$ in an attempt to write the integral as $\int A \sin^5 2x \; dx$ **M1:** Uses the answer to part (a) to find a value for I_5 and I_3 and finds $I_1 = \int_0^{\frac{\pi}{2}} \sin 2x \;\; dx = \ldots$ **A1:** $\frac{16}{15}$ cso

Question	Scheme	Marks	AOs
10(a)	Surface of revolution $= 2\pi \int x\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$ $= 2\pi \int (10 + 15t - 5t^3)\sqrt{(15 - 15t^2)^2 + (30t)^2}\ dt$	M1 A1	3.4 1.1b
	Surface of revolution = $\{2\pi\} \int \ldots \sqrt{225 - 450t^2 + 225t^4 + 900t^2}\ dt$ $= \{2\pi\} \int \ldots \sqrt{225 + 450t^2 + 225t^4}\ dt$	M1	1.1b
	$= \{2\pi\} \int \ldots \sqrt{(15 + 15t^2)^2}\ dt$ $= \{2\pi\} \int \ldots (15 + 15t^2)\ dt$	M1	2.1
	$= 2\pi \int (10 + 15t - 5t^3)(15 + 15t^2)\ dt$ $\left\{2\pi \int (150 + 150t^2 + 225t + 225t^3 - 75t^3 - 75t^5)\ dt\right\}$ $150\pi \int_0^1 (2 + 3t + 2t^2 + 2t^3 - t^5) dt$ *	A1*	1.1b
		(5)	
(b)	Surface of revolution $= 150\pi \left[2t + \frac{3}{2}t^2 + \frac{2}{3}t^3 + \frac{2}{4}t^4 - \frac{1}{6}t^6\right]_0^1$ $= 150\pi \left[\left(2(1) + \frac{3}{2}(1)^2 + \frac{2}{3}(1)^3 + \frac{2}{4}(1)^4 - \frac{1}{6}(1)^6\right) - (0)\right]$	M1	1.1b
	Surface of revolution $= 675\pi =$ awrt 2120	A1	1.1b
	Adds their surface of revolution to their area of the **circular** base $= '675\pi' + \pi \times 10^2 = \ldots$	M1	3.4
	Total surface area of the inner surface of the pot is 775π or awrt 2430	A1	1.1b
		(4)	
(c)	Number of plant pots $= \frac{120\ 000}{2 \times \text{their total surface area}}$ or Number of plant pots $= \frac{12}{2 \times \frac{\text{their total surface area}}{100^2}}$	M1	3.4
	24 (complete plant pots)	A1	1.1b
		(2)	
(d)	For example: The surface area of the outside of the plant pot will be more than the inside. There will be a (small) rim on the plant pot which is not considered There is thickness to the pot	B1	3.5b

	The surface area of the pot may not perfectly fit the curve Pot may not be smooth		
		(1)	
			(12 marks)

Notes:

(a)

M1: Uses the formula surface area $2\pi\int x\sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2+\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}\,\mathrm{d}t$ with their model.

A1: Correct formula for the inner surface area.

M1: Starts the process of manipulating into an integrable form by squaring and simplifying the expression under the square root.

M1: Completes the process by writing as something squared and cancelling.

A1*: Achieves the printed answer with no errors seen. Evidence of 75 taken out as a factor (it may be in stages) Limits just stated.

Note working with $2\pi\int y\sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2+\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}\,\mathrm{d}t$ the maximum is M0A0M1M1A0

(b)

M1: Integrates the polynomial and uses the limits of $t = 0$ and $t = 1$ the correct way around and subtracts. The limit of $t = 0$ can be implied.

A1: Correct value for the inner surface area of revolution.

M1: Adds their area of the inner surface to their area of the **circular** base

A1: Correct total inner surface area for the plant pot.

Note Correct answer for the integration $675\pi = 2120.6$ seen scores M1 A1

(c)

M1: Finds the total number of plant pots by dividing 120 000 by 2 times their total surface area or divides 12 by 2 times their surface area/100^2

A1: 24 (complete plant pots).

(d)

B1: Gives a correct limitation of the model.

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