Mark Scheme (Results)

Summer 2022

Pearson Edexcel GCE
AL Further Mathematics (9FM0)
Paper 3D Decision Mathematics 1

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General Marking Guidance

• All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.

• Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.

• Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.

• There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.

• All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.

• Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.

• When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.

• Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. These mark schemes use the following types of marks:

   - **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)

   Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - **bod** – benefit of doubt
   - **ft** – follow through
   - the symbol $\surd$ will be used for correct ft
   - **cao** – correct answer only
   - **cso** - correct solution only. There must be no errors in this part of the question to obtain this mark
   - **isw** – ignore subsequent working
   - **awrt** – answers which round to
   - **SC**: special case
   - **o.e.** – or equivalent (and appropriate)
   - **d** or **dep** – dependent
   - **indep** – independent
   - **dp** decimal places
   - **sf** significant figures
   - ∗ The answer is printed on the paper or ag- answer given

4. All M marks are follow through.

   A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but answers that don't logically make sense e.g. if an answer given for a probability is >1 or <0, should never be awarded A marks.
   be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct

two from any A or B marks gained, in that part of the question affected.

6. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response. If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is

the most complete.

7. Ignore wrong working or incorrect statements following a correct answer.

8. Mark schemes will firstly show the solution judged to be the most common response expected

from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used. If no such alternative answer is provided but the response is deemed to be valid, examiners must escalate the response for a senior examiner to review.

Question	Scheme	Marks	AOs
1(a)	$\frac{4.3+6.1+...+1.3}{10}=\frac{37.5}{10}=3.75$ so lower bound is four (balls of string)	M1 A1	1.1b 2.2a
		(2)	
(b)	Bin 1: 4.3 5.1 0.4 Bin 2: 6.1 **2.5** 1.3 Bin 3: 4.7 **3.4** **1.7** Bin 4: **5.9** 2.1	M1 **A1** A1	1.1b 1.1b 1.1b
		(3)	
			(5 marks)

Notes for Question 1

a1M1: Attempt to find the lower bound $(37.5 \pm 6.1)/10$ (a value of 3.75 seen with no working can imply this mark)

a1A1: cso – a lower bound of 4 with either a correct calculation seen or 3.75 or 'total is 37.5 and if each ball contains 10 this gives a lower bound of 4'. An answer of 4 with no working (or from part **(b)**) scores M0A0. Any incorrect working loses this mark e.g. a correct calculation followed by an incorrect value followed by 4 is A0

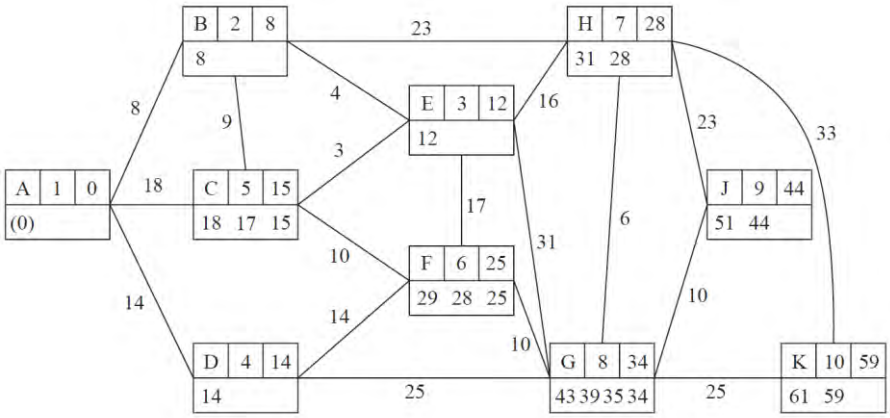
b1M1: First four items placed correctly and at least eight items placed in bins – condone cumulative totals for M1 only (the boxed values)

b1A1: First eight items placed correctly (the boxed **and** bold values), and all eleven correct values only placed in bins (so no additional/repeated values)

b2A1: cso (no additional/repeated values)

Condone working in cm provided consistent

No MR in this question – mark according to the scheme

Question	Scheme	Marks	AOs
2(a)	 **(i)** **(ii)** Path from A to K: ABEHGK **(iii)** Length of shortest path from A to K is 59 (km)	**M1** **A1** **A1** **A1ft** **A1** **A1ft**	1.1b 1.1b 1.1b 1.1b 2.2a 2.2a
		(6)	
(b)(i)	Repeated edges: AB, BE, EH	**B1**	2.2a
(b)(ii)	Length of route = 299 + 28 = 327 (km)	**B1ft**	2.2a
		(2)	
(c)	Route must start at A and finish at K therefore need to consider pairings of the nodes D, E, H and K	**M1**	3.1b
	D(AB)E + H(G)K = 26 + 31 = 57 D(FG)H + E(HG)K = 30 + 47 = 77 D(FG)K + EH = 49 + 16 = 65	**A1** **A1** **A1**	1.1b 1.1b 1.1b
	Length of new route = 299 + 57 = 356 (km)	**A1**	2.2a
		(5)	

(13 marks)

Notes for Question 2

In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded, for example at F the working values must be 29 28 25 in that order (so 29 25 28 is incorrect)

It is also important that the order of labelling is checked carefully – some candidates start with a 0 label at A (rather than 1) – which is fine. Also the order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4,… will be penalised once (see notes below) but 1, 2, 3, 5, 6,… is fine. Errors in the final values and working values are penalised before errors in the order of labelling. Condone crossed out working values if they are still legible

a1M1: A larger value replaced by a smaller value in at least **two** different Working Value boxes at either C or F or G or H or J or K

a1A1: All values at A, B, E, D and C correct. Condone lack of 0 in A's working value

a2A1: All values at F, H and G correct and the working values in the correct order. Penalise order of labelling only once per question (F, H and G must be labelled in that order and F must be labelled after A, B, E, D and C)

a3A1ft: All values in J and K correct on the follow through and the working values in the correct order. To follow through J check that the working values at J follow from the candidate's final values for the nodes that are directly attached to J (which are G and H). For example, **if** correct then the order of labelling of nodes G and H are 8 and 7 respectively so the working values at J should come from H and G in that order. The first working value at J should be their 28 (the Final value at H) + 23 (the weight of the arc HJ), the second working value at J should be their 34 (the Final value at G) + 10 (the weight of the arc GJ). Repeat the process for K (which will have working values from G and H with the order of these nodes determined by the candidate's order of labelling at G and H)

a4A1: cao (correct path from A to K – ABEHGK). Do not accept KGHEBA. Condone the length of the shortest path and the path interchanged on the answer lines in the answer book. Allow the paths written in terms of arcs, e.g. AB, BE, EH, HG, GK

a5A1ft: ft their final value at K only (if 59 stated and 59 is not the final value at K then A0)

bi1B1: cao (AB, BE, EH only but in any order) – must be stated as edges so B0 for ABEH

bii1B1ft: 327 or follow through their final value at H from **(a)** + 299

c1M1: The correct three pairings of the correct four nodes (D, E, H and K)

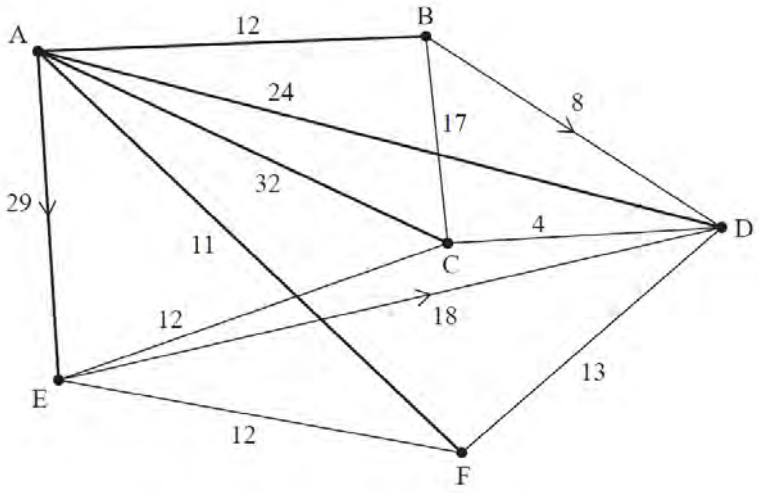
c1A1: One row correct including pairing **and** total

c2A1: Two rows correct including pairings **and** totals

c3A1: All three rows correct including pairings **and** totals

c4A1: cao (356) – dependent on all previous marks in this part

Condone lack of units throughout

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>3(a)</td><td> 
</td><td>M1

A1</td><td>1.1b

1.1b</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td>(b)</td><td>Third iteration:
<table><tr><th></th><th>A</th><th>B</th><th>C</th><th>D</th><th>E</th><th>F</th></tr><tr><td>A</td><td>-</td><td>12</td><td>29</td><td>20</td><td>29</td><td>11</td></tr><tr><td>B</td><td>12</td><td>-</td><td>17</td><td>8</td><td>29</td><td>23</td></tr><tr><td>C</td><td>29</td><td>17</td><td>-</td><td>4</td><td>12</td><td>40</td></tr><tr><td>D</td><td>24</td><td>21</td><td>4</td><td>-</td><td>16</td><td>13</td></tr><tr><td>E</td><td>41</td><td>29</td><td>12</td><td>16</td><td>-</td><td>12</td></tr><tr><td>F</td><td>11</td><td>23</td><td>40</td><td>13</td><td>12</td><td>-</td></tr></table>
Fourth iteration:
<table><tr><th></th><th>A</th><th>B</th><th>C</th><th>D</th><th>E</th><th>F</th></tr><tr><td>A</td><td>-</td><td>12</td><td>24</td><td>20</td><td>29</td><td>11</td></tr><tr><td>B</td><td>12</td><td>-</td><td>12</td><td>8</td><td>24</td><td>21</td></tr><tr><td>C</td><td>28</td><td>17</td><td>-</td><td>4</td><td>12</td><td>17</td></tr><tr><td>D</td><td>24</td><td>21</td><td>4</td><td>-</td><td>16</td><td>13</td></tr><tr><td>E</td><td>40</td><td>29</td><td>12</td><td>16</td><td>-</td><td>12</td></tr><tr><td>F</td><td>11</td><td>23</td><td>17</td><td>13</td><td>12</td><td>-</td></tr></table></td><td>M1

A1

M1

A1</td><td>1.1b

1.1b

1.1b

1.1b</td></tr>
<tr><td></td><td></td><td>(4)</td><td></td></tr>
<tr><td>(c)</td><td>Nearest neighbour routes:
E – C – D – F – A – B – E Length = 76
E – F – A – B – D – C – E Length = 59
The cycle that begins E – F –… is the better upper bound as the value is the smaller of the two</td><td>M1
A1
A1
A1</td><td>3.4
1.1b
1.1b
2.4</td></tr>
<tr><td></td><td></td><td>(4)</td><td></td></tr>
<tr><td colspan="4">(10 marks)</td></tr>
</table>

Notes for Question 3

a1M1: Correct five arcs from A

a1A1: cao no additional arcs or arrows, correct weights and arc AE directed from A to E

In the distance matrix for part (b) ignore whatever is written in the lead diagonal (top left to bottom right) and just consider the values (ignore shading etc.) – check bottom of page 9 for replaced matrices

b1M1: No change in the third row and third column with at least two values reduced correctly (no blank entries – apart from the lead diagonal)

b1A1: cao for the third iteration

b2M1: No change in their fourth row and fourth column (following their third iteration) with at least two values correctly reduced follow through from their previous iteration (no blank entries – apart from the lead diagonal)

b2A1: cso for both iterations (no follow through from an incorrect third iteration even if the fourth iteration is 'correct' so M1A0M1A1 is not possible in this part)

c1M1: Either cycle correct (must include returning to E) – nodes must be in the correct order and therefore not reversed but allow if stated in terms of arcs. If only one cycle correct then M1 only

c1A1: Both cycles correct – nodes must be in the correct order but allow if stated in terms of arcs

c2A1: Both cycles and corresponding lengths correct

c3A1: Correct reasoning that the smallest (oe) value is the better upper bound – dependent on all previous marks in this part (cso)

Question	Scheme	Marks	AOs
4(a)	Constraints: $2x+3y+4z \,„\, 13$ $x-2y+2z \ldots 8$ $3x-4z \ldots 12$ $(x, y, z \ldots 0)$	B1 B1	3.4 2.5
	Objective functions: Maximise $-2x+3y+z$ Minimise $2x-3y-z$	M1 A1	3.1a 2.2a
		(4)	
(b)	(Because M is big) the only negative in the objective row is the $2-4M$ so the pivot is from the x-column	B1	2.4
	The 3 in the a_2 row is the pivot as $\frac{12}{3}$ is less than both $\frac{8}{1}$ and $\frac{13}{2}$	B1	2.2a
		(2)	
			(6 marks)

Notes for Question 4

a1B1: One correct non-trivial inequality (allow strict inequality provided direction of inequality sign is correct) – equations with slack variables etc. scores no marks unless replaced with correct inequalities

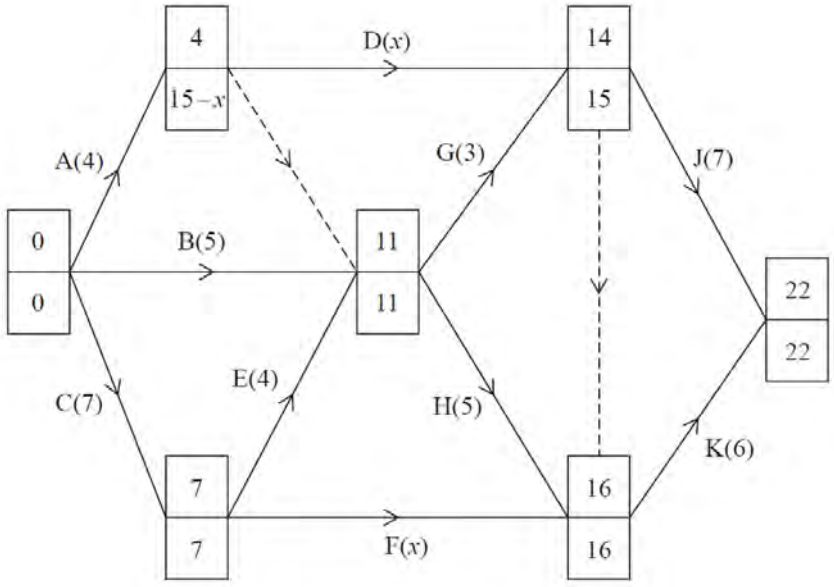
a2B1: All three non-trivial inequalities correct

a1M1: Either expression stated correctly (allow equal to (or an inequality with) any letter e.g. $P=-2x+3y+z$ but not equal to a value e.g. = 0) – ignore any mention of maximum/minimum for this mark

a1A1: Both expressions correct including max/min correctly matched with each expression (allow equal to any letter only) – do not isw if they continue and place their expression(s) equal to a value(s)

b1B1: Correct reasoning that the pivot is a value from the x-column – as a minimum must state that the $2-4M$ is the only negative (condone most negative) in the objective row (allow profit row or P row, condone 'bottom row')

b2B1: Correct justification of why the 3 in the a_2 row or the 3 in the x column is the pivot – so **must** state the correct pivot in a clear unambiguous way (so just saying the pivot is 'the 3' is B0) **and** comparing or stating that $\frac{12}{3}$ or 4 is less than/least positive for both $\frac{8}{1}$ or 8 **and** $\frac{13}{2}$ or 6.5 – **must** see all three values so do check the table for possibly stating the θ values there. However, just stating that the 3 is the pivot because it is the smallest θ value (without seeing anywhere these θ values) is B0

Question	Scheme	Marks	AOs
5(a)	<table><tr><th>Activity</th><th>IPA</th></tr><tr><td>A</td><td>-</td></tr><tr><td>B</td><td>-</td></tr><tr><td>C</td><td>-</td></tr><tr><td>D</td><td>A</td></tr><tr><td>E</td><td>C</td></tr></table> <table><tr><th>Activity</th><th>IPA</th></tr><tr><td>F</td><td>C</td></tr><tr><td>G</td><td>A, B, E</td></tr><tr><td>H</td><td>A, B, E</td></tr><tr><td>J</td><td>D, G</td></tr><tr><td>K</td><td>D, F, G, H</td></tr></table>	**B1** **B1**	1.1b 1.1b
		(2)	
(b)	(m =) 9	**B1**	2.2a
		(1)	
(c)	**(i)**  **(ii)** C, E, H and K must be critical	**M1** **A1** **B1**	1.1b 1.1b 2.2a
		(3)	
(d)	Total float for activity G is 15 – 11 – 3 = 1	**B1**	2.2a
		(1)	

(e)

Activity	Workers
A	3
B	2
C	1
D	2
E	2

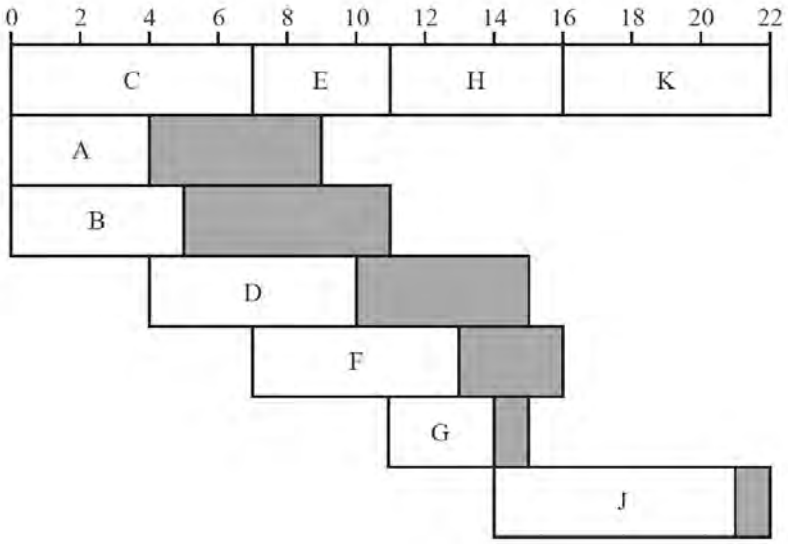
Activity	Workers
F	1
G	2
H	2
J	1
K	3

B1 1.1b

B1 1.1b

(2)

(f) $x = 6$



B1 3.1b

M1 2.1

A1 1.1b

A1 1.1b

A1 1.1b

(5)

(14 marks)

Notes for Question 5

a1B1: Four correct rows (not including the rows for A, B and C)
a2B1: All rows correct (accept blanks or dashes (etc.) for A, B and C but any letters placed in these three rows scores B0)

b1B1: cao for the value of m – allow $4 < x \,,,\, 9$ or $x \,,,\, 9$ but not $x = 9$ or $m \,,,\, 9$ unless stating the correct value too

ci1M1: Any four of the bottom boxes completed correctly
ci1A1: cao – all bottom boxes completed correctly. The $15 - x$ **must** be seen to award this mark (but may be crossed out). Condone this correct expression being replaced with their value of $15 - x$ ONLY if their value of x is **explicitly** stated either here or later in their solution

cii1B1: cao (C, E, H and K only)

d1B1: cao – a correct answer with no working (or no incorrect working) can imply this mark

e1B1: Any six values correct
e2B1: cao

f1B1: cao for the value of x (seen or implied, e.g., duration of D **and** F both correct so therefore must be consistent)
f1M1: Cascade chart with at least 8 activities labelled and at least four activities having non-zero float. A scheduling diagram (so a diagram in which no floats are evident) scores M0
f1A1: Critical activities (C, E, H and K) correct
f2A1: Activities B, G and J correct
f3A1: Activities A, D and F correct

For **(f)** the following may be useful in checking their cascade chart provided the float is shown after the corresponding activity:

Activity	Duration + Float
A	0 to 4 F: 4 to 9
B	0 to 5 F: 5 to 11
C	0 to 7 Critical
D	4 to 10 F: 10 to 15

Activity	Duration + Float
E	7 to 11 Critical
F	7 to 13 F: 13 to 16
G	11 to 14 F: 14 to 15
H	11 to 16 Critical

Activity	Duration + Float
J	14 to 21 F: 21 to 22
K	16 to 22 Critical

Qu	Scheme	Marks	AOs
6(a)	<table><tr><th>n</th><th>a</th><th>b</th><th>c</th></tr><tr><td>5</td><td>1</td><td>3</td><td>3</td></tr><tr><td></td><td>2</td><td>2</td><td>8</td></tr><tr><td></td><td>3</td><td>1</td><td>13</td></tr><tr><td></td><td>4</td><td>0</td><td>16</td></tr></table> Output: 16	**M1** **A1** **A1**	1.1b 1.1b 2.2a
		(3)	
(b) **(i)**	17 21 24 14 23 13 15 19 28 20 17 21 14 23 13 15 19 24 20 28 17 14 21 13 15 19 23 20 24 28 14 17 13 15 19 21 20 23 24 28 14 13 15 17 19 20 21 23 24 28 13 14 15 17 19 20 21 23 24 28 13 14 15 17 19 20 21 23 24 28	**M1** **A1** **A1ft** **A1**	1.1b 1.1b 1.1b 1.1b
(b) **(ii)**	Total number of comparisons: $9 + 8 + 7 + 6 + 5 + 4 = 39$	**B1**	2.2a
		(5)	
(c)	45	**B1**	2.2a
		(1)	
(d)	e.g. when $n = 3$ the total number of comparisons is 3 therefore $\lambda(3)(3-1)(3+1)(3-2) = 3$ e.g. when $n = 5$ the total number of comparisons is 45 (from **(c)**) therefore $\lambda(5)(5-1)(5+1)(5-2) = 45$	**M1**	3.1a
	For K_{50} total number of comparisons is $'\lambda'(50)(50-1)(50+1)(50-2)$	**dM1**	3.4
	749 700	**A1**	2.2a
		(3)	
			(12 marks)

Notes for Question 6

a1M1: At least three rows of cells completed with a correct first row – condone repeated values in all columns or a single value in each row

a1A1: cao – the values in the second and third rows correct

a2A1: cao – correct output following a correct fourth row (with no extra rows) – the output **must** either be stated on the given answer line or '<u>output</u> 16' must be clearly written somewhere near the table (do not bod the 16 circled, underlined, written twice, etc.)

bi1M1: Bubble sort. Consistent direction, end number (28) in place, the list containing ten numbers with the list beginning with the correct first five numbers (17 21 14 23 13). Do check these carefully as some candidates show the result of each comparison and swap in their first pass. Consider the placement of the candidate's numbers, rather than what the candidate labels each line of their pass. For example, assume that the first time that the 28 appears at the end of the list is the end of their first pass

bi1A1: Second and third passes correct – so end three numbers in place

bi2A1ft: Fourth and fifth passes correct following through from the candidate's third pass – so end five numbers in place. After their third pass their list must contain the correct 10 numbers

bi3A1: cso (correct solution only – so previous three marks must have been awarded in this part). Must show a 6^{th} pass showing no swaps/changes (give bod if the passes are not labelled but do not award this mark if it is clear that after the 5^{th} pass the list is simply being written out again (rather than a genuine 6^{th} pass taking place)). Condone if the sort continues until a 9^{th} pass has been completed (but there must be no changes in the 6^{th} to 9^{th} passes)

bii1B1: cao (for total number of comparisons)

In **(b)** starting at the right-hand end of the list is M0. Quick sort (or any other sorting algorithm e.g. shuttle) is M0. No misreads in this part – mark exactly to the scheme.

If sorting into descending order, then award M1 for 21 24 17 23 14 15 19 28 20 13 and the first A1 for **both** 24 21 23 17 15 19 28 20 14 13 **and** 24 23 21 17 19 28 20 15 14 13 **ONLY** (so two out of the first four marks) even if the list is re-ordered after the sort is complete

c1B1: cao (45)

d1M1: Considering the total number of comparisons for any positive integer value of n (greater than 2) and substitute into the given expression (**if** correct $\lambda = \frac{1}{8}$). The correct value of λ implies this mark. If using $n = 5$ then follow through their value from **(c)**. If any other value of n used, then the maximum number of comparisons must be correct e.g.

n	3	4	5	6	7	8	9	10
Comparisons	3	15	45	105	210	378	630	990

d2dM1: Using their λ and $n = 50$ to calculate the maximum total number of comparisons (dependent on the previous M mark). Writing $\frac{1}{8}(50)(50-1)(50+1)(50-2)$ implies the first two M marks

d1A1: cao (749 700) – no marks for the correct answer with **no** working

Alternative solution to (d) for those who are clearly using an algebraic method to derive the quartic expression for the maximum total number of comparisons:

If there are N values then
1st pass of bubble sort requires $N - 1$ comparisons
2nd pass of bubble sort requires $N - 2$ comparisons
3rd pass of bubble sort requires $N - 3$ comparisons and so on

Therefore, the total number of comparisons is $\sum_{r=1}^{N-1} r = \frac{1}{2}(N-1)N$ M1

Maximum number of comparisons in K_n is therefore

$$\frac{1}{2}\left(\frac{1}{2}n(n-1)-1\right)\left(\frac{1}{2}n(n-1)\right)$$

$$= \frac{1}{8}n(n-1)\left[n(n-1)-2\right]$$ dM1

$$= \frac{1}{8}n(n-1)\left(n^2-n-2\right) = \frac{1}{8}n(n-1)(n-2)(n+1) \text{ so } k = \frac{1}{8}$$

Therefore, maximum number of comparisons for K_{50} is 749 700 A1

M1: Considering the total number of comparisons that are required to sort a list containing N values (possibly will see $1 + 2 + 3 + \ldots + (n - 1)$) and using the standard series result that $\sum r = \frac{1}{2}n(n+1)$ to get the correct quadratic expression for the total number of comparisons when sorting N values (allow any letter)

dM1: Dependent on previous M mark – substituting the correct quadratic expression into the correct quadratic expression with correct algebraic working leading to $\frac{1}{8}n(n-1)(n-2)(n+1)$

A1: Correct answer of 749 700

<table>
<tr><th>Qu</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>7
(a)</td><td>$x+y$ „ $8 \Rightarrow x+y+s_1=8$
$x+2y$ „ $12 \Rightarrow x+2y+s_2=12$
$7x+2y$ „ $46 \Rightarrow 7x+2y+s_3=46$
y „ $2x+1 \Rightarrow -2x+y+s_4=1$
$P=x+ky \Rightarrow P-x-ky=0$</td><td>M1
A1

B1</td><td>3.4
1.1b

1.1b</td></tr>
<tr><td></td><td>e.g.<table><tr><td>b.v.</td><td>x</td><td>y</td><td>s_1</td><td>s_2</td><td>s_3</td><td>s_4</td><td>Value</td></tr><tr><td>s_1</td><td>1</td><td>1</td><td>1</td><td>0</td><td>0</td><td>0</td><td>8</td></tr><tr><td>s_2</td><td>1</td><td>2</td><td>0</td><td>1</td><td>0</td><td>0</td><td>12</td></tr><tr><td>s_3</td><td>7</td><td>2</td><td>0</td><td>0</td><td>1</td><td>0</td><td>46</td></tr><tr><td>s_4</td><td>–2</td><td>1</td><td>0</td><td>0</td><td>0</td><td>1</td><td>1</td></tr><tr><td>P</td><td>–1</td><td>–k</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr></table></td><td>M1

A1</td><td>3.3

2.2a</td></tr>
<tr><td></td><td></td><td>(5)</td><td></td></tr>
<tr><td>(b)</td><td>$x=2, y=5, s_1=1, s_2=0, s_3=22, s_4=0$ $(P=5k+2)$</td><td>B1</td><td>3.4</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td>(c)</td><td><table><tr><td>b.v.</td><td>x</td><td>y</td><td>s_1</td><td>s_2</td><td>s_3</td><td>s_4</td><td>Value</td><td>Row Ops</td></tr><tr><td>s_4</td><td>0</td><td>0</td><td>5</td><td>–3</td><td>0</td><td>1</td><td>5</td><td>5r1</td></tr><tr><td>x</td><td>1</td><td>0</td><td>2</td><td>–1</td><td>0</td><td>0</td><td>4</td><td>r2 + 0.4R1</td></tr><tr><td>s_3</td><td>0</td><td>0</td><td>–12</td><td>5</td><td>1</td><td>0</td><td>10</td><td>r3 – 2.4R1</td></tr><tr><td>y</td><td>0</td><td>1</td><td>–1</td><td>1</td><td>0</td><td>0</td><td>4</td><td>r4 – 0.2R1</td></tr><tr><td>P</td><td>0</td><td>0</td><td>$2-k$</td><td>$k-1$</td><td>0</td><td>0</td><td>$4k+4$</td><td>r5 $-(0.2k-0.4)$R1</td></tr></table></td><td>B1

M1

A1

B1</td><td>1.1b

2.1

1.1b

1.1b

2.4</td></tr>
<tr><td></td><td>Optimal value of P is $4k+4$ (at $x=y=4$)</td><td>M1</td><td>3.4</td></tr>
<tr><td></td><td>Second iteration not optimal $\Rightarrow -\frac{2}{5}+\frac{1}{5}k<0 \therefore k<2$</td><td>B1</td><td>3.1a</td></tr>
<tr><td></td><td>Third iteration optimal $\Rightarrow 2-k \ldots 0$ and $k-1 \ldots 0$ $(\therefore k \ldots 1)$</td><td>dM1</td><td>3.4</td></tr>
<tr><td></td><td>1„ $k<2 \Rightarrow 8$„ $P<12$</td><td>A1</td><td>2.2a</td></tr>
<tr><td></td><td></td><td>(9)</td><td></td></tr>
<tr><td colspan="4">(15 marks)</td></tr>
</table>

Notes for Question 7

a1M1: Correctly re-writing any two inequalities as equations with slack variables (can be implied by two correct rows in the Simplex tableau ignoring b.v. column). Or correctly stating all four constraints as inequalities

a1A1: Correctly re-writing all inequalities as equations with slack variables (can be implied by the four correct constraint rows in the Simplex tableau ignoring b.v. column)

a1B1: Correctly re-writes objective function (can be implied by correct row in tableau)

a2M1: Any two rows correct including consistent b.v. column entries **or** any three rows correct (ignoring b.v. column)

a2A1: cao (including consistent b.v. column) – **note that the candidate's order in which the rows appear in the tableau (and choice of letter to represent the slack variable) may be different. A correct tableau implies full marks in this part**

b1B1: cao for x, y, s_1, s_2, s_3 and s_4 only (ignore any mention of P)

c1B1: Pivot row completely correct including change of b.v.

c1M1: All **values** in one of the non-pivot rows correct (so ignore b.v. column and 'Row Ops' column) **or** one of the 'non zero and one' columns (which are s_1, s_2 or Value) correct (must have pivoted on the correct value)

c1A1: Row operations used correctly at least twice, i.e. two of the 'non zero and one' columns (s_1, s_2 or Value) correct

c2A1: cao **all** values including b.v. column – ignore 'Row Ops' column for this mark

c2B1: Correct row operations stated

(alternatives row operations are $5r1$; $r2 + 2r1$; $r3 - 12r1$; $r4 - r1$; $r5 - (k - 2)r1$)

c2M1: Their optimal value (as a linear expression in k) stated correctly following their third iteration (must have pivoted on a positive value from the s_4 column and completed the bottom row of the tableau). Condone this expression ($4k + 4$ **if** correct) being stated as part of an equation/inequality (or as part of their final answer) – sight of this expression (but must be seen **outside** of the tableau) scores this mark

c3B1: Correctly inferring that $k < 2$ either from $-\frac{2}{5} + \frac{1}{5}k < 0$ or from $4k + 4 > 5k + 2$ – just stating $k < 2$ without it being clear where this comes from is B0

c3dM1: Considering (at least) two of their linear expressions in k from their objective row (not including the Value column) after the third iteration …0 **(dependent on the previous M mark)** – note that working may be minimal here so please follow through their expressions in k from their objective row (so **if** correct, stating k „ 2 and k …1 implies this mark)

c3A1: cao for the range of values for P - this mark is dependent on a correct objective row in the tableau and the previous three marks in this part