## Pearson Edexcel

# Mark Scheme (Results) 

Summer 2022

Pearson Edexcel GCE
AL Further Mathematics (9FM0)
Paper 4C Further Mechanics 2

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## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.


## EDEXCEL GCE MATHEMATICS General Instructions for Marking

1. The total number of marks for the paper is 75 .
2. The Edexcel Mathematics mark schemes use the following types of marks:

- M marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- B marks are unconditional accuracy marks (independent of $M$ marks)
- Marks should not be subdivided.


## 3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod - benefit of doubt
- ft - follow through
- the symbol $\sqrt{ }$ will be used for correct ft
- cao - correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw - ignore subsequent working
- awrt - answers which round to
- SC: special case
- oe - or equivalent (and appropriate)
- dep - dependent
- indep - independent
- dp decimal places
- sf significant figures
-     * The answer is printed on the paper
- $\square$ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
5. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response.
If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.
6. Ignore wrong working or incorrect statements following a correct answer.
7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.


| Question | Scheme | Marks | AOs |
| :---: | :---: | :---: | :---: |
| 2 (a) | Use of $P=F v$ | B1 | 3.3 |
|  | Equation of motion ( $\left.F-3 v^{2}=60 a\right)$ | M1 | 2.1 |
|  | $\frac{200}{v}-3 v^{2}=60 v \frac{\mathrm{~d} v}{\mathrm{~d} x}$ | A1 | 2.5 |
|  | $\frac{\mathrm{d} v}{\mathrm{~d} x}=\frac{200-3 v^{3}}{60 v^{2}} \quad *$ | A1* | 2.2a |
|  |  | (4) |  |
| 2(b) | $\Rightarrow \int \frac{60 v^{2}}{200-3 v^{3}} \mathrm{~d} v=\int 1 \mathrm{~d} x \quad\left(-\frac{60}{9} \ln \left(200-3 v^{3}\right)=x(+C)\right)$ | M1 | 1.1b |
|  | $D=\left[-\frac{60}{9} \ln \left(200-3 v^{3}\right)\right]_{2}^{4}=-\frac{60}{9} \ln \left(\frac{200-3 \times 64}{200-3 \times 8}\right)$ | M1 | 1.1b |
|  | $=\frac{60}{9} \ln \frac{176}{8}=\frac{60}{9} \ln 22$ | A1 | 1.1b |
|  |  | (3) |  |
|  |  | (7) |  |
| (7 marks) |  |  |  |
| Notes: |  |  |  |
| (a) |  |  |  |
| B1 | Seen or implied <br> Not just quoted. Need at least $200=F v$ <br> Could be on its own, in an equation or on a diagram |  |  |
| M1 | Form equation of motion. Need all terms and dimensionally correct. Condone any correct form for acceleration and sign errors <br> Allow with $m$ not substituted |  |  |
| A1 | Correct equation - any equivalent form with correct acceleration |  |  |
| A1* | Obtain given answer from correct working <br> Must be as written in the question but could swap LHS and RHS |  |  |
| (b) |  |  |  |
| M1 | Separate variables and integrate to obtain $(x=) k \ln (\ldots . .$. <br> (Constant of integration not required) <br> Condone if the $x$ is not explicitly stated but M0 if it is an incorrect function. |  |  |
| M1 | Use limits correctly in an expression containing $k \ln \left(200-3 v^{3}\right)$ to find $D$ |  |  |


|  | Substitute and subtract in the correct order |
| :--- | :--- |
| A1 | Obtain exact answer from correct working <br> Any equivalent single term <br> No working seen is Max M1M0A0 |
|  |  |
|  |  |
|  |  |



| A1 | Correct unsimplified equation (accept without $g$ and/or $M$ ) Correct mass and distance <br> combination for their $\bar{x}$ |
| :--- | :--- |
| A1 | Or 0.3125 Condone 0.31 or 0.313 |


| Question | Scheme | Marks | AOs |
| :---: | :---: | :---: | :---: |
| 4(a) |  |  |  |
|  | Resolve vertically | M1 | 3.4 |
|  | $T \cos \theta=m g$ | A1 | 1.1b |
|  | $T=\left(\frac{m g}{\cos \theta}=\frac{6.8 m g}{6}\right)=\frac{17 m g}{15}$ | A1 | 1.1b |
|  |  | (3) |  |
| 4(b) | Equation of motion | M1 | 3.1b |
|  | $m r \omega^{2}=T+T \sin \theta \quad\left(m \times 3.2 a \omega^{2}=\right.$ their $\left.T\left(1+\frac{8}{17}\right)\right)$ | $\begin{aligned} & \text { A1 } \\ & \text { A1 } \end{aligned}$ | $\begin{aligned} & 1.1 \mathrm{~b} \\ & 1.1 \mathrm{~b} \end{aligned}$ |
|  | Solves for $\omega$ or $\omega^{2}$ | M1 | 1.1b |
|  | $\left(\frac{r \omega^{2}}{g}=\frac{1+\sin \theta}{\cos \theta}=\frac{6.8+3.2}{6}, \quad \omega^{2}=\frac{10 g}{6 \times 3.2 a}\right) \quad \omega=\sqrt{\frac{25 g}{48 a}}=\frac{5}{4} \sqrt{\frac{g}{3 a}}$ | A1 | 1.1b |
|  |  | (5) |  |
| (8 marks) |  |  |  |
| Notes: |  |  |  |
| (a)M1 | Need all terms. Condone sin/cos confusion |  |  |
| A1 | Correct unsimplified equation. |  |  |
| A1 | Correct answer only <br> 1.1 mg or better (1.13...mg) <br> Do not ignore subsequent working if they try to combine this with a tension in $A R$ |  |  |
| (b)M1 | Equation for circular motion. Need all terms and dimensionally correct. Condone $\sin /$ cos confusion and sign errors. <br> Any correct form for acceleration |  |  |
| $\begin{aligned} & \mathrm{A} 1 \\ & \mathrm{~A} 1 \end{aligned}$ | Unsimplified equation with at most one error Correct unsimplified equation |  |  |


|  | Allow M1A1A0 for $m r \omega^{2}=T^{\prime}+($ their $(a)) \sin \theta$ |
| :--- | :--- |
| M1 | Clear attempt to substitute for trig and tension or divide their two equations to solve for <br> $\omega$ or $\omega^{2}$ in terms of $a$ and $g$ <br> Independent M mark but requires an equation using tension and trig. |
| A1 | Any equivalent form <br> $0.72 \sqrt{\frac{g}{a}}$ or better $\quad(0.7216 \ldots)$ |


| Question | Scheme | Marks | AOs |
| :---: | :---: | :---: | :---: |
| 5(a) | Using sector: distance $O G=\frac{2 \times 3 a \sin \frac{\pi}{4}}{3 \times \frac{\pi}{4}}$ | B1 | 1.1b |
|  | Using Pythagoras: $2 d^{2}=\frac{32 a^{2}}{\pi^{2}} \quad\left(d^{2}+d^{2}=O G^{2}\right)$ Or using trigonometry: Distance from $O C=O G \cos 45^{\circ}=O G \sin 45^{\circ}$ | M1 | 2.1 |
|  | $d=\sqrt{\frac{16 a^{2}}{\pi^{2}}}=\frac{4 a}{\pi} *$ | A1* | 2.2a |
|  |  | (3) |  |
| 5(a) alt | Using semicircle of radius $3 a: \quad \bar{y}=\frac{4 \times 3 a}{3 \pi}\left(=\frac{4 a}{\pi}\right)$ | B1 | 1.1b |
|  | Moments about diameter: $\frac{9 \pi a^{2}}{2} \times \frac{4 a}{\pi}=2 \times \frac{9 \pi a^{2}}{4} \times d$ | M1 | 2.1 |
|  | $\Rightarrow d=\frac{4 a}{\pi} \quad *$ | A1* | 2.2a |
|  |  | (3) |  |
| (b) | $A B C O$ $O D E F$ $O D C$ |  |  |
|  | Mass ratio 9 9 $\frac{9 \pi}{4}$ |  | 1.2 |
|  | From FC |  |  |
|  | Moments about $F C$ : | M1 | 3.1a |
|  | $-9 \times \frac{3 a}{2}+9 \times \frac{3 a}{2}+\frac{9 \pi}{4} \times \frac{4 a}{\pi}=\left(18+\frac{9 \pi}{4}\right) \bar{x}(=9 a)$ | A1 | 1.1b |
|  | $\bar{x}=\frac{4 a}{8+\pi}$ | A1 | 1.1b |
|  |  | (4) |  |
|  |  |  |  |



| A1 | Correct unsimplified equation for their axis |
| :--- | :--- |
| A1 | Or equivalent with no errors seen <br> Accept $0.36 a$ or better $(0.3590 \ldots a)$ |
| (c)B1ft | Allow use of symmetry seen or implied. <br> Accept $\bar{y}=\bar{x}$ <br> (From FE, $\left.\bar{y}=\frac{28 a+3 \pi a}{8+\pi}\right)$ Accept $+/-$ |
| M1 | Correct strategy to find a relevant angle <br> $(\theta$ or $90-\theta)$ Need to substitute their values of $\bar{x}$ and distance from $F \neq \frac{4 a}{\pi}$. |
| A1ft | Correct unsimplified expression for a relevant angle. Follow their $\bar{x}$ and $\bar{y}$ |
| A1 | 6.1 or better (6.10067...) <br> The question defines $\theta$ as measured in degrees. 0.106 can score B1M1A1ftA0 <br> Do not ISW |
|  |  |


| Question | Scheme | Marks | AOs |
| :---: | :---: | :---: | :---: |
| 6(a) | Mass of cone $=\int_{0}^{9} \pi y^{2} \lambda x \mathrm{~d} x=\pi \lambda \int_{0}^{9} \frac{x^{3}}{9} \mathrm{~d} x$ | M1 | 3.4 |
|  | $=\pi \lambda\left[\frac{x^{4}}{36}\right]_{0}^{9} \quad\left(=\frac{729 \pi \lambda}{4}(\mathrm{~kg})\right)$ | A1 | 1.1b |
|  | Moments: $\int_{0}^{9} \pi y^{2} \lambda x \times x \mathrm{~d} x=\pi \lambda \int_{0}^{9} \frac{x^{4}}{9} \mathrm{~d} x$ | M1 | 3.4 |
|  | $=\frac{\pi \lambda}{45}\left[x^{5}\right]_{0}^{9} \quad\left(=\frac{\pi \lambda}{5} \times 9^{4}\right)$ | A1 | 1.1b |
|  | $\Rightarrow d=\frac{\frac{\pi \lambda}{5} \times 9^{4}}{\frac{\pi \lambda}{4} \times 9^{3}}$ | DM1 | 2.1 |
|  | $d=\frac{36}{5}=7.2(\mathrm{~cm})$ | A1 | 1.1b |
|  |  | (6) |  |
| (b) | Remains at rest $\Rightarrow$ centre of mass at centre of plane surface | B1 | 2.1 |
|  | Moments about diameter of plane surface: | M1 | 3.1 b |
|  | $(9-d) W\left\{=\left(9-\frac{36}{5}\right) W\right\}=\frac{3}{8} \times 3 \times k W$ | A1ft | 1.1b |
|  | $k=\frac{8}{5}$ | A1 | 1.1b |
|  |  | (4) |  |
| (10 marks) |  |  |  |
| Notes: |  |  |  |
| (a) | NB: Some candidates are confusing the mass and the volume. For the first M1A1: <br> - If they have a correct method for the mass and they tell you that this is mass, award the marks. <br> - If they have a correct method for the mass say nothing, but use it correctly, award the marks. <br> - If they have a correct method for the mass, say nothing, and use it as the moment, then M0 because this implies that they do not think it is the mass. |  |  |
| M1 | Use the model to find the mass of the cone. Allow without limits. |  |  |
| A1 | Correct integration. Correct limits seen or implied Substitution not required. |  |  |


|  | Allow $2 / 2$ if $\pi$ not seen and consistent with (b) if attempted |
| :---: | :---: |
| M1 | Use the model to find the moment of the cone (usual rules for integration) Allow without limits |
| A1 | Correct integration. Correct limits seen or implied Substitution not required. <br> Allow $2 / 2$ if $\pi$ not seen and consistent with (a) |
| M1 | Complete method to find the distance of the centre of mass from the vertex. A complete method requires the two preceding M marks. <br> They need to get as far as a value for $d$. <br> If they have a method that comes directly to this stage you might not see the $\lambda$ or $\pi$ |
| A1 | Correct only <br> If all you see is $\Rightarrow d=\frac{9^{5}}{45} \div \frac{9^{4}}{36}$ or even $\Rightarrow d=\frac{9}{5} \times 4$ then award $6 / 6$ Allow $6 / 6$ if $\pi$ not seen throughout but otherwise correct |
| (b)B1 | Correct deduction for location of c of m Stated or implied by their moments equation |
| M1 | Moments about diameter of plane face(s) <br> M0 if the moments equation contradicts the centre of mass being on the interface M0 if using volume in place of mass |
| A1ft | Correct unsimplified equation. Follow their 7.2 <br> Alternative moments equations: <br> Using vertex: $W \bar{x}+k W\left(9+\frac{3}{8} \times 3\right)=(W+k W) \times 9$ <br> Using base: $W(12-\bar{x})+k W\left(3-3 \times \frac{3}{8}\right)=(W+k W) 3$ <br> If they are working with the axis at an angle they will possibly have trig terms which should cancel. |
| A1 | Correct only |
|  |  |
|  |  |


| Question | Scheme | Marks | AOs |
| :---: | :---: | :---: | :---: |
| 7(a) | Conservation of energy: | M1 | 3.1b |
|  | $\frac{1}{2} m u^{2}=\frac{1}{2} m \nu^{2}+m g \times \frac{2 a}{5}(1-\cos \theta)$ | A1 | 1.1b |
|  | Equation of motion towards $O$ | M1 | 3.1 b |
|  | $T-m g \cos \theta=\frac{5 m v^{2}}{2 a}$ | A1 | 1.1b |
|  | Complete method to find $T$ in terms of $u, a$ and $\theta$ | DM1 | 2.1 |
|  | $\begin{gathered} T=m g \cos \theta+\frac{5 m}{2 a}\left(u^{2}-\frac{4 a}{5} g(1-\cos \theta)\right) \\ =3 m g \cos \theta-2 m g+\frac{5 m u^{2}}{2 a} \quad * \end{gathered}$ | A1* | 2.2a |
|  |  | (6) |  |
| (b) | Require $T \geq 0$ when $\theta=\pi: \frac{5 m u^{2}}{2 a} \geq m g(2+3)$ | M1 | 2.1 |
|  | $u^{2} \geq 2 a g, \quad$ minimum $u=\sqrt{2 a g}$ | A1 | 1.1b |
|  |  | (2) |  |
| (c) | $\theta=\frac{\pi}{2}, u=2 \sqrt{a g} \Rightarrow T=-2 m g+\frac{5 m}{2 a} \times 4 a g$ | B1 | 1.1b |
|  |  |  |  |
|  | Magnitude of acceleration $=g \sqrt{64+1}$ | M1 | 2.1 |
|  | $=\sqrt{65} g$ | A1 | 1.1b |
|  |  | (3) |  |
| (d) | Consider the uniformity / dimensions of the package String might be extensible. include the weight of the string | B1 | 3.5c |
|  |  | (1) |  |
|  |  |  |  |


| (12 marks) |  |
| :---: | :---: |
| Notes: |  |
| (a)M1 | Need all terms. Dimensionally correct. Condone sign errors and $\sin /$ cos confusion Allow with $\frac{2 a}{5} \cos \theta$ in place of $\frac{2 a}{5}(1-\cos \theta)$ |
| A1 | Correct unsimplified equation |
| M1 | Need all terms. Dimensionally correct. Condone sign errors and sin/cos confusion |
| A1 | Correct unsimplified equation |
| M1 | Complete method, e.g. using conservation of energy and the circular motion, to form sufficient equations to obtain an expression without $v$ A complete method requires the two preceding M marks. |
| A1* | Obtain given result from correct working |
| (b)M1 | Identify correct condition for complete circle and solve for $u$. Condone working from $T=$ 0 |
| A1 | Allow $u \geq \sqrt{2 a g}$ <br> Condone $u>\sqrt{2 a g}$, and $u=\sqrt{2 a g}$ |
| (c)B1 | Correct $T$ or $v^{2}$ seen or implied |
| M1 | Use of Pythagoras with their horizontal component of acceleration |
| A1 | Correct only, or $8.1 \mathrm{~g}(8.062 \ldots g)$ or better |
| (d) B1 | Any valid suggestion relating to the model. <br> Allow negatives of statements within the model <br> e.g. not model the package as a particle. <br> B 0 if multiple suggestions including one incorrect. <br> B0 for accuracy of $g$ as this is not part of the description of the model. |
|  |  |
|  |  |


| Question | Scheme | Marks | AOs |
| :---: | :---: | :---: | :---: |
| 8(a) | At equilibrium: $0.5 \mathrm{~g}=\frac{25 e}{1.25}, e=\frac{0.5 \times 10 \times 1.25}{25}=\frac{1}{4}$ | B1 | 3.3 |
|  | For taut string, when distance $x$ from equilibrium, equation of motion | M1 | 2.1 |
|  | Alternative for M1: <br> Conservation of energy using a known point $(E$ or $B)$ and a general position From $E: \frac{25 e^{2}}{2 \times 1.25}+K E($ constant $\neq 0)+0.5 g x=\frac{25(e+x)^{2}}{2 \times 1.25}+\frac{1}{2} 0.5 v^{2}+0($ differentiate wrt $x$ for M1 $\quad \Rightarrow 0.5 g=\frac{25(e+x)}{1.25}+\frac{1}{2} v \frac{\mathrm{~d} v}{\mathrm{~d} x}$ | ion: <br> GPE) an |  |
|  | $\frac{25(e+x)}{1.25}-0.5 g=-0.5 \ddot{x}$ | A1ft | 1.1b |
|  | $\ddot{x}=-40 x \quad$ hence SHM* | A1* | 2.2a |
|  | Periodic time: | M1 | 3.4 |
|  | $T=\frac{2 \pi}{\sqrt{40}}=\frac{\pi}{\sqrt{10}} *$ | A1* | 2.2a |
|  |  | (6) |  |
| (b) | Max KE $=2.5=\frac{1}{2} \times \frac{1}{2} \times \max v^{2} \quad \Rightarrow \max v^{2}=10$ | B1 | 1.2 |
|  | Max speed $=a \omega: \sqrt{10}=a \sqrt{40}$ | M1 | 3.4 |
|  | $A B=1.25+\frac{1}{4}+\frac{1}{2}=2(\mathrm{~m}) *$ | A1* | 1.1b |
|  |  | (3) |  |
| (b) alt | Energy : $\frac{25 e^{2}}{2.5}+2.5+0.5 g a=\frac{25(e+a)^{2}}{2.5}$ | B1 |  |
|  | Solve for $a$ | M1 |  |
|  | $A B=1.25+\frac{1}{4}+\frac{1}{2}=2(\mathrm{~m}) \quad *$ | A1* | 1.1b |
|  |  | (3) |  |
| (c) | $a=0.5, x=0.5 \cos \sqrt{40} t$ | B1 | 2.2a |
|  | $-0.25=0.5 \cos \sqrt{40} t \quad \Rightarrow t=0.3311 \ldots$ | M1 | 3.1a |
|  | $v^{2}=40\left(0.5^{2}-0.25^{2}\right)=\frac{15}{2}$ | M1 | 3.4 |
|  | Total time $=2 \times 0.3311 \ldots+\frac{2 \times \sqrt{7.5}}{10}$ | DM1 | 3.1a |
|  | $=1.2(\mathrm{~s})$ or better | A1 | 2.2a |


|  |  |  |
| :--- | :--- | :--- | :--- |


|  | If they use suvat to find the time as a projectile it must be a complete method e.g. <br> $\sqrt{\frac{15}{2}}=-\sqrt{\frac{15}{2}}+g t$ or a combination of $v^{2}=u^{2}+2 a s$ and $s=u t+\frac{1}{2} a t^{2}$ |
| :--- | :--- |
| A1 | $=1.2(\mathrm{~s})$ or better Condone an answer to $>2$ s.f. <br> Not scored if they have used 9.8. |
|  |  |
|  |  |

